Magnetoresistance Fluctuations in Mesoscopic Wires and Rings

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A simulation of the magnetoresistance of very small wires shows sample-specific, aperiodic structure due to the non-self-averaging nature of quasi-one-dimensional conductors in this size range. This explains earlier failures to observe flux-periodic magnetoresistance in very small rings. Analysis of the temperature and size dependence of the effect gives good agreement with experiment. A prediction is made for the experimental conditions in which oscillations periodic with flux hc/e will be observed.

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Physicists are accustomed to dealing with two classes of quantum-mechanical systems: (1) microscopic systems in which the energy levels are discrete and their spacing is typically much greater than kT (at low temperatures); (2) macroscopic systems in which the level spacing is always much less than kT. However, recently it has become possible to study experimentally solid-state systems in the intermediate "mesoscopic" size regime. In these systems the energy-level spacing is only a few orders of magnitude smaller than kT at low temperatures. I argue that quite generally one should see novel quantum-mechanical fluctuation phenomena in the transport coefficients of such systems. These *time-independent* sample-specific fluctuations are a direct consequence of the differing microscopic configurations of two mesoscopic systems with the same average physical properties. The fluctuations will be large if the states of the mesoscopic system are localized, as in a semiconductor, and small if the states are extended, as in a metal. Large fluctuations of this type have already been observed experimentally in one-dimensional semiconductor devices, where they appear as reproducible, noiselike structures in the conductance as a function of electron density.[1] Similar but much weaker structure has been seen very recently in the magnetoresistance of small metal wires and rings.[2,3]

In this paper I present a theory of the origin of this structure based on a detailed numerical simulation that agrees well with these experiments; the experiments should therefore be seen as the first observation of mesoscopic fluctuations in a metallic system. I also predict the experimental conditions necessary for observation of *periodic* magnetoresistance oscillations in normal-metal rings, despite the presence of these fluctuations. It must also be emphasized here that while there has been a great deal of theoretical work on resistance fluctuations in strongly localized, purely one-dimensional systems, surprisingly, there has been no theoretical work at all on fluctuations in weakly disordered wires where the transport is diffusive.

The experimental impetus for studying very small rings and wires was the observation by Sharvin and Sharvin of very weak periodic oscillations in the magnetoresistance of normal-metal cylinders,[4] with the periodicity of the superconducting flux quantum $hc/2e$. Working on much smaller systems (rings of 3000-Å diameter and 500-Å width) Webb, Washburn, Umbach, and Laibowitz[2] and Blonder[3] observed sample-specific, aperiodic, reproducible structure in the magnetoresistance which was two orders of magnitude larger than the Sharvin and Sharvin effect and which persisted without decrease in amplitude out to 8 T. The same structure was also seen in wires (Fig. 1), showing that, unlike either the Sharvin and Sharvin effect or the usual Bohm-Aharonov effect (which has flux period hc/e), it had nothing to do with the system's topology. The structure grew slowly with decreasing temperature roughly as $T^{-1/2}$, and had an amplitude of (0.01–0.1)% compared to the background resistance. An asymmetry in the structure was observed when the magnetic field direction was reversed.

In order to understand the experiments, I study a model first treated by Lee and Fisher[5,6] which consists of a nearest-neighbor tight-binding model on an infinite two-dimensional strip containing a finite disordered region of N sites in length and M sites in width. The Hamiltonian in the site representation is