Abstract

Single Electron Tunneling Through Discrete Semiconductor Impurity States

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1998

With the advent of epitaxial crystal growth techniques such as molecular beam epitaxy (MBE) and metal-organic chemical vapor deposition (MOCVD), and the development of microfabrication and nanoscale lithography techniques (like electron-beam lithography), the design and implementation of many new experimental systems has become possible. The realization of granular electronic systems, which exhibit single electron effects, such as low dimensional semiconductor and ultra small metallic systems, has focused attention on the basic physical properties of the discrete electronic states. In this dissertation I investigate one such system consisting of individual semiconductor impurities with discrete electronic states. These impurities are located in the quantum well regions of a semiconductor heterostructure resonant tunneling diode and their eigenstates are probed by measuring the electron tunneling current through these states. Quantum effects due to the narrow well width of these diodes change the energy and other physical properties of all the electronic states (including the impurity states) in this region. I focus on one particular physical property, the effective spin $g*$ factor, of electrons in this region and accurately determine it by studying the spin splitting of the impurity states in an applied magnetic field. An analysis of the electron tunneling current through this two state system enables the determination of the tunneling rates of the two potential barriers of the resonant tunneling diode individually. I also investigate a time dependent phenomenon observed in this tunneling system. A two level fluctuating current (random telegraph signal) associated with individual single electron tunneling channels is observed. Possible mechanisms for these fluctuations are discussed. Once the physical properties of the impurity are understood, it can be used as a probe to investigate the local properties of the semiconductor contacts. The impurity states serve as narrow, localized, spectroscopic probes and give information about the local density of states fluctuations in the contacts.
Single Electron Tunneling Through Discrete Semiconductor Impurity States

A Dissertation
Presented to the Faculty of the Graduate School of Yale University
in Candidacy for the Degree of Doctor of Philosophy

by
Mandar Ramesh Deshpande
May 1998

Advisor: Professor Mark A. Reed
Acknowledgments

Dedicated to my parents

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Contents

List of Tables ii
List of Figures iii
List of Abbreviations and Symbols xi

1 Introduction 1
1.1 Motivation ................................................................. 1
1.2 Outline and overview of the thesis ........................................ 4
1.3 Historical development of the impurity system ........................... 5

2 Basic theory of resonant tunneling diodes, quantum dots and impurity systems 9
2.1 Introduction ................................................................. 9
2.2 Quantum Wells - one dimensional quantum confinement ............... 12
  2.2.1 Quantum size effects ................................................. 12
  2.2.2 Resonant Tunneling Diode I(V) Characteristics ..................... 13
2.3 Quantum Dots - three dimensional quantum confinement ............. 19
  2.3.1 Energy states of a quantum dot .................................... 19
  2.3.2 I(V) characteristics of a quantum dot ............................... 23
2.4 Impurity system - Coulomb potential confinement ..................... 25
  2.4.1 Energy states of a single impurity ................................ 26
  2.4.2 High binding energy states: impurity pairs ....................... 32
  2.4.3 Coulomb charging energy in the impurity system .................. 37
  2.4.4 I(V) characteristics of the impurity system ..................... 38
2.5 Summary ........................................................................ 43

3 Experimental methods 44
3.1 Electrical measurement set up ............................................ 44
  3.1.1 dc I(V) characterization .............................................. 44
  3.1.2 Characterization of time varying signal ............................... 48
3.2 Measurements at Low Temperatures ..................................... 50
  3.2.1 Cryostat Systems ..................................................... 50
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Details about the heterostructure of the various samples under investigation.</td>
</tr>
<tr>
<td>4.2</td>
<td>Comparison of theoretically predicted current step height by the bandprofile model and the experimentally measured average of the various observed current step heights for the three different device types.</td>
</tr>
<tr>
<td>4.3</td>
<td>Observed number of localized states in different devices.</td>
</tr>
<tr>
<td>4.4</td>
<td>Expected number of impurity pairs in different devices.</td>
</tr>
<tr>
<td>4.5</td>
<td>Observed number of current steps in different devices up to a bias 30 mV lower than the turn on voltage which correspond to binding energies higher than 15 meV.</td>
</tr>
<tr>
<td>4.6</td>
<td>Observed number of current steps in different devices up to a bias 60 mV lower than the turn on voltage which correspond to binding energies higher than 30 meV.</td>
</tr>
<tr>
<td>5.1</td>
<td>$\alpha$ and $g^*$ values as calculated for the various devices and current steps.</td>
</tr>
<tr>
<td>6.1</td>
<td>Switching times for different closely spaced bias locations corresponding to the second dc current step in forward bias.</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Comparison of quantum effects in nanostructures. Adapted from reference [47]. ................................................. 2

2.1 Plot of bandgap vs lattice constant for various semiconductors. The solid lines connecting various compounds indicate direct bandgaps, while the dashed lines indicate indirect bandgaps. .................................................. 11

2.2 Schematic illustration of quantum size effects in a *quasi-two dimensional* region. (a) The conduction band showing only discrete allowable energies in the confinement direction, but it still has a continuous density of states in the transverse direction. (b) The two-dimensional density of states, $D_{2d}$, showing discrete step increase at energies corresponding to each discrete energy level in the confinement direction. Also shown is the three-dimensional density of states times the length, $D_{3d}L$, calculated for a layer with a thickness equal to that of the quantum well. .................................................. 13

2.3 Schematic of a resonant tunneling diode (RTD). ............................ 14

2.4 (a) Model of the conduction band of a resonant tunneling diode (RTD), (b) Transmission coefficient for the structure shown in (a). ............................... 15

2.5 Band plots of a resonant tunneling structure under different applied bias. By convention, the region to the left of the tunneling structure is referred to as the emitter. The dotted line in the emitter represents the quasi-Fermi level for the emitter. (a) No applied bias. (b) Biased to the peak resonant current (1 V), (c) Biased past resonance (1.5 V). The I(V) characteristics are shown in figure 2.6 ................................. 16

2.6 Expected I(V) characteristics for the structure shown in figure 2.5. $V_{peak}$ corresponds to (b) and $V_{valley}$ corresponds to (c). The dotted line connecting the two points corresponds to the negative differential resistance region when the device is oscillating. The gray dotted line corresponds to the expected I(V) characteristic of a single tunneling barrier with thickness equal to the sum of the two barriers. ................................. 17
2.7 Illustration of the operation of a resonant tunneling diode. (a) Model conduction band diagram at zero bias showing the quantum well eigenstate of energy $E_0$ measured relative to the emitter conduction band bottom. (b) Band diagram under an applied bias $V$. The quantum eigenstate is pulled down in energy by an amount $\alpha V$. (c) Spherical Fermi surface for a degenerately doped emitter. Conservation of lateral momentum during tunneling allows only those emitter electrons whose momenta lie on a disk of constant $k_z$ as shown by the shaded disk. Current flows through the device when the disk lies in the region between the north pole and the equator with the peak being at the equator.

2.8 Schematic of a quantum dot device.

2.9 Schematic of an impurity system showing an impurity state in the quantum well.

2.10 Calculated dependencies of the on-center and on-edge hydrogenic donor binding energies versus the well thickness $L$ in a quantum well with an infinite barrier height ($V_0 = \infty$). $R_b$ and $a_b$ are the bulk effective Rydberg and Bohr radius respectively. (Adapted from G. Bastard [1]).

2.11 Calculated dependence of the on-edge hydrogenic donor binding energy in quantum wells versus the GaAs well width $L$ for different barrier heights $V_0 = 212$ meV, 318 meV, 424 meV and $\infty$. (Adapted from C. Priester[42]).

2.12 Calculated dependence of the hydrogenic donor binding energy in a quantum well of width $L = 94.8$ Å, versus the impurity position $z_i$ for (a) infinite barrier case ($V_0 = \infty$) and for a finite barrier case ($V_0 = 318$ meV). There is an interface at $z_i = L/2$. (Adapted from S. Chaudhuri[5]).

2.13 Dependence of the parameter $\beta$ in the variational wavefunction of the $H^+_2$ problem upon the positive charge separation $R$. $R$ is scaled in the units of Bohr radius.

2.14 Electronic energy of a $H^+_2$ ion system in bulk GaAs as a function of the impurity separation $R$. $R$ is scaled in the units of the Bohr radius ($a_b$) and the energy in the units of Rydberg ($R_b$), the radius and the energy of the lowest state of the single impurity ($H^+$) problem which in GaAs semiconductor bulk are 101 Å and 5.5 meV respectively. The top and the right scale show the radius and the energy in real units respectively for bulk GaAs.

2.15 Expected lateral radius of the $H^+_2$ ion system as a function of the impurity separation. The lengths are in the units of the bulk single impurity radius ($a_b = 101$ Å).

2.16 Coulomb repulsion energy of two singly charged impurities in a 44 Å quantum well, electronic energy of an electron in the potential of these two impurities and the total energy of the impurities-electron system as a function of the impurity separation, $R$. 

20

21

25

30

31

32

34

35

36

37
2.17 (top): Schematics of the I(V) characteristics, at zero temperature, through a single impurity as predicted by equation 2.41. An impurity channel turns on at a bias when the impurity state crosses the Fermi level in the emitter and the current gradually decreases to zero as the impurity level crosses the bottom of the conduction band in the emitter. These bias locations are determined by the voltage to energy conversion factor ($\alpha$) and the energy of the impurity level at zero bias as shown. (bottom): The I(V) characteristics of a device having a random distribution of impurities. The energy and the current through a given impurity channel is a function of the impurity location.

3.1 (a) Preliminary I(V) measurement setup using the SPA only. (b) The use of a battery powered reduction box to improve the voltage resolution by a factor of 10. SPA acts as the voltage source, current meter and a data storage device in either cases.

3.2 Schematic for the low-noise voltage/current preamplifier (bias box).

3.3 Simplified schematic showing active device measurement components.

3.4 Schematic of instrumentation amplifiers used in figures 3.2 and 3.3.

3.5 Schematic of the measurement set up for RTS characterization.

4.1 Cross-sectional transmission electron microscope (TEM) image of device 2014 showing the quantum well and the barriers.

4.2 Simulated conduction band diagram of device 2014 at an applied bias of 100 mV. The dashed lines represent the Fermi levels in the contacts. The solid line in the well is the calculated energy location of the first quantum eigenstate. The small line in the well is a schematic notation for a possible impurity state in the well which is bound to the quantum eigenstate.

4.3 I(V) characteristics (zero magnetic field) at 1.4 K of the quantum well device showing the main resonance peaks (top). The magnified lower bias region shows two step-like structures attributed to single electron tunneling through two separate localized states due to impurities (bottom).

4.4 I(V) characteristics of all the devices on 2014-A with different lateral widths (as marked) at 1.4 K. Note that current steps are observed in all of them and the step magnitudes do not show any systematic dependence upon the area of the device. The step current magnitudes do not scale with the area of the device but they are of the same order of magnitude for devices with vastly different areas.

4.5 I(V) characteristics showing the observed current steps in different devices belonging to each of the three different epitaxial heterostructures under investigation.

4.6 I(V) characteristics of device 2014-B (8) in forward bias at 1.4 K showing the triangular nature as predicted by theory.
4.7 I(V) characteristics of device 2014-A (8) in forward bias at 35 mK and at 4.2 K. Observe the effect of thermal smearing of the I(V) characteristics at 4.2 K but note that the absolute magnitude of the current at 4.2 K gives an estimate of the number of current steps and hence the number of impurity states in the device. .......................... 64

4.8 I(V) characteristics of six different devices on chip 2014-A. The curves are labeled by their respective lateral size in μm and the current is scaled by their respective lateral area. .......................... 65

4.9 Probability of finding at least one pair of impurities with a given separation $R$ assuming an average concentration of one impurity per $(μm)^2$ area of the device for devices with different lateral widths. .......................... 67

4.10 I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures showing the Fermi level broadening and the Fermi fit (equation 4.3) to these I(V) traces for $V < V_{th}$ (solid lines). .......................... 71

4.11 I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures and the extrapolation of the Fermi fits (equation 4.3) to these I(V) traces as obtained in figure 4.10 to voltages greater than the threshold. The experimentally measured current can be seen to be less than the theoretically expected current on the plateau. .......................... 71

4.12 I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures showing the Fermi level broadening and the corrected fit (equation 4.4) to these I(V) traces for $V < V_{th}$ (solid lines), which takes into account the finite occupancy of the state. .......................... 73

4.13 I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures and the extrapolation of the corrected fits (equation 4.4) to these I(V) traces as obtained in figure 4.12 to voltages greater than the threshold ($V \geq V_{th}$). .......................... 74

4.14 I(V) characteristics of the first current step edge in forward bias of the device 2015-C (32) at different temperatures and the corrected fits (equation 4.4) to these I(V) traces. The fits are done over the entire bias range spanning the step. .......................... 75

4.15 Current-voltage characteristics in a dilution fridge with mixing chamber temperature of 35 mK, in magnetic field (0-9 Tesla in 0.094 Tesla increments) parallel to the current for the forward bias direction of device 2014-A (8). Traces are offset by a constant current value for clarity. .......................... 78

4.16 A plot of the bias location of the first current step in forward bias of 2014-A (8) versus the magnetic field parallel to current up to 11 T in a dilution fridge with mixing chamber temperature of 35 mK. .......................... 79

4.17 Current-voltage characteristics in magnetic field (0-9 Tesla in 0.094 Tesla steps) perpendicular to the current for device 2014-A (8) in forward bias direction. .......................... 80

4.18 Maximum current values for the first current step of device 2014-A (8) in forward bias as a function of perpendicular and parallel magnetic field. .......................... 81
4.19 Schematics of the tunneling process through a localized state in the quantum
well illustrating how the state acts as a “spectroscopic probe” to investigate
the electronic states in the emitter below the Fermi level. The 3d bulk density
of states in the emitter has a smooth, \( \sqrt{E} \) dependence on energy upon which
the fluctuations in local density of states are superimposed (solid lines). In a
magnetic field Landau bands are formed (dashed line) which are also modified
due to the local density fluctuations.

4.20 I(V) data in parallel field, ranging from 6 Tesla (bottom) to 9 Tesla (top).
Curves are vertically offset by a constant current value for clarity.

4.21 Fan diagram showing peak voltage location versus magnetic field parallel to
the current for the forward bias direction.

4.22 Fan diagram showing peak voltage location versus magnetic field perpendicular
to the current for the forward bias direction.

5.1 Schematic of the band structure near \( k = 0 \) showing the conduction band
and the valence bands.

5.2 Theoretical predictions of the behavior of the effective spin \( g^* \) factor for an
Al\(_{0.35}\)Ga\(_{0.65}\)As / GaAs / Al\(_{0.35}\)Ga\(_{0.65}\)As quantum well as a function of the
well width (\( L \)) as discussed by Ivchenko and Kiselev[30]. The dotted line is
the result of a one band calculation which ignores the asymmetry.

5.3 I(V) characteristics of the first current step edge of the 85 Å barrier device
(\( T_{\text{mix}} = 35 \) mK) in reverse bias at 0T and at 11 T under both magnetic field
orientations (field parallel to current and field perpendicular to current). The
three curves are given some arbitrary offset along the x-axis for clarity.

5.4 I(V) characteristics of the 85 Å barrier device at \( T_{\text{mix}} = 35 \) mK showing
six different current steps in zero field (dashed line) and with a field of 9 T
oriented perpendicular to current (solid line).

5.5 I(V) characteristics (\( T_{\text{mix}} = 35 \) mK) of the first current step edge in forward
bias of the 85 Å barrier device in increasing magnetic fields, (field perpendicular
to current), from 0 T (bottom) to 9 T (top). The successive field curves
are offset a constant amount along the y-axis for clarity.

5.6 The experimental spin splitting versus magnetic field for the first current step
of the 85 Å barrier device for the different bias and magnetic field orientations
at \( T_{\text{mix}} = 35 \) mK. The solid lines are linear fits to the data.

5.7 Comparison of the measured \( g^* \) factors with the theoretical predictions of
Ivchenko and Kiselev[30] assuming that the sign of the measured quantities
is positive.

5.8 I(V) characteristics at 9 T showing the spin-split first current step edge in
forward bias at different temperatures. Both fragments of the split edge
exhibit Fermi broadening with the temperature.
5.9 I(V) characteristics at \( T_{\text{mix}} = 35 \text{ mK} \) of the first current step edge of the 85 Å barrier device in forward bias (left) and reverse bias (right) at 0 T (dashed line) and 9 T (solid line). The magnetic field is oriented perpendicular to the current direction. \( I_1 \) and \( I_2 \) mark the current values at 9 T as shown. \( I_1 \) gives the current of the first fragment while \( I_2 \) is the net current of both fragments of the split step edge.

5.10 Schematic of electron tunneling through the two state system.

5.11 Tunneling rates \( T_{\text{em}} \) and \( T_{\text{id}} \) as a function of the magnetic field strength perpendicular to current for the 85Å barrier device in forward bias orientation.

5.12 Tunneling rates \( T_{\text{em}} \) and \( T_{\text{id}} \) as a function of the magnetic field strength perpendicular to current for the 85 Å barrier device in reverse bias orientation.

5.13 Tunneling rates \( T_{\text{em}} \) and \( T_{\text{id}} \) as a function of the magnetic field strength parallel to current for the 85Å barrier device in forward bias orientation.

5.14 Tunneling rates \( T_{\text{em}} \) and \( T_{\text{id}} \) as a function of the magnetic field strength parallel to current for the 85Å barrier device in reverse bias orientation.

6.1 The dc (top) and the ac (bottom) I(V) characteristics of the device in forward bias at \( T_{\text{mix}} = 35 \text{ mK} \).

6.2 dc and ac characteristics of the reverse bias second current step. Note that the ac signal is not symmetrical about the dc step.

6.3 The time dependence of the current at fixed bias voltages in forward bias at \( T_{\text{mix}} = 35 \text{ mK} \). The top curve is at 90.00 mV corresponding to the second current step in the dc I(V) characteristics, while the bottom curve is at 85.85 mV corresponding to the first current step. The two curves are vertically offset for clarity.

6.4 The time dependence of the current at a fixed bias voltage of 93.15 mV corresponding to the second dc current step in reverse bias at \( T_{\text{mix}} = 35 \text{ mK} \).

6.5 Histogram (symbols) of the on times at a fixed bias voltage of 89.9 mV in forward bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives \( \tau_{\text{on}} = 14 \text{ mSec} \).

6.6 Histogram (symbols) of the off times at a fixed bias voltage of 89.9 mV in forward bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives \( \tau_{\text{off}} = 17 \text{ mSec} \).

6.7 Histogram (symbols) of the on times at a fixed bias voltage of 93.15 mV in reverse bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives \( \tau_{\text{on}} = 2 \text{ mSec} \).

6.8 Histogram (symbols) of the off times at a fixed bias voltage of 93.15 mV in reverse bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives \( \tau_{\text{off}} = 91 \text{ mSec} \).
6.9 Power spectrum of the RTS signal at three specific biases corresponding to the second current step in forward bias at $T_{\text{mix}} = 35$ mK. The spectra show the expected Lorentzian shape as discussed in the text with a 20 dB/decade roll off and a knee at 21 Hz. The spectrum at 85 mV is at a bias where no RTS is observed and shows the background noise in the measurement system.

6.10 The temperature dependence of the switching current amplitude at a fixed bias voltages of 90.00 mV in forward bias corresponding to the peak in the rms ac signal. The symbols are actual data points while the solid line is a fit to the data as discussed in the text.

6.11 Schematic of the “Two state model” for the observed Random Telegraph Signal. I propose the existence of two closely spaced tunneling levels and claim that the observed RTS is due to random switching between these two tunneling levels.

6.12 A plot of the expected switching current (equation 6.4) according to the two state model.

6.13 Coulomb repulsion energy of two singly charged impurities in a 44 Å quantum well, electronic energy of an electron in the potential of these two impurities and the total energy of the impurities-electron system as a function of the impurity separation, $R$. When there is no electron, the energy of the system is just the Coulomb repulsion energy of the two singly charged impurities. When there is an electron present in the state, the Coulomb repulsion gets screened and the total energy of the impurities-electron system is as shown (from figure 6.13). $R_a$ and $R_b$ mark the two possible separations between the two impurities. 1, 2, 3, 4 mark the four possible states of the impurity pair with or without electron and with separation $R_a$ or $R_b$. ($R_a - R_b$) is shown to be large in this figure for clarity.

6.15 Schematic representation of the Poisson tunneling process. We can see that the probability for the tunneling event not to occur for a time $\tau_0$ exponentially decreases as the time $\tau_0$ increases.
# List of Abbreviations and Symbols

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0d</td>
<td>Zero dimensional</td>
</tr>
<tr>
<td>1d</td>
<td>One dimensional</td>
</tr>
<tr>
<td>(100)</td>
<td>A specific crystal plane designated by its Miller indices</td>
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<tr>
<td>2d</td>
<td>Two dimensional</td>
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<tr>
<td>2DEG</td>
<td>Two dimensional electron gas</td>
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<tr>
<td>3d</td>
<td>Three dimensional</td>
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<tr>
<td>A</td>
<td>Area</td>
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<td>Å</td>
<td>Angström</td>
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<td>Voltage to Energy conversion factor</td>
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<td>Bohr radius</td>
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<td>ac</td>
<td>alternating current</td>
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<td>AlGaAs</td>
<td>Aluminum Gallium Arsenide</td>
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<tr>
<td>B</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>β</td>
<td>Slater’s variational parameter</td>
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<tr>
<td>c</td>
<td>Concentration of impurities per (μm)² area of the device</td>
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<tr>
<td>C</td>
<td>Capacitance</td>
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<td>C</td>
<td>Centigrade</td>
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<tr>
<td>ΔE</td>
<td>Change in energy</td>
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<td>ðE</td>
<td>Energy spacing between single-particle states</td>
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<td>ΔIₘᵥ</td>
<td>Average step current magnitude</td>
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<td>Voltage splitting</td>
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<td>dc</td>
<td>direct current</td>
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<td>Eₕ</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>ε₀</td>
<td>Permitivity of free space</td>
</tr>
<tr>
<td>f(ΔE)</td>
<td>Fermi function</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
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<tr>
<td>g₀</td>
<td>Spin Lande g factor of free electrons</td>
</tr>
<tr>
<td>g*</td>
<td>Effective spin g factor of electrons in a crystal</td>
</tr>
</tbody>
</table>
Effective spin $g$ factor of electrons in a quantum well with magnetic field oriented perpendicular to the growth direction of the well.

Effective spin $g$ factor of electrons in a quantum well with magnetic field oriented parallel to the growth direction of the well.

$\Gamma$ Tunneing rate

GaAs Gallium Arsenide

$\hbar$ Planck's constant

$\hbar / 2\pi$ Planck's constant divided by $2\pi$

$H_2^+$ Hydrogen molecular ion

He Helium

HP Hewlett Packard

$I$ Current

$I_{th}$ Threshold current

$J$ Current density

$k$ Wavevector

$k$ Boltzmann's constant

$K$ Kelvin

$L$ Quantum well thickness

$m^*$ Effective mass of electron

$m_0$ Mass of free electron

$\mu_B$ Bohr Magneton

MBE Molecular Beam Epitaxy

MOCVD Metalorganic Chemical Vapor Deposition

MOSFET Metal Oxide Semiconductor Field Effect Transistor

$n^+$ Doping level of a semiconductor

$N$ Number of impurity states in a device

$N(V_1, V_2)$ Number of impurity states in a device in the bias range $(V_1, V_2)$

$N(E)$ Density of states

$\Omega$ Ohms

$\Phi$ Potential energy

PARC Princeton Applied Research Corporation

qed quantum electrodynamics

QPC Quantum point contact

$R$ Separation between impurities that form a pair

$R_0$ Rydberg

$rms$ root mean square

RTD Resonant tunneling diode

RTS Random Telegraph Signal

$S$ Spin of electron

SET Single Electron Transistor

Si Silicon
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPA</td>
<td>Semiconductor Parameter Analyzer</td>
</tr>
<tr>
<td>Ψ</td>
<td>Wavefunction</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>T</td>
<td>Tesla</td>
</tr>
<tr>
<td>T_{mix}</td>
<td>Mixing chamber temperature</td>
</tr>
<tr>
<td>T(E)</td>
<td>Transmission probability</td>
</tr>
<tr>
<td>T_b</td>
<td>Tunneling rate of bottom barrier</td>
</tr>
<tr>
<td>T_{cl}</td>
<td>Collector barrier tunneling rate</td>
</tr>
<tr>
<td>T_{em}</td>
<td>Emitter barrier tunneling rate</td>
</tr>
<tr>
<td>T_{t}</td>
<td>Tunneling rate of top barrier</td>
</tr>
<tr>
<td>TEM</td>
<td>Transmission Electron Microscopy</td>
</tr>
<tr>
<td>τ</td>
<td>Lifetime of quasi-bound quantum state</td>
</tr>
<tr>
<td>τ_{av}</td>
<td>Average electron tunneling time</td>
</tr>
<tr>
<td>τ_{off}</td>
<td>Average time the random telegraph signal is low</td>
</tr>
<tr>
<td>τ_{on}</td>
<td>Average time the random telegraph signal is high</td>
</tr>
<tr>
<td>U_C</td>
<td>Single electron charging energy</td>
</tr>
<tr>
<td>V</td>
<td>Voltage</td>
</tr>
<tr>
<td>V_0</td>
<td>Potential barrier height</td>
</tr>
<tr>
<td>V_{th}</td>
<td>Threshold voltage</td>
</tr>
<tr>
<td>V_{to}</td>
<td>Turn on voltage</td>
</tr>
<tr>
<td>W</td>
<td>Lateral size of the device</td>
</tr>
<tr>
<td>x</td>
<td>Fraction of Al content in AlGaAs</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Motivation

With the advent of epitaxial crystal growth techniques such as molecular beam epitaxy (MBE) and metal-organic chemical vapor deposition (MOCVD), and the development of nanoscale patterning techniques such as electron beam lithography, the design and implementation of nanometer scale systems that exhibit new and interesting classical and quantum mechanical effects has become possible. The motivation for creating such structures has been two-fold. Firstly, to create nano-scale laboratories to explore physics in a new regime, and secondly to develop a new technology with important applications in the vast field of scaled devices.

The first motivation has and continues to guide a very broad and active research field. Discoveries such as the integer\cite{34} and the fractional quantum hall effects\cite{71} are cases where new and unexpected experimental discoveries led to major theoretical work. The observation of Bloch oscillations in a superlattice structure\cite{73} is an example where theoretically expected physics has finally seen experimental realization. The second motivation has also yielded important technological outputs\cite{75}. Semiconductor quantum well lasers and the recent introduction of blue light emitting diodes are examples of these. One research field that generates major physics and technological interests has been the field of single electron transport systems. We refer to U. Meirav and E. B. Foxman for a review of single electron phenomena in semiconductors\cite{39}. As the system size reduces new clas-
Figure 1.1: Comparison of quantum effects in nanostructures. Adapted from reference [47].

Classical (charge quantization) and quantum mechanical (energy quantization) effects become important. Charge is not a continuum but is quantized in the units of single electronic charge. This leads to the observation of new phenomena, like Coulomb blockade[21], in the electronic transport properties of mesoscopic systems where the classical energy necessary to charge the system with the lowest possible charge becomes significant. Similarly as the system size decreases, size quantization leads to discrete electronic energy levels which exhibit themselves in devices like quantum dots[44]. From a technological point of view this field of single electron devices has gained significance as a possible alternative technology for smaller and faster electronic devices. The present day semiconductor technology and the scaling of the transistor is some day going to reach its physical limits and search is on for an alternate technology to go beyond that. A variety of experimental systems have been explored in this respect and it is important to understand the physics of these systems to come up with a viable new technology.
The operational voltage and temperature range of any new electron device technology is a very important factor in determining its feasibility. Figure 1.1 summarizes the temperature and voltage operation ranges for various nanostructure systems. The horizontal dashed white line marks 300K (room temperature operation), and the vertical dashed white line ($\Delta E_q/e$) shows the voltage operation limits imposed by the separation of quantized states (either through confinement or through charging). The majority of these structures (waveguides, quantum interference devices, quantum point contacts (QPC), metal single electron transistors (SET), and depletion-defined lateral structures) are restricted to operation at cryogenic temperatures (less than 100K; less than 1K in some cases). Only epitaxial heterojunction tunneling devices and molecular electronic devices have potential of achieving room temperature operation. The field of molecular devices is as yet quite new and basic research in a variety of fields such as chemistry, material science and engineering will have to be undergone before the challenges can be met to demonstrate a molecular single electron transistor (SET). The field of epitaxial heterojunction tunneling devices, on the other hand, has been active for quite some years now since the first introduction of resonant tunneling devices[4]. These devices have found room temperature applications in niche areas like high frequency oscillators and for possible use in multivalued logic systems. These however are not single electron devices as they contain many, degenerate electronic states which are occupied by many electrons at a given time. The quantum confinement in these devices is along one direction only and the freedom along the other two directions results in the above mentioned degeneracy. Demonstration of a three dimensionally confined device exhibiting single electron effects (charging and/or energy quantization) at room temperature is still not possible as the energy separation between the discrete electronic states is still lower than what is required for room temperature operation. Nevertheless, much research has undergone since the first demonstration of discrete electronic states in a three dimensionally confined semiconductor nanostructure system, called as quantum dots, by Reed and coworkers[44]. This research has now focused attention on the basic physical properties of the discrete electronic states in such systems. The basic tunneling process involved in all
these systems, that of a single electron tunneling into and out of a single discrete state is under investigation. The effects of a finite occupancy of an electron on the discrete state are being explored. The influence of confinement on the basic properties of the discrete state, like its spin $g$ factor, are being studied. In order to focus directly onto the discrete state it is necessary to have a clean system with no additional unknown complications in the parts of the system other than the discrete state. Unfortunately for the conventional system of quantum dots[44], the fabrication process employed to create a discrete state also imposes some additional unknown confinement potential in the emitter and collector contact electrodes which complicates the analysis.

This thesis dissertation focuses on a new, clean and simple system which is physically similar to a quantum dot system but does not have the above mentioned problem and hence allows one to concentrate on the properties of the discrete state directly. This system consists of discrete semiconductor impurities in the quantum well regions of a resonant tunneling diode. The coulomb potential of these impurities results in the formation of three dimensionally confined, discrete hydrogenic energy states bound to the quantum well eigenstate. Since the lateral size of these devices is large, there is no lateral potential in the emitter and the collector contacts.

1.2 Outline and overview of the thesis

In chapter 2 I will discuss the basic theory of resonant tunneling diodes, (RTDs), and of quantum dots and analyze the current voltage, $(I(V))$, characteristics of these devices. Then I will introduce the impurity system which is physically similar to the quantum dot system. I will discuss the binding energy of an electron in the potential of a single or a pair of impurities in the quantum well. The details of the theory concerning the impurity system are discussed in the relevant chapters that follow. Chapter 3 outlines the experimental methods used in this study. In chapter 4 I will presents the experimentally measured $I(V)$ characteristics in the various devices and compare it with our model. Sharp steps in the pre-threshold region of the $I(V)$ characteristics are observed which are attributed to single electron tunneling
through localized impurity states. I investigate the thermal broadening of these current steps which show the effect of the finite occupancy and the spin degeneracy of the impurity states. An understanding of these effects enables the determination of the electron tunneling rates through the two potential barriers of the double barrier system individually. This chapter also describes the use of these impurity states as narrow spectroscopic probes to investigate the local density of states fluctuations in the contact electrodes. Chapter 5 investigates the Zeeman spin-splitting of these localized, impurity states in a magnetic field. This leads to an accurate determination of the spin $g^*$ factor of electrons in the quantum well region. An understanding of the tunneling current through this two state spin system also enables the determination of the electron tunneling rates through the two potential barriers of the double barrier system individually. These two independent measurements of the tunneling rates in chapters 4 and 5 are found to be in good agreement with each other. I will also discuss the tunneling rates as a function of the magnetic field strength. Chapter 6 discusses some time dependent phenomena observed in tunneling through these discrete states. We observe a random telegraph signal, (RTS), at the threshold of some of the current steps. The amplitude of this RTS signal is observed to be a strong function of bias and temperature and increases dramatically as the temperature decreases, however, the characteristic times of this signal do not show any measurable dependence on temperature or bias. Quantitative understanding of the amplitude dependence based upon a “two state model” is achieved. I will present the observations and discuss the plausible microscopic models that could explain the fluctuations. Finally, chapter 7 will summarize this thesis work.

1.3 Historical development of the impurity system

At this stage it is important to take stock of the historical development of this small research field of “impurity systems”. I would also like to put in perspective the work done in this thesis with respect to the work done by other groups and elaborate on what is new in this thesis.

The field of resonant tunneling diodes, (RTDs), was pioneered by Chang, Esaki and
Tsu by their seminal work in 1973-74[4,70]. Interest was renewed in these devices with the advancement in epitaxial growth techniques, like MBE, and the work of Sollner et al. in 1983-84[64,65]. The research then focussed on reducing the dimensionality of these devices to achieve three dimensionally confined single electron states. This was first achieved by Reed et al. who introduced the vertical quantum dots[44]. The activity then concentrated on getting a three terminal resonant tunneling device. One approach to this consisted of putting a gate electrode on the top surface of a resonant tunneling diode and laterally shrinking the size of the device by the fringing gate potential[9, 26]. It was in these sub-micron devices that Dellow et al. of the University of Nottingham first reported in 1992 the observation of some features in I(V) characteristics which they attributed to single electron tunneling through localized impurity states[9].

Independent of the work at Nottingham group, resonant tunneling diodes were being investigated in Professor Mark Reed’s laboratory at Yale at the same time. In 1992 some interesting, unexplained features were observed in the low temperature I(V) of some triple barrier, double quantum well devices by N. H. Dekker, J. L. Huber, J. W. Sleight and M. A. Reed. In early 1993 I started work at the laboratory investigating some resonant tunneling diodes and I observed some reproducible features in the pre-threshold regions of the I(V) characteristics of those devices. These features were attributed to single electron tunneling through localized impurity states. We presented our data in 1993[11] which was the first report of the observation of impurity states in macroscopic, large area (4 (μm)² to 4096 (μm)²) resonant tunneling devices. Subsequently the Nottingham group also published their observations of the impurity states in macroscopic, large area devices[55]. Both the groups realized the significance of these observations, in particular the similarity of the “impurity system” to the quantum dot system and have pursued research in this system for the last five years.

Though resonant tunneling devices are being studied for many years no body observed the presence of single localized impurity states in them till 1992. A variety of factors may have been responsible for that. All RTDs would not show impurity states and only
devices with certain range of well and barrier thicknesses would exhibit single electron tunneling through the impurity states. The availability of good quality devices with low background currents is essential to observe single electron tunneling effects. Small area (\(\sim 100 \, (\mu m)^2\)) devices are needed to get the number of impurities in the device to be small. Also investigation at cryogenic temperatures is needed to observe the small current steps. The focus of research in the semiconductor industries has been to obtain devices with high currents and those working at room temperatures which is contrary to the above requirements. Thus it is an accident that the Nottingham group and I happen to observe the impurity related tunneling events in resonant tunneling diodes.

There are certain differences in the structures of the devices investigated by the two groups. The notable difference being that the devices studied by the Nottingham group have a two dimensional emitter and low Fermi energy in the emitter while the samples investigated at Yale have three dimensional emitter with high Fermi energy. The Nottingham group investigated some devices which have an intentional delta doped layer of Si impurities in the center of the quantum well. These devices showed an enhancement in the number of features in I(V) characteristics as compared to the undoped samples. This proves that these features, even in undoped quantum well samples, are indeed due to impurities[55]. They were also the first ones to introduce the idea of pairs of impurities to account for the higher binding energy states[23]. They also observed Fermi edge singularity in resonant tunneling through these localized states in 1994[22]. This was facilitated because of their two-dimensional emitter. The samples that I investigated only showed a weak Fermi edge singularity effect which was attributed to the three-dimensional nature of the emitter.

Investigations at Yale led to the observation of an oscillatory structure on the current plateaus in 1994 which was termed as the “fine structure”. This was attributed to fluctuations in the local density of states in the contact electrode[12,13]. Similar “fine structure” was observed by Su et al.[67] and by Sleight et al.[61] in quantum dot systems. The impurity system is the best system to investigate this structure because of its simplicity. This structure was thoroughly investigated later in 1995-96 by Schmidt et al. of K. von Klitzing's
group in Stuttgart[56,57,58].

My research then focused on the investigation of the spin degeneracy of the localized impurity states and Zeeman splitting. Spin splitting has not been observed in other quantum dot systems and it has become a topic of debate. In horizontal quantum dots, the situation is complicated due to the presence of a relatively large number of electrons in the dot which leads to strong electron-electron interaction[39]. In vertical quantum dots however, where the number of electrons in the dots is small, one does expect to observe spin splitting under proper experimental conditions. The observation of spin-splitting in the impurity system thus gained significance[14,15,16]. It also introduced a new and accurate technique to determine the spin $g^*$ factor of electrons in the confined quantum well region. The availability of a two-state system also enabled the determination of the electron tunneling rates across the two barriers of the device individually. The spin degeneracy also affects the zero magnetic field $I(V)$ characteristics as is observed from investigating the thermal broadening of the current steps. A Fermi function fit to this broadening as done by Su et al.[67] and Sleight et al.[61] in quantum dot systems is observed to be inadequate. The corrected function which takes into account the degeneracy and the finite occupancy of the localized state is observed to give a good fit to the data.

Finally research at Yale led to the observation of random telegraph signal in the impurity system. An understanding of this fluctuating signal may be important from the point of view of future single electron devices.
Chapter 2

Basic theory of resonant tunneling diodes, quantum dots and impurity systems

This chapter discusses the basic theory behind the experimental system. The field of compound semiconductor heterostructures is first introduced and then the operation of a resonant tunneling device where electrons are confined along one direction is explained. The basic theory of a quantum dot is then discussed where electrons are confined in all three dimensions. Finally the impurity system is introduced and compared with the quantum dot system.

2.1 Introduction

The fields of III-V and II-VI semiconductors has seen rapid developments in the past few decades. As the name suggests, these are compound semiconductors with one element from group III of the periodic table and another from group V (or from group II and VI respectively). Gallium Arsenide (GaAs) is a common III-V semiconductor and it has certain advantages over a more conventional semiconductor, such as Silicon (Si). GaAs has a direct bandgap as opposed to an indirect bandgap in Si and hence it has important applications in the field of optoelectronics like semiconductor lasers and light emitting diodes. GaAs conduction band electrons have a lower effective mass $m^*$ making it attractive for faster device
operations. One important advantage of compound semiconductors has been the feasibility to form ternary and quaternary alloys and thus the ability to tune the semiconductor properties like its bandgap. $\text{Al}_x\text{Ga}_{1-x}\text{As}$ is a typical example of a ternary alloy semiconductor where Aluminum (Al) and Gallium (Ga) (both from group III of the periodic table) are randomly distributed in a specific ratio.

Another important advantage of these compound semiconductors became apparent with the advent of epitaxial crystal growth techniques like molecular beam epitaxy (MBE) and metal-organic chemical vapor deposition (MOCVD). These techniques allow atomic level precision while growth, enabling the production of good quality semiconductor heterostructures. As the name suggests, heterostructures consist of layers of different compound semiconductors or different alloy semiconductors. The advanced growth techniques allow fabrication of structures where the interface between two layers is precise and smooth on an atomic scale. One important criteria for the ability to grow good quality epitaxial heterostructures is the lattice constant of the different layers. If the lattice constant is not the same, strain is introduced in the system at the interface which causes dislocations and limits the thickness of the overgrown epitaxial layer and the quality of the interface. Figure 2.1 shows a plot of the bandgap of the various semiconductor compound and alloy systems as a function of the lattice constant. Material systems that lie near or at the same point along a horizontal line in figure 2.1 are best suited for the growth of heterostructures. One can see that AlAs and GaAs have very similar lattice constants. Hence one of the most common heterostructure material system has been the GaAs \ system. At an interface between two distinct alloys, the difference in bandgap results in an abrupt change in the conduction band and the valence band energies. By combining thin layers of different materials, arbitrary electronic potential profiles can be created. This manipulation has been termed bandgap engineering\cite{75}. If the layers are thin, approaching the electron wavelength in the material, quantum size effects become important. In $\text{Al}_x\text{Ga}_{1-x}\text{As}$ systems this thickness is ($\leq 200\text{Å}$). The next section discusses these quantum size effects in one dimensions.
Figure 2.1: Plot of bandgap vs lattice constant for various semiconductors. The solid lines connecting various compounds indicate direct bandgaps, while the dashed lines indicate indirect bandgaps.
2.2 Quantum Wells - one dimensional quantum confinement

2.2.1 Quantum size effects

The simplest structure to investigate quantum size effects is a quantum well, which consists of a thin layer of a semiconductor buried in a second semiconductor such that the conduction band edge of the thin layer lies below the conduction band edge of the surrounding material. When the thickness of the layer becomes small, quantum mechanics restricts the allowed energies of the electrons in the layer, along the confinement direction, to discrete values. As the well width decreases, the separation of these allowed electronic energies increases. When this separation becomes larger than other energy scales in the system (for example the thermal energy, kT) then the electronic structure of the thin layer can no longer be considered continuous but is called quasi-two dimensional.

For real materials systems (e.g. GaAs/Al$_x$Ga$_{1-x}$As) the conduction band offsets, which act as barrier heights, are large enough that to a first approximation the infinite barrier case accurately describes the position of the lowest lying energy states. The discrete energy levels, shown in figure 2.2a, are given by

\[ E_n = \frac{(\pi \hbar)^2}{2 m^* L^2} n^2, \]  

where \( m^* \) is the bulk effective mass of the material and \( n \) is an integer.

It is important to note that while the allowed energies in the confinement direction (referred throughout this work as the \( z \)-direction) are discrete, there is no such modification along the transverse direction and therefore the total dispersion relation for the quantum well is given by

\[ E_n(k) = E_n + \frac{\hbar^2 k_{\parallel}^2}{2 m^*}, \]  

where \( k_{\parallel} = \sqrt{k_x^2 + k_y^2} \) is the magnitude of the wave vector in the transverse direction.

For a bulk material, the 3d-density of states is given by

\[ D_{3d} = \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} E^{1/2}. \]
Figure 2.2: Schematic illustration of quantum size effects in a quasi-two dimensional region. (a) The conduction band showing only discrete allowable energies in the confinement direction, but it still has a continuous density of states in the transverse direction. (b) The two-dimensional density of states, \( D_{2d} \), showing discrete step increase at energies corresponding to each discrete energy level in the confinement direction. Also shown is the three-dimensional density of states times the length, \( D_{3d}L \), calculated for a layer with a thickness equal to that of the quantum well.

In the quasi 2-D case, as is implied by the dispersion relation, for each value of \( n \), there is a continuous density of states due to the transverse wave vectors, given by

\[
D_{2d} = \frac{m^*}{\pi \hbar^2}.
\]  

Each discrete energy level along \( z \), adds a constant density of states creating a step-like function for the total density of states as shown in figure 2.2b. One important point to note is that the lowest allowed energy state has a finite non-zero energy because of the confinement. As the width of the well increases, the energy spacing of the confined states decrease and the quasi 2d-density of states approaches the 3d-density of states.

2.2.2 Resonant Tunneling Diode I(V) Characteristics

Resonant States

If the confining barrier thickness is finite (as shown in figure 2.3), the eigenstates of the quantum well are no longer stationary but are called quasi bound or resonant states. The eigenstates penetrate through the barrier and they have a finite lifetime. The wavefunctions
Well

Collector

Barriers

Figure 2.3: Schematic of a resonant tunneling diode (RTD).

in the quantum well couple with the states outside of the well, at the same energy, as shown in figure 2.4a. Any electron incident on such a structure at an energy equal to one of the quantum well eigenenergies will see an increased transmission probability as it is able to couple to the quantum eigenstate and tunnel through the structure. Hence such a structure is referred to as a resonant tunneling diode (RTD). The energies of the resonant states can be determined by calculating the transmission coefficients for an electron through the tunneling structure as described by L. L. Chang, L. Esaki, and R. Tsu[70,4] and summarized in the Ph.D thesis of Dr. J. L. Huber[29]. The states lie at energies of enhanced transmission through the structure. The results are similar to that shown in figure 2.4b.

This particular model of the devices is what is known as the coherent tunneling picture. The transmission coefficient is calculated by considering a wavefunction across the entire device such that an electron coherently tunnels across the entire structure[48]. These devices can also be modeled in what is known as the sequential tunneling picture[36]. In this picture, tunneling is a two step process where the electron first tunnels in the well and then tunnels out. The electron has a finite occupancy in the well. It has been shown that if the transmission coefficient is much more narrow than the supply of incoming electrons, the coherent and the sequential pictures are indistinguishable[74]. This is the case for the
Figure 2.4: (a) Model of the conduction band of a resonant tunneling diode (RTD). (b) Transmission coefficient for the structure shown in (a).

structures to be discussed.

**Qualitative description of I(V) characteristics**

Placing a voltage bias across an RTD allows the electronic structure of the quantum well to be probed[4, 64, 65, 70]. Figure 2.5a shows an RTD with no applied bias. At low bias across the device, the quantum eigenstate is above the emitter Fermi level and hence the transmission probability for those electrons to tunnel through is low. As the bias across the device is increased, the quantum eigenstate is pulled down in energy towards the emitter Fermi level. When it crosses the Fermi level, electrons at the same energy in the emitter as the eigenstate in the well experience an increased transmission coefficient and are able to tunnel through the structure. As the bias increases the current increases, peaking when the quantum eigenstate approaches the bottom of the emitter conduction band. Once the eigenstate is pulled below the emitter conduction band edge, the emitter electrons no longer see the increased transmission probability, resulting in a sharp decrease
Figure 2.5: Band plots of a resonant tunneling structure under different applied bias. By convention, the region to the left of the tunneling structure is referred to as the emitter. The dotted line in the emitter represents the quasi-Fermi level for the emitter. (a) No applied bias. (b) Biased to the peak resonant current (1 V). (c) Biased past resonance (1.5 V). The I(V) characteristics are shown in figure 2.6.
Figure 2.6: Expected $I(V)$ characteristics for the structure shown in figure 2.5. $V_{\text{peak}}$ corresponds to (b) and $V_{\text{valley}}$ corresponds to (c). The dotted line connecting the two points corresponds to the negative differential resistance region when the device is oscillating. The gray dotted line corresponds to the expected $I(V)$ characteristic of a single tunneling barrier with thickness equal to the sum of the two barriers.

in current. Physically, cutoff of the resonant current occurs because of conservation of transverse momentum and this will be discussed in more detail later. Further increasing the bias will bring the next quantum eigenstate into resonance, resulting in another peak in the $I(V)$ characteristics. Superimposed on the resonant current, is a nonresonant background current associated with tunneling through the entire structure taken as a single tunneling barrier such that $I_{\text{nonres}} \sim e^{V_{\text{bias}}}$. The $I(V)$ characteristic expected from the structure of figure 2.5 is shown in figure 2.6.

Density of states model of resonant tunneling

To get a quantitative model of the $I(V)$ characteristics one has to take into account the density of states of the electrons that can tunnel through. This was first proposed by Luryi[36]. The following discussion follows the expanded treatment by Ohno[40]. This model assumes that the current arises from carriers elastically scattered into the well, which implies both conservation of energy and conservation of momentum parallel to the confining
interfaces (perpendicular to the direction of current flow). It does not assume coherence of the electron wavefunction.

A general expression for tunneling current density at bias $V$ is obtained by summing over all available electron energies in the emitter as follows:

$$J(V) = e \int f(E) N(k) v(k) T(E, V) \, dk,$$

(2.5)

where $f(E)$ is the Fermi distribution function which can be assumed to be a step function (1 up to $E_f$ and 0 for higher energies), $N(k)$ is the density of carriers available for tunneling, $v(k)$ is the carrier velocity, and $T(E, V)$ is the transmission probability at bias $V$ for an electron with energy $E$ to tunnel through the device. All energies are measured relative to the bottom of the emitter conduction band. For a double barrier tunneling structure, the available carrier densities and velocities in the emitter, assuming a 3 dimensional emitter, are given by $N(k) = 2 \left( \frac{1}{2 \pi} \right)^3$ and $v(k) = \frac{1}{h} \frac{\partial E}{\partial k_z}$, where the current is taken to flow in the $z$-direction. A reasonable approximation for the transmission probability $T(E, V)$ is to assume a Lorentzian lineshape,

$$T(E, V) = \frac{|T_0|^2}{\pi} \frac{\Gamma^2}{(E - (E_O - \alpha V))^2 + \Gamma^2}.$$  

(2.6)

Here $E_O$ is the energy of the quantum well eigenstate relative to the bottom of the emitter conduction band at zero bias, $\alpha$ is the voltage to energy conversion factor which determines by how much the quantum eigenstate is pulled down in energy relative to the emitter when a bias $V$ is applied, $T_0$ is a constant and $\Gamma$ is the resonance width. If $\Gamma$ is much smaller than the incoming supply of electrons (i.e. the emitter Fermi energy), it can be further simplified to

$$T(E, V) \approx |T_0|^2 \Gamma \delta(E - (E_O - \alpha V)).$$  

(2.7)

This allows equation (2.5) to be written as

$$J(V) = e \frac{|T_0|^2 \Gamma}{2\pi^2 \hbar} \int k_\parallel dk_\parallel \int dk_z \frac{\partial E}{\partial k_z} \delta(E - (E_O - \alpha V)),$$

(2.8)
The second integral integrates to 1 if $0 < (E - \alpha V) < E_F$ imposing the condition $E = (E_O - \alpha V)$ and $k_z = (\sqrt{2} m^* (E_O - \alpha V)) / \hbar$. Thus,

$$J(V) = \frac{e |T_O|^2 \Gamma}{2\pi^2 \hbar} \int k_{||} dk_{||}, \quad (2.9)$$

where the integration limits for $k_{||}$ are determined by those available electrons which can conserve transverse momentum ($k_{||}$) during the tunneling process. As discussed by Luryi[36] and shown graphically in figure 2.7, this consists of those emitter electrons which lie within the Fermi sphere on a disk with a constant $k_z = (\sqrt{2} m^* (E_O - \alpha V)) / \hbar$. Thus $\int k_{||} dk_{||} = \pi (k^2_{||} - k_z^2)$ and the expression for the current density becomes,

$$J(V) = \begin{cases} 
0 & V \leq (E_O - E_F) / \alpha \\
\frac{e m^* |T_O|^2 \Gamma}{\pi \hbar^3} (\alpha V - (E_O - E_F)) & (E_O - E_F) / \alpha \leq V \leq E_O / \alpha \\
0 & V \geq E_O / \alpha .
\end{cases} \quad (2.10)$$

This model assumes that the emitter is purely three dimensional, with no confinement effects in the accumulation layer at the emitter/well interface.

### 2.3 Quantum Dots - three dimensional quantum confinement

Consider a quantum well diode, as described above, which has confinement along one direction, the $z$ direction. One can physically reduce the lateral size of the device (as shown in figure 2.8) by processes like reactive ion etching. As the lateral size gets smaller, quantum size effects along the lateral dimensions ($x - y$) become important. The electron energies along all directions are now discrete giving a truly three dimensionally confined system. This is called a quantum dot[44,61,67,68,69].

#### 2.3.1 Energy states of a quantum dot

**Quantum size effects**

In a quantum dot as described above, the potential an electron experiences along the $z$ direction is due to the semiconductor heterostructure band alignments and is modeled as a finite square well potential as in a quantum well. The potential the electron experiences
Figure 2.7: Illustration of the operation of a resonant tunneling diode. (a) Model conduction band diagram at zero bias showing the quantum well eigenstate of energy $E_0$ measured relative to the emitter conduction band bottom. (b) Band diagram under an applied bias $V$. The quantum eigenstate is pulled down in energy by an amount $\alpha V$. (c) Spherical Fermi surface for a degenerately doped emitter. Conservation of lateral momentum during tunneling allows only those emitter electrons whose momenta lie on a disk of constant $k_z$ as shown by the shaded disk. Current flows through the device when the disk lies in the region between the north pole and the equator with the peak being at the equator.
along the lateral \((x - y)\) directions depends critically upon the highly anisotropic reactive ion etching process. If the lateral shape of the dot is asymmetric then, each eigenstate of the system in general is nondegenerate (except for the spin degeneracy). If the dot has cylindrical symmetry then the various eigenstates reflect that symmetry. An additional effect to be considered is the Fermi level pinning of the exposed side walls of the device. The Fermi level in the exposed semiconductor material gets pinned at a fixed value near the mid gap. This causes bending of the bands and depletion of carriers from the region near the surface. The effective lateral size of the electrically active device is thus smaller than its physical dimensions. The potential an electron experiences along the lateral directions due to the band bending and depletion is often modeled as a cylindrically symmetric parabolic potential\([44,67]\) as,

\[
\Phi(r) = \frac{1}{2} m^* \omega_0^2 r^2.
\]  

(2.11)

Schrödinger’s equation for such a structure is best expressed in cylindrical coordinates. There is no \(\theta\) dependence, and since the potential is separable along the radial and the \(z\) directions, \((\Phi(r, z) = \Phi(r) + \Phi(z))\) separation of variables is possible. The single electron eigenenergies are given by

\[
E_N = E_z + E_{n, \ell}.
\]  

(2.12)
$E_z$ comes from the vertical quantization, and this will almost always be $E_0$, the quantum well ground state in the vertical direction. $E_{n,l}$ are the eigenenergies that result from the parabolic potential.

$$E_{n,l} = (M + 1) \hbar \omega_0,$$  
(2.13)

where $M = (2n + |l|)$, the radial quantum numbers $n = 0, 1, 2, ...$, and the azimuthal quantum numbers $l = 0, \pm 1, \pm 2, ...$. For typical quantum dots, $E_0 \gg \hbar \omega_0$ and typical values of energy level separation, $\hbar \omega_0$ are on order tens of meV.

**Coulomb charging effects**

Coulomb charging effects become important as the lateral area of the device decreases. To charge a cylindrical double barrier device by a single electron needs a charging energy,

$$E_C = \frac{e^2}{2C},$$  
(2.14)

where $C$ is the effective capacitance which can be assumed to be the effective semi-classical geometric capacitance of the two barriers[61,69]

$$C = C_e + C_c \approx \epsilon_0 k \pi a^2 \left( d_e^{-1} + d_c^{-1} \right).$$  
(2.15)

Here $k$ is the dielectric constant, $a$ is defined as the device’s effective electrical diameter, and $d_e$ and $d_c$ are the emitter and collector barrier thickness. If one assumes a quantized energy spacing given by a 0-D box, $\Delta E \approx 8\hbar^2 / m^* a^2$, and the ratio of the charging energy, $E_C$, to the quantization energy, $\Delta E$, is given by

$$\frac{E_C}{\Delta E} \approx \frac{d_{eff} e^2 m^*}{4\pi \epsilon_0 k \hbar^2} = \frac{d_{eff}}{a_B},$$  
(2.16)

where $d_{eff} = (d_e^{-1} + d_c^{-1})^{-1}$ and $a_B \simeq 100 \text{ Å}$ is the effective Bohr radius in GaAs. This means that for $d_{eff} = 100 \text{ Å}$, the ratio is unity and single electron charging should be apparent in vertical quantum dot structures.
2.3.2 I(V) characteristics of a quantum dot

The electronic structure of the quantum dot can be probed by studying the two terminal current-voltage (I(V)) characteristics as in a quantum well resonant tunneling diode. If the heterostructure barriers along the z direction have finite thickness then the quantum dot eigenstates couple to the electronic states in the emitter and the collector regions on either side of the dot along the z direction.

Let us first consider the simple case when the Coulomb charging energy of the dot is small compared to the size quantization energy. Any electron incident on such a structure at an energy equal to one of the quantum dot eigenenergies will see an increased transmission probability as it is able to couple to the quantum dot eigenstate and tunnel through the structure. As bias is applied across the device, the quantum dot states are pulled down in energy towards the emitter Fermi level. As a level crosses the Fermi energy, electrons in the emitter having the same energy as the quantum dot state experience enhanced transmission probability and thus there is a sharp increase in current. This current is due to single electrons tunneling one at a time through the quantum dot eigenstate. As the bias increases slightly, to first order, the current through a given quantum dot state remains the same causing a current plateau or step in the I(V) characteristics. A further increase in bias brings another quantum dot state below the Fermi level in the emitter introducing another channel for electrons to tunnel through. The I(V) characteristics of the device thus resembles a stair case structure, the bias locations of the steps corresponding to the energy spectrum of the quantum dot.

To get a quantitative expression for the I(V) characteristics one has to take into account the emitter density of states as summarized in the Ph. D. thesis of Dr. J. W. Sleight[60]. Figure 2.8 shows that the basic fabrication process to create quantum dots has resulted in the imposition of a lateral confining potential in the emitter and the collector contact. There is no confinement along the z direction in these regions and hence quasi 1d sub-bands
are formed. The energy of an electron in the $n$th subband of the emitter can be written as

$$E_{n,k_z} = E_n + \frac{\hbar^2 k_z^2}{2 m^*},$$

(2.17)

The current versus applied bias for such a structure is given by

$$I(V) = e \int_0^E \sum_n f(E) N_n(E) v_n(E) T_n(E, V) dE,$$

(2.18)

Here $T_n(E, V)$ is the transmission probability through the double-barrier structure at an applied bias $V$ for an electron with energy $E$ in the $n$th subband in the emitter. $f(E)$ is the Fermi distribution function for electrons in the emitter given by,

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}},$$

(2.19)

where $k$ is the Boltzmann constant and $T$ is the temperature, $N_n(E)$ is the 1d density of states of the $n$th subband in the emitter given by

$$N_n(E) = \frac{1}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_n)}},$$

(2.20)

and $v_n(E)$, the velocity of electrons at energy $E$ in subband $n$ is given by

$$v_n(E) = \frac{1}{\hbar} \frac{\partial E}{\partial k_z} v_n(E) = \sqrt{\frac{2(E - E_n)}{m^*}},$$

(2.21)

One can see that for the case of 1d emitter subbands, the product $N_n(E)v_n(E) = 1/\pi \hbar$ is a constant. Once again one can assume a Lorentzian line shape for the transmission probability, with resonance width $\Gamma_e$ and transmission coefficient $T_{0,n}(V)$,

$$T_n(E, V) = \frac{|T_{0,n}(V)|^2}{\pi} \frac{\Gamma_e^2}{(E - (E_O - \alpha V))^2 + \Gamma_e^2},$$

(2.22)

and assuming $\Gamma \ll kT$ (the thermal width) gives a current contribution of,

$$\Delta I(V) = \frac{e}{\pi \hbar} \left[ \sum_n |T_{0,n}(V)|^2 \right] \Gamma_e f(E_O - \alpha V),$$

(2.23)

for each spin state of the quantum dot with energy below the Fermi energy and above the band edge in the emitter[69].
Figure 2.9: Schematic of an impurity system showing an impurity state in the quantum well.

This simple picture gets modified when the Coulomb charging energy becomes comparable to the size quantization energy. An additional bias, corresponding to the charging energy of a single electron, is necessary for the threshold of conduction. The energy spectrum of the dot also depends upon the number of electrons occupying the dot states. The step structure of the I(V) characteristics gets modified and the bias locations of the steps no longer just correspond to the energy spectrum of the uncharged dot but depend upon the occupancy of the dot.

2.4 Impurity system - Coulomb potential confinement

There is another way in which one can create a three dimensionally confined experimental system consisting of discrete, single electron states. This is by having a small number of donor impurities in the quantum well regions of resonant tunneling diodes. The Coulomb potential of the ionized donor atoms give rise to shallow, hydrogenic bound states (figure 2.9). These localized states are physically similar to discrete quantum dot states. The I(V) characteristics of this system is thus similar to that of a quantum dot system. The impurity system has certain advantages over the quantum dot system. It is a truly 3d - 0d - 3d tunneling system unlike a 1d - 0d - 1d system of the fabricated quantum dot. There is no
fabrication imposed unknown potential in the emitter and collector regions. The Coulomb potential experienced by the electron due to the impurity is a known potential and the impurity eigenstates and energies are better characterized. It is possible to get devices with only a few isolated impurities and thus investigate the basic properties of a single, discrete state without having to worry about other states and their occupancy. The physical extent of the impurity state in a GaAs quantum well is of the order of 100 Å which is smaller than the lateral extents of fabricated quantum dot eigenstates. An understanding of the basic physics of this impurity system is thus valuable for the understanding of the fabricated quantum dot system.

2.4.1 Energy states of a single impurity

Single impurity binding energy in bulk semiconductors

In GaAs semiconductors a Silicon atom at the Gallium site acts as an electron donor. Si has one extra electron more than Gallium which it can donate to the lattice as it is loosely bound. The impurity site thus has an unbalanced positive charge which introduces a Coulomb potential which is screened by the available free carriers in the semiconductor. The attractive Coulomb potential gives rise to hydrogen like energy states which can bind a free carrier from conduction band. In order to determine the energy states of such a semiconductor hydrogenic atom problem one can follow the treatment of G. Bastard[2].

The wave function of a donor state in a semiconductor can be expressed in the form (from W. Kohn[35]),

\[ \psi(\mathbf{r}) = \sum_{i=1}^{N} \alpha_i F_i(\mathbf{r}) \phi_i(\mathbf{r}), \]  

where \( N \) is the number of equivalent conduction band minima, \( \phi_i(\mathbf{r}) \) are the Bloch wave functions and \( F_i(\mathbf{r}) \) are the envelope functions. In GaAs for the conduction band which is non-degenerate and isotropic with parabolic dispersion relations (\( N = 1 \) and single effective mass \( m^* \)) the envelope functions, \( F(\mathbf{r}) \) of the impurity states fulfill the equation,

\[ \left[ \frac{p^2}{2m^*} - \frac{e^2}{4\pi\varepsilon_0kr} \right] F(\mathbf{r}) = E F(\mathbf{r}), \]  

where \( p \) is the momentum, \( m^* \) is the effective mass, \( \varepsilon_0 \) is the permittivity in free space, \( r \) is the distance, \( E \) is the energy, and \( F(\mathbf{r}) \) is the envelope function.
where \( p \) is the momentum, \( m^* \) is the carrier effective mass, \( k \) is the dielectric constant of the semiconductor, \( e \) is the electronic charge, \( \epsilon_0 \) is the permittivity of space, and \( E \) is the energy of the impurity state. The ground bound state of this system is the 1S hydrogenic wavefunction,

\[
F_{1S}(r) = \frac{1}{\left(\pi a_0^2\right)^{1/2}} \exp\left(-r/a_b\right),
\]

where \( a_b \) is the effective three dimensional Bohr radius of the semiconductor hydrogenic impurity:

\[
a_b = \frac{4 \pi \epsilon_0 k \hbar^2}{m^* e^2} = 0.53 \times k \times \frac{m_0}{m^*} \text{Å}.
\]

The binding energy of this 1S state, \( R_b \) is given by

\[
R_b = \frac{m^* e^4}{32 \pi^2 \epsilon_0^2 k^2 \hbar^2} = \frac{1}{k^2} \times \frac{m^*}{m_0} \times 13600 \text{meV}.
\]

In GaAs \( k = 12.85 \) and \( m^*/m_0 = 0.067 \) giving \( a_b \approx 101 \text{Å} \) and \( R_b \approx 5.5 \text{meV} \).

The binding energy of hydrogenic donors in bulk GaAs is very small compared to its band gap of 1.5 eV. Hence this state is called as a shallow donor state. An electron occupying this state has an energy \(-5.5 \text{meV}\) as measured from the bottom of the conduction band.

**Single impurity binding energy in quantum wells**

In contrast to the bulk material, the binding energy of an impurity in a quantum well depends upon the characteristic of the well, in particular its width \( (L) \) and its barrier height \( V_0 \). The impurity binding energy increases as the well width decreases as long as the penetration of the quantum well wavefunction \( \chi(z) \) in the barriers remain small. This seems surprising at first since one intuitively associate higher kinetic energy (and hence lower binding energy) with the localization of a particle in a finite region of space. This is true of the energy value of the ground state of the quantum well \( (E_1(L)) \) and so also of the ground state of the impurity \( \epsilon(L) \) when measured with respect to some fixed reference like the bottom of the well. But the binding energy of the impurity \( E_b(L) = E_1(L) - \epsilon(L) \) actually increases as \( L \) decreases since for the impurity state the confinement causes the electron to stay near to the attractive center and thus experience a higher potential energy.
while this Coulomb attractive force has no effect upon the energy states of the quantum eigenstate. In the limiting case of an infinite barrier \( (V_0 = \infty) \) and zero well width \( (L = 0) \) one reaches the two dimensional limit when the ground state binding energy of a hydrogenic impurity in a GaAs quantum well is \( 4R_0 = 22 \text{ meV} \).

Another important feature of impurities in quantum wells is that the impurity binding energy explicitly depends upon the precise location of the impurity along the growth axis \( (z \text{ direction}) \) as there is no translational invariance along that axis. This energy depends upon whether the impurity is at the center of the well or at the edge of the well or within the barrier. The wave function for an impurity at the center of the well \( (z = 0) \) approaches the 1S wave function of the bulk. But the wave function for an impurity at the edge of the well \( (z = L/2) \) approaches a truncated \( 2p_z \) wave function as the barrier potential forces the impurity wave function to almost vanish at the interface. Thus the on edge impurity has a lower binding energy than the on center impurity.

**Approximate solution of the hydrogenic impurity problem in a quantum well**

This section discusses an approximate solution of the impurity problem in a quantum well. Let us assume the conduction bands of the materials to be parabolic in \( k \) and neglect the effective mass discontinuities at the interface as well as the difference in the dielectric constants of the two materials. The effective Hamiltonian of the system is given by

\[
H = H_0 + V_{\text{imp}} = \frac{p_z^2}{2m^*} + \frac{p_x^2}{2m^*} + \frac{p_y^2}{2m^*} + V_0 \Theta(z^2 - L^2/4) - \frac{e^2}{4\pi\epsilon_0k\sqrt{\rho^2 + (z - z_i)^2}},
\]

where \( V_0 \) is the barrier height, \( \Theta(x) \) is the step function \( (\Theta(x) = 1 \text{ if } x \geq 0 \text{ and } 0 \text{ otherwise}) \) and \( z_i \) is the location of the impurity along the growth axis. In the absence of the impurity the eigenstates of \( H_0 \) are separable in \( (x, y) \) and \( z \) giving,

\[
H_0|\nu, k_\perp > = (E_\nu + \frac{\hbar^2k_{\perp}^2}{2m^*})|\nu, k_\perp >, \tag{2.30}
\]
where $\nu$ labels the quantum well eigenstates (energy $E_\nu$) and $k_\perp = (k_x, k_y)$. Since the $|\nu, k_\perp>$ basis is complete one can expand the impurity wavefunction $\psi_{\text{loc}}$ in the form

$$|\psi_{\text{loc}}> = \sum_{\nu, k_\perp} c(\nu, k_\perp) |\nu, k_\perp>.$$  \hspace{1cm} (2.31)

Thus the Coulomb potential couples the states of different subbands. In a quasi bidimensional case when the inter subband separation is large one can neglect such a coupling giving $c(\nu, k_\perp) = c_\nu(k_\perp) \delta_{\nu,0}$. Thus the impurity wavefunction displays a separable form

$$<r|\psi_{\text{loc}}> = \chi_{\nu_0}(z) \phi(\rho),$$  \hspace{1cm} (2.32)

where $\chi_{\nu_0}(z)$ is the quantum well eigenstate wavefunction with energy $E_{\nu_0}$. The wave function $\phi(\rho)$ is the solution of the two dimensional Schrödinger equation

$$\left[ \frac{p_x^2 + p_y^2}{2m^*} + V_{\text{eff}}(\rho) \right] \phi(\rho) = (\epsilon - E_{\nu_0}) \phi(\rho),$$  \hspace{1cm} (2.33)

where $V_{\text{eff}}$ is the effective in plane Coulomb potential:

$$V_{\text{eff}}(\rho) = -\frac{e^2}{4\pi\epsilon_0 h} \int dz \chi_{\nu_0}^2(z) \frac{1}{\sqrt{\rho^2 + (z - z_i)^2}}.$$  \hspace{1cm} (2.34)

A solution of this equation is sought variationally with the simplest choice for the ground state being the trial wave function,

$$\phi_0(\rho) = \frac{1}{\lambda} \sqrt{\frac{2}{\pi}} \exp(-\rho / \lambda),$$  \hspace{1cm} (2.35)

where $\lambda$ is a variational parameter. The results of such a calculation for an infinite barrier quantum well ($V_0 = \infty$) are shown in figure 2.10 for an impurity located at the center as well as for an on edge impurity. Here the binding energy is in the units of the bulk impurity Rydberg ($R_b = 5.5$ meV for GaAs) and the well width is in the units of the bulk impurity Bohr radius ($a_b = 101$ Å for GaAs). For this experimental system where the well width is 44 Å, this figure gives the binding energy for on-center impurity to be $\approx 16$ meV and the binding energy for on-edge impurity to be $\approx 10$ meV.

So far one has assumed that the barriers are infinite. For real systems this is not true and the binding energy of an impurity does depend upon the barrier height $V_0$. For finite
bars, as the well width decreases beyond a certain limit ($\approx 0.2 \ a_b$) the impurity energy becomes comparable to the barrier height and the impurity wavefunction gets more and more delocalized. Thus the effect of the Coulomb potential gets smaller and hence the binding energy decreases. Figure 2.11 depicts this effect for an on-edge impurity[42] for various barrier heights. Note that for the samples under study where $L = 44$ Å, an infinite barrier approximation is a reasonable approximation.

The impurity binding energy monotonically decreases when the impurity location $z_i$ moves from the center to the edge of the well and finally deep into the barrier. A donor placed in the barrier material can still bind an electron in the well region as the electron experiences the Coulomb attraction of the donor at a distance. Figure 2.12 depicts the effect of the location of the donor on the binding energy for a well of width $L = 94.8$ Å for two different barrier heights. Note that a donor deep within a barrier is more effective in binding an electron in the well when the barrier is finite as then the electron is able to couple to the donor attractive center more effectively.
Figure 2.11: Calculated dependence of the on-edge hydrogenic donor binding energy in quantum wells versus the GaAs well width $L$ for different barrier heights $V_0 = 212$ meV, $318$ meV, $424$ meV and $\infty$. (Adapted from C. Priester[42]).
Figure 2.12: Calculated dependence of the hydrogenic donor binding energy in a quantum well of width $L = 94.8$ Å, versus the impurity position $z_i$ for an infinite barrier case ($V_0 = \infty$) and for a finite barrier case ($V_0 = 318$ meV). There is an interface at $z_i = L/2$. (Adapted from S. Chaudhuri[5]).

2.4.2 High binding energy states: impurity pairs

So far in this section it was assumed that the impurities are isolated from each other. I mentioned that the single impurity binding energy for the devices under consideration, which have a well width of 44 Å ranges from 10 meV to 16 meV depending upon the location of the impurity in the well relative to the well center. The impurities are randomly distributed in the well region and hence it is plausible, particularly in large area devices, that two impurities lie close to each other. Such two impurities would give rise to an increase in the binding energy of the electron. If $R$, the distance between two such impurities, is of the order of the single impurity Bohr radius, ($a_b \approx 100$ Å), then one expects the binding energy of the electron to be substantially modified from its single impurity value.
**Impurity pair in bulk semiconductors**

The problem of determining the binding energy in bulk semiconductor for an electron in the potential of a pair of two impurities is identical to that of a Hydrogen molecular ion ($\text{H}_2^+$) problem. The ground state wave function for the electron in the field of a single positive charge (Hydrogen atom problem, H system) is given by,

$$
\psi(r) = (\pi a_b^3)^{-1/2} \exp(-r/a_b),
$$

where $a_b$ is the Bohr radius. If there are two positive charges (Hydrogen molecule ion problem, $\text{H}_2^+$ system), then as a first approximation the electron wave function can be taken as the linear combination of the two atomic orbitals (LCAO). It can be shown that this LCAO wavefunction does not give the physically expected results in the limiting case of the positive charge separation going to zero. A better solution is obtained by considering the variational approach as is described by Slater [59]. This solution gives a good physical insight into the problem and also approximates the exact solution quite satisfactorily. Since the mass of the electron is much smaller than the mass of the positive charges one can assume that the two charges are at a fixed distance $R$ and obtain the electron wavefunction as a function of $R$. The variational wavefunction for the electron in the potential field of two positive charges located at positions A and B separated by a distance $R$ can be written as,

$$
\psi = N \left( \exp(-\beta r_a) + \exp(-\beta r_b) \right),
$$

where $r_a$ and $r_b$ are the distances of the electron from the two positive charges respectively, $\beta$ is a variational parameter and $N$ is a normalization constant. $\beta$ can be determined by requiring that the electronic energy be minimized for any fixed $R$. Figure 2.13 shows the dependence of $\beta$ upon the separation $R$. Note that $\beta$ shows the expected behavior in the two limiting cases and varies smoothly from a value of 1 ($R = \infty$, H problem) to a value of 2 ($R = 0$, He$^+$ problem). Given this $\beta$, one can determine the energy of the electron in the potential of the two positive charges as a function of $R$. This energy is termed the “electronic energy”. This is the energy (kinetic + potential) of the electron in
the field of the two positive charges. To obtain the total energy of the system one has to add to this the Coulomb repulsion energy of the two positive charges. The situation in a bulk semiconductor is exactly identical to the free H$_2^+$ ion situation except that the energy and the distance gets scaled by the appropriate factors of the effective mass, $m^*$, and the dielectric constant, $k$. The results of such a calculation are shown in figure 2.14. Note that as $R$ goes to zero the energy goes to 4 Rydberg which is the case for the He$^+$ system.

**Impurity pair in a quantum well**

The “electronic energy” would get modified due to the confinement effect if the impurity pairs are located in the quantum well. In order to determine the effect of confinement I use the scaling of the single impurity binding energy with well width as determined by Bastard[1] (figure 2.10). For a fixed well width, this gives the dependence of the energy of a hydrogenic state in the quantum well upon its bulk radius. The greatest binding energy will be if both impurities are located in the center of the well. The impurity separation $R$ is thus along a direction perpendicular to the quantum well growth direction. Figure 2.10
Figure 2.14: Electronic energy of a H$_2^+$ ion system in bulk GaAs as a function of the impurity separation $R$. $R$ is scaled in the units of the Bohr radius ($a_b$) and the energy in the units of Rydberg ($R_b$), the radius and the energy of the lowest state of the single impurity (H$^+$) problem which in GaAs semiconductor bulk are 101 Å and 5.5 meV respectively. The top and the right scale show the radius and the energy in real units respectively for bulk GaAs.
Figure 2.15: Expected lateral radius of the $\text{H}_2^+$ ion system as a function of the impurity separation. The lengths are in the units of the bulk single impurity radius ($a_b = 101$ Å).

shows that the binding energy of a single impurity depends upon the ratio $a_b / L$ where $a_b$ is the radius of the single impurity wave function in bulk and $L$ is the well width. I assume that the same scaling holds for the $\text{H}_2^+$ ion system except that $a_b$ is replaced by the lateral radius of the electron wavefunction in the two impurity system. This lateral wave function extent can be calculated from Slater’s variational wavefunction 2.37 to be,

$$\langle x^2 \rangle = \frac{1}{\beta^2} \frac{(1 + [1 + \beta R + \frac{2}{15} (\beta R)^2 + \frac{1}{15} (\beta R)^3] \exp(-\beta R))}{(1 + [1 + \beta R + \frac{1}{3} (\beta R)^2] \exp(-\beta R))},$$  

(2.38)

and is shown in figure 2.15 as a function of the impurity separation $R$. Thus using the lateral radius as obtained in figure 2.15 and using the scaling of energy as a function of well width, one can obtain the energy for this system which has a fixed well width of 44 Å. This energy is shown in figure 2.16 as a function of the impurity separation $R$. It is thus possible to obtain higher binding energy states up to 50 meV in this system. This figure also shows the Coulomb repulsion energy between the two impurities and the total energy of the combined system of two impurities and one electron.
Figure 2.16: Coulomb repulsion energy of two singly charged impurities in a 44 Å quantum well, electronic energy of an electron in the potential of these two impurities and the total energy of the impurities-electron system as a function of the impurity separation, $R$.

2.4.3 Coulomb charging energy in the impurity system

In the impurity system the lateral size of the device is large (a few microns) and hence the charging energy of one electron is negligible. If the impurity concentration in the device is low, and the impurity states are separated by a large distance ($\sim \mu m$), then one can assume that each impurity channel is independent of the others as the presence or absence of an electron in one would not have any effect on the others. But charging energy does show its influence on tunneling of electrons through the states associated with a single impurity. A given hydrogenic impurity has many eigenstates. In particular the ground state of the impurity is spin degenerate in zero magnetic field. Though one has two states with the same energy, only one of them can be occupied by an electron at a given time. This has important consequences on the I(V) characteristics as will be discussed later.
2.4.4 I(V) characteristics of the impurity system

The discrete impurity states can be probed by studying the two terminal current-voltage, (I(V)), characteristics as in a quantum dot. One expects the I(V) characteristics of the impurity system to be similar to that of the quantum dot system (section 2.3.2). One important difference between the fabricated quantum dot system and the impurity system is that the emitter and the collector contact electrodes in the impurity system do not have any lateral confinement and are three dimensional.

Let us first try and see what expression one gets for the current through the device by doing the analysis as in section 2.2.2. The current density versus applied bias for a double barrier, single quantum well structure is given by (as in equation 2.5),

$$ J(V) = e \int f(E) N(k) v(k) T(E, V) dk, \quad (2.39) $$

where $f(E)$ is the Fermi distribution function 2.19, $N(k)$ is the 3d density of states in the emitter, $v(k)$ is the velocity of an electron in the emitter along the $+z$ direction and $T(E, V)$ is the transmission probability through the double barrier structure at an applied bias $V$ for an electron with energy $E$ in the emitter. This transmission probability depends upon the overlap between the impurity wave function and the emitter electron wavefunctions. Assuming that the resonance width is much smaller than temperature, $(\Gamma \ll kT)$, once again one can approximate a delta function transmission probability for an electron in the emitter to tunnel through the impurity state,

$$ T(E, V) \approx |T_V|^2 \Gamma \delta(E - (E_O - \alpha V)), \quad (2.40) $$

where $E_O$ is the energy of the impurity state relative to the bottom of the emitter conduction band at zero bias, $\alpha$ is the voltage to energy conversion factor which determines by how much the impurity state is pulled down in energy relative to the emitter when a bias $V$ is applied, $T_V$ is the transmission coefficient which depends upon the bias and $\Gamma$ is the resonance width. This allows equation 2.39 to be written as,

$$ J(V) = e \int_0^{E_f} f(E) \frac{2 \hbar k}{m^*} |T(V)|^2 \Gamma \delta(E - (E_O - \alpha V)) d^3k $$
Let us try and understand what this equation (2.41) predicts. At low temperatures, at a given bias when a particular impurity state aligns with the Fermi level in the emitter, the current through the device would show a sharp step increase (as the Fermi function is a step function at low temperatures). As the bias increases this current would decrease linearly with bias (figure 2.17). At a higher bias another impurity state (most likely from another distinct impurity) would align with the Fermi level causing further step increase in the current. The I(V) characteristics would thus look like a random saw tooth step structure as shown in figure 2.17. This equation has been derived in what is known as the coherent tunneling picture.

To get the total current one has to integrate equation 2.41 over the lateral (x,y) dimensions. Assuming that the impurity is located at a specific location (x_0,y_0) one expects only the electronic states in the emitter contact near (x_0,y_0) to contribute to the current. Hence one does not expect the current to scale with the area of the device. I would also like to note that the above expression for current (equation 2.41) is valid only in the regime when the current is supply limited. It assumes that the electron spends negligible time in the localized state in the well. This is not the case when the two barriers of the device have approximately the same thickness. The electron then spends a finite time in the well and hence prevents other electrons from tunneling within that time. The current is thus saturated and hence not supply limited. Under such a situation the current would not be directly proportional to emitter density of states and one would not expect to see the linear decrease in current with bias as expressed in equation 2.41 and depicted in figure 2.17.

\[
\begin{align*}
= & \frac{4\pi\hbar|T(V)|^2\Gamma}{(2\pi)^3 m^*} \int_{0}^{\infty} \frac{k^3}{\Gamma} \left(c\left(\frac{\hbar k}{2m^*}\right) - \frac{\hbar k}{2m^*}\right) \int_{0}^{\pi} \cos \theta \sin \theta d\theta \\
= & \frac{\epsilon}{\pi^2 \hbar |T(V)|^2 \Gamma} f(E_O - \alpha V) k_0^2 \\
= & \frac{\epsilon m^*}{\pi^2 \hbar} f(E_O - \alpha V) (E_O - \alpha V) |T(V)|^2 \Gamma, \\
\end{align*}
\]

where \( k_0 = \frac{\sqrt{2 m^* (E_O - \alpha V)}}{\hbar} \).
Figure 2.17: (top): Schematics of the I(V) characteristics, at zero temperature, through a single impurity as predicted by equation 2.41. An impurity channel turns on at a bias when the impurity state crosses the Fermi level in the emitter and the current gradually decreases to zero as the impurity level crosses the bottom of the conduction band in the emitter. These bias locations are determined by the voltage to energy conversion factor ($\alpha$) and the energy of the impurity level at zero bias as shown. (bottom): The I(V) characteristics of a device having a random distribution of impurities. The energy and the current through a given impurity channel is a function of the impurity location.
In general however, the current through a single localized state can be expressed as,

\[ \Delta I(V) = e f / \tau(V), \]  

(2.42)

where \( \tau(V) \) is called as the life time of the impurity state. This \( \tau \) depends upon the bias \( V \), the local density of states near the impurity location and the transmission coefficient which depends upon the overlap between the emitter state wavefunctions and the impurity wavefunction.

In order to consider the effects of the finite occupancy of the electron in the well one has to look at the tunneling process in the sequential tunneling picture. In this picture the tunneling process is separated into two separate processes. One is tunneling from the emitter to the localized state and another is tunneling from the localized state to the collector. In order to derive an expression for current in this picture let us define, \( T_{em} \) and \( T_{cl} \) to be the electron tunneling rates for tunneling across the emitter and the collector barriers respectively. These would depend upon the barrier thickness, the bias and also upon the available density of states in the respective contact electrodes to which the localized state can couple to. Let us also define \( p \) to be the occupancy of the electron in the localized state. Figure 2.18 describes schematically the tunneling process under a typical bias condition. In steady state the rate of electrons tunneling in the localized state is the same as the rate of electrons tunneling out. The rate of inflow from the emitter is equal to \( T_{em} \) times the probability that an energy state \( E \) in the emitter is occupied (which is \( f(E) \)), times the probability that the localized state is unoccupied (which is \( 1 - p \)). Once the electron is in the localized state it can either tunnel back to the emitter at a rate \( p \left( 1 - f(E) \right) T_{em} \) or it can tunnel out into the collector at a rate \( p T_{cl} \). One can see from the band alignment in figure 2.18 that one does not have to worry about the occupancy of collector states as the Fermi level in the collector is much below the localized state energy. Hence we have,

\[ f(E) \left( 1 - p \right) T_{em} = p \left( 1 - f(E) \right) T_{em} + p T_{cl}, \]

(2.43)

which gives,

\[ p = f(E) \frac{T_{em}}{T_{em} + T_{cl}}, \]

(2.44)
which is just the partial tunneling rate. The tunneling current through a single localized state under such circumstances is given by,

\[ \Delta I = e p_T = e f(E) \frac{T_{em} T_{cl}}{T_{em} + T_{cl}}. \]  \(2.45\)

If one has single electron tunneling through single localized states it is not possible to experimentally determine both \(T_{em}\) and \(T_{cl}\) independently and hence it is not possible to distinguish between equations 2.42 and 2.45 if one assumes \(1 / \tau = T_{em} T_{cl} / (T_{em} + T_{cl})\).

In general though the ground state consists of not one but two states due to the spin degeneracy. In this situation one would naively expect the current to be double that of equation 2.45. This is not correct as only one state can be occupied at a given time. The probability of either one of the two states to be occupied is \((2p - p^2)\) where \(p^2\) denotes the probability of simultaneous occupancy of both the states. Hence the current through the system is given by,

\[ \Delta I = e (2 - p) p_T = (2 - f(E) \frac{T_{em}}{T_{em} + T_{cl}}) e f(E) \frac{T_{em} T_{cl}}{T_{em} + T_{cl}}. \]  \(2.46\)

If \(p\) is small compared to 2 then the current is just twice that of single state current and is proportional to the Fermi function \(f(E)\). But if \(p\) is not negligible, then the current would
not be directly proportional to the Fermi function. It should be possible to see this effect of degeneracy and finite $p$ if one investigates the thermal broadening of the current steps.

2.5 Summary

In this chapter the basic theory behind resonant tunneling diodes (RTDs) and quantum dots was reviewed. Quantum size effects in these systems and the theoretically expected $I(V)$ characteristics were studied. The impurity system was introduced as a system physically similar to a quantum dot system. The confining potential in this system is the known Coulomb potential and there is no unknown potential in the contact leads. It has thus the advantage of being a clean and simple system, facilitating direct investigation of the discrete localized states. The binding energy of an electron in a single impurity Coulomb potential in bulk GaAs semiconductor is found to be 5.5 meV while the wavefunction radius is approximately 100 Å. In a quantum well this binding energy increases and it is theoretically calculated to be between 10 and 16 meV, depending upon the location of the impurity in the 44 Å wide quantum well. The possibility of finding a pair of impurities was explored and the enhancement in the binding energy of an electron in the potential of the pair was estimated. It was shown that binding energies as high as 50 meV are possible for the experimental system under investigation. Finally the theoretically expected $I(V)$ characteristics for the impurity system was discussed and the effect of a finite occupancy of the localized state on the nature of the $I(V)$ characteristics was considered.
Chapter 3
Experimental methods

In this chapter I will document the experimental techniques used to carry out the low temperature high resolution measurements. The electronic measurement set up will be first described and then the low temperature cryostats.

3.1 Electrical measurement set up
3.1.1 dc I(V) characterization
preliminary measurements

The devices studied in this dissertation consists of two terminal resonant tunneling diodes. Preliminary measurements are done on these samples at room temperature and at 77 K. The sample chips are mounted on a standard 16 pin DIP or a special Yale 8 pin non-magnetic header which can then be mounted on a cryostat or on a dip stick for measurements at 4.2 K or lower. An HP 4145-B, Semiconductor Parameter Analyzer (SPA), is used to measure the two terminal current-voltage (I(V)) characteristics (figure 3.1a). The SPA is used as the voltage source, current meter and also as the data storage equipment. The major advantages of using the SPA are its programmability, the ease of storing data and a very good current resolution (≤ 50 fA). Its major disadvantages are a voltage resolution of only 1 mV and the limit of storing only 1000 points per trace.

To improve upon these measurements a reduction box was designed which increased the voltage resolution to 0.1 mV. The circuit of this box is shown in figure 3.1b. This box is used
Figure 3.1: (a) Preliminary I(V) measurement setup using the SPA only. (b) The use of a battery powered reduction box to improve the voltage resolution by a factor of 10. SPA acts as the voltage source, current meter and a data storage device in either cases.
in conjunction with the SPA. It consists of an operational amplifier wired as a reduction circuit to reduce the voltage of the SPA by a factor of 10. The SPA is used as the voltage source, current meter and also as the data storage equipment as above. This simple circuit gives quick results but it was discovered later that the operational amplifier in the reduction box adds high frequency noise to the measurement. This noise is responsible for broadening sharp features in the I(V) characteristics like the sharp current steps at low temperatures. In order to improve upon this and also to improve the voltage resolution below 0.1 mV a bias box was used that was designed and constructed by Dr. Jeff Sleight[60].

**Low noise, high resolution bias box**

Figure 3.2 shows a schematic of the preamplifier[10]. The basis of operation is as follows. A symmetric bipolar voltage signal, typically on the order of tens of volts, is supplied by the SPA as the input to the preamp. This signal is immediately attenuated in the preamp by a selectable factor of two, twenty, or two hundred. This results in a corresponding reduction in the signal to noise ratio of the input signal. The preamp is battery powered to reduce 60Hz
noise, the ground of the box is isolated from the ground of the SPA to kill grounding loop pickup, and capacitors to the box ground serve to shunt high frequency noise components.

The attenuated signal is then routed across two bias resistors, in series with the sample, as shown in figure 3.3. The voltage across the sample is determined from the output of the instrumentation amplifier, IA1. The current is determined by measuring the total bias voltage with IA3, and subtracting from it the voltage across the sample, given by IA2, by using the output of IA2 for the reference signal of IA3. These outputs of IA1 and IA3 are measured simultaneously by using two channels of the SPA as voltage meters and thus again SPA is used as source, meter and for data storage.

The instrumentation amplifiers (IA1, IA2, and IA3) consist of three discrete OP-amps, using two Burr-Brown OPA111BM and one Burr-Brown INA105BM (shown in figure 3.4). These instrumentation amplifiers have $\mu$V level voltage noise and approximately $10 \text{fA}$ level current noise in the frequency range where the measurements are made.

Extreme care was taken in the construction of this unit, as very high resistance bias resistors are used for $R_b$. These resistors are mounted on a machined teflon board, and much of the internal wiring is teflon coated (teflon resistance is greater than $10^{15}$ Ohms). All inputs to the instrumentation amplifiers are brought in on flying teflon coated leads (not attached to the printed circuit board, which can have resistances of only $10^{12}$ Ohms). Also, high resistance ceramic selector switches are used.

Even the resolution provided by the bias box is not sufficient to determine the spin
splitting energy which will be discussed in detail in chapter 5. This measurement requires a high resolution as well as a high dynamic range as a small voltage difference of $\approx 100 \mu V$ has to be measured with the absolute magnitude of the voltage being around 100 mV. Since only the relative voltage difference of the two spin states are of interest the following simple trick can be used to achieve the required resolution. A constant voltage source Yokogawa instrument can be used to subtract the high voltage background from the signal and the difference then amplified using a low noise Princeton Applied Research Corporation (PARC) 113 differential amplifier. This technique can also be used to obtain a higher resolution in the current measurements if needed. With this technique a resolution of less than $10 \mu V$ was achieved.

### 3.1.2 Characterization of time varying signal.

As will be discussed in chapter 6 the current through the impurity channels is observed to fluctuate with time in a random fashion. This fluctuation is termed as the random telegraph signal. In order to characterize this signal the measurement set up is modified as shown schematically in figure 3.5. A voltage ramp is used as a voltage source and a voltage divider circuit is used to get a reduction in voltage of 100. The current through the device is monitored by measuring the voltage across the sense resistor $R_s$ which is in series. Two different sense resistors 100 Kohm and 1 M ohm are used. The dc I(V) characteristics is
measured in this set up by using a Princeton Applied Research Corporation (PARC) 113 preamplifier in the dc mode to amplify the voltage across $R_s$ and then using a X - Y chart recorder to plot it versus the source (ramp) voltage. To monitor the time varying random telegraph signal two different techniques are used. In the first technique a PARC 124A lock-in amplifier in the flat band, ac voltmeter mode is used to monitor the voltage across $R_s$. The output of the lock-in is plotted on a chart recorder along with the source voltage. This characterization gives information about the bias dependence of the fluctuating signal. In another technique the source voltage is kept constant and the PARC 113 is used in the band pass mode (0.03 Hz to 300 Hz or even higher) to amplify the fluctuating signal only across $R_s$ and the output of which was recorded and analyzed using a HP 3561 spectrum analyzer. The spectrum analyzer can record the time trace of the signal like any oscilloscope and it can also get the fourier transform of the signal to obtain the frequency spectrum. The recorded data on the spectrum analyzer is then transferred to a computer for storage and further analysis.
3.2 Measurements at Low Temperatures

The electrical characterization of quantum-size-effect structures typically necessitates measurements at low temperatures. It is relatively easy to obtain temperatures from 300 K to 1.2 K in a commercially available cryostat using $^4$He as a cryogen. Temperatures below 1.2 K are generally not possible without the use of more elaborate refrigeration techniques. A simple way to get to 300 mK is to use $^3$He instead of $^4$He, however this adds greatly to the complexity of the cryogenic system. Due to the high cost of $^3$He, it must be recovered. Therefore a closed-cycle pumping system is required. Using a mixture of $^3$He and $^4$He, a dilution refrigerator, which will be discussed later, typically obtains temperatures in the 10 mK range.

3.2.1 Cryostat Systems

Janis Supervaritemp

For the measurements done between room temperature and 1.2 K, a Janis Research Supervaritemp cryostat is used. This system consists of an outer liquid nitrogen dewar, separated from an inner liquid helium dewar by a vacuum wall. The system can be operated in a variety of modes. The sample can be immersed in He, and by varying the pressure over the sample with a Walker regulator, temperatures between 1.4 K and 4.2 K can be obtained. Temperatures higher than 4.2 K are generally reached by heating the He as it passes through the capillary into the sample reservoir. Although this may be used to regulate temperatures from 4.2 K to 300 K, in practice, it is usually used up to 120 K. At this point the He flow is shut off. The system temperature rises slowly enough to allow for acquisition of data every ten degrees, with no significant thermal averaging.

Oxford Kelvinox Dilution Refrigerator

For measurements under 1.2 K, an Oxford Kelvinox 25 dilution refrigerator is used. The measured cooling power of the Yale dilution refrigerator is 100 $\mu$W at 100 mK, and a base temperature of 27 mK has been obtained. The sample is mounted on either the Yale 8 pin
header, a 16 pin commercial DIP, or in a custom 24 pin socket. A 9 Tesla superconducting magnet is available for magnetotransport studies. 11 Tesla is obtained by pumping on the lambda plate attached to the magnet, thus reducing the magnet temperature.

Again for this system, computer control is important, as low noise I(V)’s take 5-10 minutes to acquire. Stepping from 0 to 9 Tesla at fine increments (i.e., 2000 Gauss) takes 40 hours, or more. For the magnetotransport experiments, the control program that ran the Janis was modified to also control the magnet supply in the dilution refrigerator. In this manner, I(V,B) traces are acquired with almost no user intervention (except to monitor the dilution refrigerator).

The dilution refrigerator is also under computer control, which allows monitoring of temperature sensors at the various important points in the dilution refrigerator (the still, the mixing chamber and sample, and the 1K pot). Details of this and other aspects concerning the operation of the refrigerator, and the specifics concerning the Yale refrigerator, are covered in the Oxford manual[41] and in Dr. Mark Keller’s dissertation[33].
Chapter 4

Single electron tunneling through impurity states.

This chapter discusses the details of the experiments done and presents the preliminary data for the impurity system under investigation. The first section, section 4.1, gives the details of the growth and fabrication of the devices investigated. It introduces the Bandprofile model which simulates these quantum well devices. In section 4.2, the I(V) characteristics of the various devices are presented. In section 4.3, the temperature dependence of the observed characteristics are investigated and in section 4.4 the effects of magnetic field are studied. The following section, section 4.5, demonstrates the use of these narrow, localized energy states due to the impurities as spectroscopic probes to investigate the local properties in the emitter contact. Final section summarizes this chapter.

4.1 Sample growth and simulation

The samples under investigation in this dissertation are resonant tunneling diodes or double barrier single well heterostructures as were introduced in the theory chapter (chapter 2). They were grown on Si-doped n+ GaAs (100) conductive substrates using a Riber molecular beam epitaxy (MBE) system[46]. As part of another independent study, these samples were precisely characterized by a variety of techniques during and after growth. Cross-sectional transmission electron microscopy (TEM) was used to independently determine the layer
thicknesses of the quantum well and the barriers. Capacitance-voltage profiling provided an independent determination of the doping density in the contacts. Photoluminescence test patterns were grown epitaxially, on top of the active region, to provide for the determination of the Al-content in the barriers by measuring the AlGaAs bandgap.

The active region of the samples consist of the following epitaxially grown layers. A 0.5 μm Si-doped GaAs buffer and bottom contact layer, a (nominally) 150 Å undoped GaAs spacer layer, an undoped Al$_x$Ga$_{1-x}$As bottom barrier, an undoped GaAs quantum well, an undoped Al$_x$Ga$_{1-x}$As top barrier of nominally the same width as the bottom barrier, another 150 Å undoped GaAs spacer layer, and a 0.5 μm Si-doped GaAs top contact. On top of this, further layers were grown as photoluminescence test devices. These layers consist of a 0.5 μm undoped Al$_x$Ga$_{1-x}$As layer, an undoped 50 Å GaAs quantum well, a 0.1 μm undoped Al$_x$Ga$_{1-x}$As layer and a 100 Å GaAs cap layer. The heterostructures were grown at a constant temperature of 600 C. Samples with different barrier widths were investigated and they were grown sequentially to insure a constant unintentional impurity background.
Prior to device fabrication, the top photoluminescence diagnostic layers were removed by a chemical etch so that contact could be made to the upper n$^+$ GaAs layer. Square mesa devices were fabricated using standard photolithography and chemical etching techniques. Silicon-Nitride / Polyimide was used as passivation after the etching. The top and the bottom contacts were standard Nickel-Germanium-Gold alloyed metal ohmic contacts. Figure 4.1 shows the cross-sectional TEM image of one of the heterostructures. The sample names and the details of their heterostructure, as determined from the various characterization techniques, are listed in table 4.1. The different chips (die) of a given epitaxial heterostructure are labeled as A, B, C and so on. On a given chip there are square mesas with lateral dimensions (W), from 2 $\mu$m to 64 $\mu$m. Thus a particular device under investigation would be labeled as 2014-A (8) where the last (8) corresponds to the lateral size (W), in $\mu$m of the square mesa device.

A 1-d Bandprofile model developed by W. R. Frensley et al.[19,46] is used to simulate these devices. This program uses the material properties of each of the heterostructure layers including effective mass, band gap, band offsets and the doping levels to simulate the band diagram of the device along the growth direction at any given bias. It can then solve the Schrödinger equation in the well region to determine the quasibound eigenstates of the quantum well, their energies and lifetimes. Figure 4.2 shows a simulation of the device 2014 at an applied bias of 100 mV. This 1-d model gives useful insight into the device structure. It gives an estimate of the Fermi energy in the emitter and collector contacts.

<table>
<thead>
<tr>
<th>Sample name</th>
<th>well width</th>
<th>barrier width</th>
<th>Al content</th>
<th>contact doping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Å)</td>
<td>(Å)</td>
<td>(x %)</td>
<td>(10$^{18}$ cm$^{-3}$)</td>
</tr>
<tr>
<td>2013</td>
<td>48 ($\pm$ 5)</td>
<td>118 ($\pm$ 5)</td>
<td>27.7 ($\pm$0.6)</td>
<td>1.7 ($\pm$ 0.2)</td>
</tr>
<tr>
<td>2014</td>
<td>44 ($\pm$ 5)</td>
<td>85 ($\pm$ 5)</td>
<td>26.4 ($\pm$0.6)</td>
<td>1.7 ($\pm$ 0.2)</td>
</tr>
<tr>
<td>2015</td>
<td>44 ($\pm$ 5)</td>
<td>65 ($\pm$ 5)</td>
<td>27.7 ($\pm$0.6)</td>
<td>1.4 ($\pm$ 0.4)</td>
</tr>
</tbody>
</table>

Table 4.1: Details about the heterostructure of the various samples under investigation.
It numerically determines the lifetime of the quantum eigenstate which gives an estimate of the magnitude of the current through the device. It is possible to also use the model to determine the energy locations of the various observed features in the I(V) characteristics by comparing those with this model and thus do a spectroscopic study of the various features in the I(V).

4.2 Experimental Results

4.2.1 Tunneling thorough the quantum eigenstate

Two terminal dc I(V) characteristics are measured for the devices at low temperatures using techniques described in the experimental methods chapter 3. Figure 4.3 (top) shows the I(V) characteristics for a typical device (2014-A (8)) at 1.4 K, showing the main quantum well resonance peaks (top) in both forward and reverse bias orientations. In forward bias, electron injection is from the top barrier of the heterostructure while in reverse bias the electron injection is through the bottom barrier of the heterostructure. The characteristics show the familiar peaks in current due to tunneling through the quantum well eigenstate as discussed in section 2.2.2. Note the slight asymmetry in the I(V) characteristics. The
Figure 4.3: I(V) characteristics (zero magnetic field) at 1.4 K of the quantum well device showing the main resonance peaks (top). The magnified lower bias region shows two step-like structures attributed to single electron tunneling through two separate localized states due to impurities (bottom).
peak current and voltage magnitudes are smaller in forward bias than in reverse bias. This indicates that there is a slight asymmetry in the device growth suggesting that one barrier is slightly thicker than the other barrier. Since the peak current is smaller in forward bias, I conclude that the emitter barrier in forward bias is slightly thicker than the collector barrier. This is also consistent with the inference drawn from the peak bias locations. A thicker emitter barrier in forward bias implies that the the voltage to energy conversion factor, $\alpha$, would be larger in forward bias than in reverse bias. A larger $\alpha$ implies that the peak voltage location in forward bias would be lower than in reverse bias. Since in forward bias electron injection is from the top barrier, the top barrier is the emitter barrier in forward bias. It can thus be inferred that the top barrier of the device is slightly thicker than the bottom barrier. This asymmetry affects most of the device characteristics and I will refer to it as the “forward-reverse” asymmetry. I(V) characteristics of different devices on a given chip (die) scale with the area of the device and they all show the same “forward-reverse” asymmetry. The magnitude of this asymmetry is different for devices on different chips. Devices on 2014-B shows an asymmetry similar to that of devices on 2014-A, (figure 4.3), while devices on 2014-C and 2014-D show a much lesser asymmetry. Similarly, devices on 2015-A show some asymmetry while those on 2015-B and 2015-C are symmetric. This indicates that these variations are due to the slight differences in growth conditions at different die locations on a given wafer. These die to die variations are consistent with the independent study of these variations as mentioned in section 4.1[46]. Device 2014-A (8) is the most extensively studied device in this thesis and hence an understanding of the “forward-reverse” asymmetry is essential for this thesis.

4.2.2 Tunneling thorough localized states

Magnification of the current in the low bias region of the I(V) characteristics (figure 4.3 bottom), shows two sharp current steps for both forward and reverse bias directions. This step structure is observed to be sample specific, but for a given sample it is exactly reproducible from one voltage sweep to another and independent of the voltage sweep direction.
Figure 4.4: I(V) characteristics of all the devices on 2014-A with different lateral widths (as marked) at 1.4 K. Note that current steps are observed in all of them and the step magnitudes do not show any systematic dependence upon the area of the device. The step current magnitudes do not scale with the area of the device but they are of the same order of magnitude for devices with vastly different areas.

The steps are reproduced even after numerous thermal cycling of the sample, except for some threshold voltage shifts.

Figure 4.4 shows the I(V) characteristics for the different area devices on a single chip (2014-A). Note that current steps are observed in all devices and the step magnitudes do not show any systematic dependence upon the area of the device. The current magnitudes do not scale with the area of the device but they are of the same order of magnitude for devices with vastly different areas. This indicates that the observed features are due to tunneling through some localized states in the device.

Note that in the I(V) characteristics in figure 4.3 (bottom), two steps are observed in both forward and reverse bias orientations. Similar approximately symmetric behavior is observed for the other devices investigated. This indicates that the localized states to which these features are attributed, lie within the quantum well and not in either of the two barriers. A localized state within a barrier is asymmetrically located with respect to the
Current steps are observed in devices with different barrier thicknesses. Figure 4.5 shows the observed current steps in three different devices with different epitaxial heterostructures. Note that the step current magnitude are different for the devices with different barrier thicknesses. This can be understood in the framework of the discussion in the theory chapter (section 2.4.4). The current step magnitude for tunneling through single localized states in the quantum well is given by (equation 2.42), \( \Delta J(V) = e f / \tau \) where \( f \) is the Fermi distribution function and \( \tau \) is the life time of the localized state in the well. \( \tau \) depends upon the barrier widths of the devices. The larger the barrier, the greater the life time and hence lower current. A first order approximation to \( \tau \) is the life time of the 1d quantum eigenstate. This can be estimated by the bandprofile program introduced earlier.

A comparison of the expected current step magnitudes and the average of the various observed current step heights is shown in table 4.2. Note that there is a good agreement between the observed step heights and the predicted height according to the 1-d bandprofile model. This agreement confirms the attribution of these current steps to single electron

<table>
<thead>
<tr>
<th>Device name</th>
<th>barrier thickness (Å)</th>
<th>( \Delta I ) (pA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td>2013</td>
<td>118</td>
<td>0.45</td>
</tr>
<tr>
<td>2014</td>
<td>85</td>
<td>120</td>
</tr>
<tr>
<td>2015</td>
<td>65</td>
<td>1450</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of theoretically predicted current step height by the bandprofile model and the experimentally measured average of the various observed current step heights for the three different device types.
Figure 4.5: I(V) characteristics showing the observed current steps in different devices belonging to each of the three different epitaxial heterostructures under investigation.
tunneling through localized states in the quantum well. The assumption to use the quantum eigenstate lifetime as $\tau$ for the localized states is only an approximation. The actual lifetime of the localized states depend upon the exact location of the state in the well. A state located near the emitter barrier will have a smaller lifetime than a state located at the center of the well. If these states are randomly distributed in the well one expects a random distribution in the measured current step heights for the devices of a given epitaxial layer structure, which is observed.

Figure 4.5 shows that the current in sample 2013 is small due to it's thick barriers. Since it is difficult to do a thorough investigation of this device, I only investigate samples 2014 and 2015 in detail. Note that the well widths in 2014 and 2015 are the same and thus the major difference between these two samples is their barrier widths.

It was predicted in the theory chapter and depicted in figure 2.17 that, one expects the I(V) characteristics of single electron tunneling from a 3-d emitter through a 0-d localized state to be triangular in shape. A triangular shape is observed in some of the devices as can be seen in figure 4.6. As discussed in section 2.4.4, from this figure one can estimate the Fermi energy in the emitter contact to be approximately 30 meV. This is comparable to the 40 meV as estimated from device modeling by Bandprof (figure 4.2). This experimental determination is however not reliable as the triangular shape of the I(V) characteristics is not observed for most current steps. This indicates that the current is not supply limited but is saturated by the lifetime of the impurity state as was discussed in section 2.4.4. The triangular nature of the I(V) characteristics were observed only in the forward bias orientations and never in the reverse bias orientations, another case of the “forward-reverse” asymmetry as mentioned earlier. The asymmetry is because the collector barrier in reverse bias is thicker than the emitter barrier. This implies that the current is less sensitive to the density of states in the emitter and less likely to be supply limited in reverse bias orientation than in forward bias. Hence it is less likely to observe a triangular shape in reverse bias.

Note also the presence of an oscillatory structure on the current plateaus. This structure is different for different devices but is reproducible for a given device even after thermal
Figure 4.6: I(V) characteristics of device 2014-B (8) in forward bias at 1.4 K showing the triangular nature as predicted by theory.

cycling. As can be seen in figure 4.5 this structure is observed in devices with different barrier thicknesses. I will denote this structure as the “fine structure” and investigate it in more detail in section 4.5. This structure is attributed to fluctuations in the local density of states in the emitter. These fluctuations in the local density dominate the smooth, monotonic dependence of the bulk density on energy and hence it is difficult to quantify the triangular or non-triangular nature of the current steps.

All the observations presented in this section so far indicate that the observed current steps are due to tunneling through individual, localized states in the quantum well. We attribute these states to the presence of donor impurities in the quantum well. These impurities are randomly distributed in the well. Though the quantum well region is nominally undoped, one expects a finite number of impurities to be present there due to the low but finite background impurity concentration in the MBE chamber during growth. In the theory chapter, (section 2.4), I discussed the binding energy of an electron in the potential of a single or a pair of impurities. It is possible to do a spectroscopic analysis and determine the binding energy of the observed features in the I(V) characteristics. The next section
(section 4.2.3) presents a statistical analysis of the observed number of current steps and their binding energies. That analysis conclusively proves that the observed current steps in these devices are due to tunneling through localized states due to donor impurities.

4.2.3 Statistical analysis of the observed number of localized states

In this section I discuss the experimental determination of the total number of observed localized states in devices of different lateral dimensions and in a given bias range. From this survey one can determine the localized state concentration in the devices and their binding energies. This then allows one to attribute the low binding energy localized states to single impurity states and the high binding energy localized states to impurity pairs.

The devices under investigation have nominally undoped quantum wells. From a background impurity concentration in the MBE growth chamber of $10^{14}$ impurities per cm$^3$ one estimates a 44 Å quantum well to have an impurity concentration of $\approx 0.5$ impurities per $(\mu$m)$^2$ lateral area of the device. The devices under study have a square cross section with lateral dimensions of 2, 4, 8, 16, 32 and 64 μm and hence respective lateral areas of 4, 16, 64, 256, 1024 and 4096 $(\mu$m)$^2$. In small size devices like the 2 or 4 μm devices, it is possible to just count the number of current steps in the I(V) characteristics with each step corresponding to a single impurity state in the quantum well. In larger devices however, the number of states is high and it is not possible to distinguish individual current steps except at very low temperatures and lower biases. Figure 4.7 shows the I(V) characteristics of the 8 μm device at the lowest temperature investigated and also at 4.2 K. We observe that the current at the lowest temperature shows discrete current steps. The current at 4.2 K shows the effect of thermal smearing of the I(V) characteristics. One can see from this figure that the absolute magnitude of the current at 4.2 K gives an estimate of the number of current steps that would be observed at low temperature and hence gives an estimate of the number of localized states in the device. One can thus determine the number of localized states in a variety of devices from their I(V) characteristics at 4.2 K.

By investigating the low temperature, (< 100 mK), I(V) of a few devices one can de-
Figure 4.7: I(V) characteristics of device 2014-A (8) in forward bias at 35 mK and at 4.2 K. Observe the effect of thermal smearing of the I(V) characteristics at 4.2 K but note that the absolute magnitude of the current at 4.2 K gives an estimate of the number of current steps and hence the number of impurity states in the device.

termine the average step current magnitude (ΔI_{av}) for devices of given barrier thicknesses. Using this value one can determine the number of localized states in other devices from their 4.2 K I(V) characteristics. One can also determine the number of current steps in a given bias range and thus determine the number of localized states with a given binding energy from the equation,

\[ N(V_1, V_2) = \frac{(I(V_2) - I(V_1))}{\Delta I_{av}}, \]  

(4.1)

where \( N(V_1, V_2) \) is the estimated number of impurity states in the bias range from \( V_1 \) to \( V_2 \) where \( V_1 < V_2 < V_{t_0} \). \( V_{t_0} \) is defined to be the turn on voltage of the resonant tunneling diode. This method is not a good procedure for small size devices. However for that case it is not needed as one can resort to simple counting. The basic assumption in this procedure is that all the current at the biases under consideration (below the turn on voltage) is attributed to tunneling through impurity related localized states. This is a good assumption since the barrier thicknesses of these devices is large (65 Å and 85 Å). The
Figure 4.8: I(V) characteristics of six different devices on chip 2014-A. The curves are labeled by their respective lateral size in µm and the current is scaled by their respective lateral area.

The background current in these devices is thus negligible compared to the impurity related current in the relevant bias range.

In order to get the statistics of the number of localized states correct it is important to determine the turn on voltage, \( V_{to} \), accurately. It is defined to be the voltage at which the quantum eigenstate in the well crosses the Fermi level in the emitter. This is determined by investigating the I(V) characteristics of the different devices on a given chip scaled by their respective areas. The current at biases greater than the turn on voltage is due to tunneling through the quantum eigenstate and is expected to scale with the area of the device. The turn on voltage is determined as the bias at which some deviation is observed from the scaling behavior or there is a change in slope. Figure 4.8 shows the I(V) characteristics of six different devices on a given chip (2014-A). Note that except for the 2 µm device the I(V) characteristics of the other devices overlap, when scaled by their respective areas.

Once \( V_{to} \) for a chip is known, one can determine the number of localized states in the various devices on that chip. In this section I restrict myself to the determination of the
A collection of data from the various measured devices is shown in table 4.3. The average current step height for the 2014 devices is measured to be 80 pA while it is 1 nA for the 2015 devices. The turn on voltage for all the 2014 chips is observed to be 160 mV while that for all the 2015 chips is 170 mV.

There is a large error in the estimation of the total number of localized states. I estimate an error of about ±10% in the determination of the average current step height $\Delta I_{av}$. There is also an error of about ±2 mV in the estimation of $V_t$. This corresponds to an error of about ±25% in the determination of the total number of localized states. Thus there is an error of about ±35% in the estimation of the number of localized states for a given device. Since I am averaging over a certain number of devices, $N_{dev}$, of a given lateral size, I estimate the net error to be $\pm(35/\sqrt{N_{dev}})$%. This error is reported in the last column of table 4.3.

We can see from table 4.3 that the devices have approximately $1 \pm 0.25$ localized states per $(\mu m)^2$ area of the device. This is consistent with the estimated number of impurities
from the background doping concentration in MBE growth.

**Likelihood of impurity pair states**

I will now discuss the likelihood of observing higher binding energy localized states which can be attributed to impurity pairs in the quantum well. From the experimentally determined concentration of the impurity states, the average separation between two impurities can be determined to be approximately one µm. It is thus unlikely to find a pair of impurities separated by 250 Å in a small area device. This is statistically possible though, for a large area device with lateral dimensions of 32 or 64 µm. Using a given concentration \( c \) of the impurities and assuming a random distribution of impurities, one can determine the probability of finding at least one pair of impurities with a given separation \( R \) between the two impurities of the pair. The probability of finding a given impurity in a region of area \( A \) is \( A / W^2 \) where \( W \) is the lateral width of the device. The probability of not finding it in that region is \( (1 - A / W^2) \). If there are \( N \) total number of impurities then the probability of not having any pair with separation less than \( R \) implies not having any impurity within the area \( \pi R^2 \) around any other impurity. Thus the probability of having at least one pair is expressed as\([23,49]\)

\[
P(R) = 1 - \prod_{i=0}^{N-1} (1 - \frac{i\pi R^2}{W^2})
\]

(4.2)

where \( N = c W^2 \) is the total number of impurities in the device. Figure 4.9 shows how this probability varies with the separation \( R \) for the devices with different lateral widths. Here I assume \( c = 1 (\mu m)^{-2} \) as can be concluded from table 4.3. This figure shows that it is very likely to observe at least a pair of impurities in devices with large lateral widths. It however does not estimate the total number of pairs that exist in a given device.

In order to estimate that one can follow the analysis of Griffiths et al.\([25]\). If the impurity concentration \( c \) is small then the probability of finding an impurity in a region of area \( A \) is \( cA \). The probability of finding a pair (one impurity in region \( A \) and another in an area \( \pi R^2 \) around it) is \( cA\pi R^2 \). Thus the ratio of pairs to total number of impurities can be estimated to be \( c\pi R^2 \). Table 4.3 (and also table 4.4), shows the experimentally determined
Figure 4.9: Probability of finding at least one pair of impurities with a given separation $R$ assuming an average concentration of one impurity per $(\mu m)^2$ area of the device for devices with different lateral widths.

total number of impurities for a device of a given area. From that data and using the above analysis one can estimate the expected number of pairs in a given device as is shown in table 4.4.

Now I compare this expectation with the experimental data. Table 4.5 lists the observed total number of current steps in the bias range zero to 30 mV lower than the turn on voltage ($N(0, V_{lo} - 30 mV)$). Similarly 4.6 lists the observed total number of current steps in the bias range zero to 60 mV lower than the turn on voltage ($N(0, V_{lo} - 60 mV)$). Once again we estimate the error in this experimental determination to be around $\pm \frac{35}{\sqrt{N_{dev}}} \%$.

Since the devices under investigation are symmetric double barrier devices one expects the voltage to energy conversion factor, $\alpha$, to be approximately 0.5. (This factor will be accurately determined in the next section for a few devices.) A current step at bias 30 mV below the turn on voltage is thus attributed a binding energy $\alpha \times 30 = 15$ meV while a current step at bias 60 mV below the turn on voltage is attributed a binding energy $\alpha \times 60 = 30$ meV.
Table 4.4: Expected number of impurity pairs in different devices.

<table>
<thead>
<tr>
<th>chip name</th>
<th>device lateral size (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2014-A</td>
<td>0</td>
</tr>
<tr>
<td>2014-B</td>
<td>0</td>
</tr>
<tr>
<td>2014-C</td>
<td>3</td>
</tr>
<tr>
<td>2014-D</td>
<td>0</td>
</tr>
<tr>
<td>2015-A</td>
<td>0</td>
</tr>
<tr>
<td>2015-B</td>
<td>0</td>
</tr>
<tr>
<td>average</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.5: Observed number of current steps in different devices up to a bias 30 mV lower than the turn on voltage which correspond to binding energies higher than 15 meV.
Table 4.6: Observed number of current steps in different devices up to a bias 60 mV lower than the turn on voltage which correspond to binding energies higher than 30 meV.

In the theory chapter, (section 2.4.2), I discussed the binding energy of an electron in the potential of a pair of impurities in the quantum well. From figure 2.16 one can associate a separation $R < 1000$ Å for binding energies greater than 15 meV and similarly $R < 250$ Å for binding energies greater than 30 meV. Thus a comparison between the experimental tabulation of observed high binding energy current steps in table 4.5 with the expected number from the theory as tabulated in row 2 of table 4.4 is meaningful. Similarly one can compare table 4.6 with row 3 of table 4.4. One can see that the experimentally observed number of high binding energy current steps is in good agreement with the probabilistic estimate of impurity pairs. The analysis in this section thus corroborates our attribution of the low bias current steps to tunneling through high binding energy states associated with impurity pairs.
Figure 4.10: I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures showing the Fermi level broadening and the Fermi fit (equation 4.3) to these I(V) traces for $V < V_{th}$ (solid lines).

4.3 Variable temperature measurements: thermal broadening of the current steps

This section investigates the temperature dependence of the current steps in the I(V) characteristics. As discussed in the theory chapter (chapter 2) and expressed in equation 2.42 or in 2.45, the magnitude of the current step in tunneling through a single electronic state is given by $\Delta I(V) = e f / \tau$ where $\tau$ is the life time of the 0-d state, $f$ is the Fermi distribution function and $\alpha$ is the voltage to energy conversion factor. The sharpness of the current plateau edge is thus expected to decrease as the temperature increases, due to the broadening of the emitter Fermi distribution function. The current (equation 2.42) can be expressed as,

$$I(V,T) = 2I_{th} f(V,T) = \frac{2I_{th}}{1 + \exp[\alpha e(V_{th} - V)/kT]}, \quad (4.3)$$

where $e$ is the electron charge, $k$ is Boltzmann’s constant, and $V_{th}$ and $I_{th}$ are the threshold voltage and current at the observed common intersection point of the various I(V) curves at different temperatures. Note that when $V = V_{th}$, $f = 1/2$ and $I = I_{th}$ irrespective
Figure 4.11: I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures and the extrapolation of the Fermi fits (equation 4.3) to these I(V) traces as obtained in figure 4.10 to voltages greater than the threshold. The experimentally measured current can be seen to be less than the theoretically expected current on the plateau.

of the temperature. The only free parameter is $\alpha$, which can be determined from a fit (figure 4.10) of the above function to the I(V) traces at zero magnetic field. The fits are done for the region $V \leq V_{th}$ only because of the presence of the oscillatory structure on the current plateaus referred to as the “fine structure”. The fits are done for data taken at different temperatures, from 0.5 K to 5 K, and $\alpha$ is determined for each. The average value of $\alpha$ along with the root mean square (rms) error is reported. For the device 2014-A (8) $\alpha = 0.48 \pm 0.02$ for forward bias and $0.42 \pm 0.02$ for reverse bias. This is one more example of the “forward-reverse” asymmetry mentioned in section 4.2.1. $\alpha$ is smaller in reverse bias than in forward bias as was predicted in that section from observing the asymmetry in the peak current and voltage values. For the device 2015-C (32) $\alpha = 0.40 \pm 0.02$ for forward bias.

If the localized electronic level is spin degenerate however, the expression for the current gets modified due to the finite occupancy of the electron within the state as is discussed in the theory chapter 2. There are now two electronic states instead of just one as was
considered above. Coulomb charging prevents simultaneous occupation of both the spin states and the step current gets saturated at a lower current value if the occupancy is high. This effect can be seen if the fits obtained for equation 4.3 are extrapolated for voltages greater than the threshold as shown in figure 4.11. The experimental plateau current value is smaller than that predicted by the Fermi fit. This indicates that the investigated impurity states are indeed spin degenerate and this must be taken into account.

The current in this case of tunneling through two degenerate states can be expressed as in equation 2.46,

\[ \Delta I = A (2p_0 f - (p_0 f)^2), \]  

where \( A = e T_{cl} \) is a constant and \( p_0 = T_{em} / (T_{em} + T_{cl}) \) is the occupancy of the electron in either one of the two localized degenerate states when \( f = 1 \). \( p_0 \) depends upon the relative tunneling rates of the two potential barriers. If the occupancy \( p_0 \) is small such that \( p_0^2 \ll 2p \) then equation 4.4 reduces to equation 4.3. Note also that for \( V \leq V_{th} \) when \( f \leq 1/2 \) the occupancy which is \( p_0 f \) is even smaller. Hence a fit done to the temperature data for \( V \leq V_{th} \) with equation 4.3 is a reasonable approximation. It is however possible to get a better fit to the temperature data if one uses equation 4.4 instead of equation 4.3. Figure 4.12 shows a fit of equation 4.4 to the data. \( V_{th} \) is obtained from the common intersection point of the curves at different temperatures. \( A \) and \( p_0 \) are obtained from the measured current values of plateau current \( (I_{\text{plateau}}) \) and the threshold current \( I_{th} \) in the following way. Experimentally \( I_{\text{plateau}} \) is determined to be the current on the step far away from the threshold where the curves at all temperatures overlap. There is some uncertainty in the determination of \( I_{\text{plateau}} \) due to the fluctuating structure on the plateau which will be discussed in more detail later. When \( V \gg V_{th} \) then \( f = 1 \) and

\[ \Delta I = I_{\text{plateau}} = A (2p_0 - (p_0)^2) \]  

and when \( V = V_{th} \) then \( f = 1/2 \) and

\[ \Delta I = I_{th} = A (p_0 - (p_0/2)^2) \]
Figure 4.12: I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures showing the Fermi level broadening and the corrected fit (equation 4.4) to these I(V) traces for $V < V_{th}$ (solid lines), which takes into account the finite occupancy of the state.

Solving these two equations simultaneously from the measured values of $I_{\text{plateau}}$ and $I_{th}$ one gets $A$ and $p_0$. From the data shown in figure 4.12, $A = 81.4$ pA and $p_0 = 0.35$. Thus once again the only free parameter in the fit is $\alpha$ which then can be accurately determined. Once again the fits are done only for bias voltages $V \leq V_{th}$ because of the presence of the “fine structure” on the current plateaus for $V \geq V_{th}$. The $\alpha$ values obtained from this fit for this step is $0.50 \pm 0.01$ which is within the experimental error of that obtained from the previous fit ($0.48 \pm 0.02$). This is consistent as both the fits are done at $V \leq V_{th}$ when the occupancy is small and equation 4.4, reduces to that of equation 4.3.

An extrapolation of these fits to bias voltages $V \geq V_{th}$ however now accurately determines the current plateau values as can be seen from figure 4.13.

It is also possible to determine the occupancy for the current steps in reverse bias. For this device, (2014-A (8)), $p_0 = 0.57$ in reverse bias. Similarly a higher occupancy is observed in reverse bias from the magnetic field measurements. This asymmetry is once
Figure 4.13: I(V) characteristics of the first current step edge in forward bias of the device 2014-A (8) at different temperatures and the extrapolation of the corrected fits (equation 4.4) to these I(V) traces as obtained in figure 4.12 to voltages greater than the threshold \((V \geq V_{th})\).

again a result of the “forward-reverse” asymmetry mentioned in section 4.2 attributed to a slight asymmetry in the growth of the device. The higher \(p_0\) for reverse bias is consistent with the lower \(\alpha\) value in reverse bias and the larger magnitudes of the main resonance peak current and bias values in reverse bias as compared to forward bias. I will discuss this asymmetry in more detail when I investigate the magnetic field effects in chapter 5.

The importance of this corrected fit becomes even more apparent while investigating the thermal broadening of current steps on which the fine structure is not very prominent. In such cases the fit can be carried out over the entire range of the data unlike the case described above where it was restricted to \(V \leq V_{th}\). This is shown in figure 4.14 for the device 2015-C (32) in forward bias. From the data, \(p_0 = 0.57\), \(A = 0.56\) nA and \(\alpha = 0.42\) for this particular step. Note that doing a Fermi fit only (equation 4.3) to the data of figure 4.14 would be very inaccurate. The ratio of the current at the threshold, the common intersection point of all the curves, and the current on the plateau is \(I_{th} / I_{plateau} = 0.275\).
Figure 4.14: I(V) characteristics of the first current step edge in forward bias of the device 2015-C (32) at different temperatures and the corrected fits (equation 4.4) to these I(V) traces. The fits are done over the entire bias range spanning the step.

nA / 0.458 nA = 0.6. This is much larger than 0.5 as one would have expected if one had a perfect Fermi function behavior.

**Electron tunneling rates**

From the experimental values of $A$ and $p_0$ one can determine the electron tunneling rates $T_{em}$ and $T_{cl}$. $p_0 = T_{em} / (T_{em} + T_{cl})$ and $A = e T_{cl}$. Thus for the first current step in forward bias of device 2014-A (08) (figure 4.12), $T_{cl} = 509$ MHz and $T_{em} = 274$ MHz. This conclusively proves the assertion regarding the “forward-reverse” asymmetry that for 2014-A (8) in forward bias the emitter barrier is thicker than the collector barrier. Similarly for the first current step in forward bias of device 2015-C (32) (figure 4.14, $T_{cl} = 3.6$ GHz and $T_{em} = 4.7$ GHz. Similar measurements of the tunneling rates can also be obtained from the magnetic field studies. Those measurements however need really low temperature (less than 300 mK) and high magnetic fields. The calculation described here requires relatively moderate temperatures (greater than 1 K) and hence is important.
Summary of thermal broadening effects

I would like to summarize the discussion in this section. Thermal broadening of the current steps in the I(V) characteristics is observed which is attributed to Fermi level broadening. A Fermi function fit to the data can be done to obtain the voltage to energy conversion factor \( \alpha \). It is observed however that a Fermi fit does not accurately represent the data. It cannot account for the saturated lower current values due to the spin degeneracy and finite occupancy of the impurity state. This effect is explained by introducing another parameter, \( p_0 \), the occupancy of the impurity state. The corrected fits then predict the current broadening quite accurately.

We will see in chapter 5 that we have to take into account the finite occupancy of the impurity state once again to understand the tunneling current through the impurity in a magnetic field. One can determine the occupancy \( p_0 \) from those measurements also however that gives \( p_0 \) in the presence of a magnetic field. Those results however can be extrapolated to zero field giving for this particular current step the occupancy \( p_0 = 0.3 \) from magnetic field studies. This is in good agreement with the measured value here of \( p_0 = 0.35 \) from the temperature studies. From the measured occupancy and the plateau current magnitudes one can also determine the electron tunneling rates through the emitter and the collector barriers independently.

A single parameter \( p_0 \), the occupancy of the impurity state, thus explains the non-Fermi thermal broadening and also the asymmetry in the magnetic field behavior. These two independent measurements thus give an added proof that the theory is correct.

4.4 Magnetotunneling measurements: diamagnetic shifts and current suppression

Magnetic field is an important tool used in the investigation of transport properties of a device. This section investigates the effects of a magnetic field on the current steps in the I(V) characteristics of the devices under investigation. The Hamiltonian of the impurity
system gets modified in the presence of a magnetic field and can be written as,

\[ H = \frac{1}{2m^*}(\mathbf{p} + e\mathbf{A})^2 + V(z) + V_{imp}(\mathbf{r}), \]  

(4.7)

where \( V(z) \) is the quantum well potential, \( z \) being the growth direction, \( V_{imp}(\mathbf{r}) \) is the potential due to the impurity and \( \mathbf{A} \) is the vector potential due to the magnetic field. I ignore the spin of the electron in this section. It is considered in detail in chapter 5. Let us first consider the case of the magnetic field parallel to the current that is parallel to the growth direction of the quantum well, \( \mathbf{B} = B_z \hat{k} \). Choosing the symmetric gauge we can write,

\[ \mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r} = \frac{1}{2}(-yB_z \hat{i} + xB_z \hat{j}). \]  

(4.8)

Equation 4.7 then can be written as,

\[ H = \frac{1}{2m^*}\mathbf{p}^2 + V(z) + V_{imp}(\mathbf{r}) + \frac{1}{2m^*}eB_z L_z + \frac{1}{8m^*}e^2B^2_z (x^2 + y^2), \]  

(4.9)

where \( L_z = (xp_y - yp_x) \) is the angular momentum component along the \( z \) direction. Since the magnetic potential is small compared to the quantum well and the impurity potentials it is possible to do a perturbative analysis to determine the change in the energy, \( \Delta E \) of the system due to the magnetic field. A first order calculation gives,

\[ \Delta E = \frac{eB_z}{2m^*} < L_z > + \frac{e^2B^2_z}{2m^*} < x^2 + y^2 >, \]  

(4.10)

where \( < O > \) denotes the expectation value of an operator \( O \) in the eigenstate of the system without magnetic field. For the ground state of a single impurity one expects \( < L_z > \) to be equal to zero by symmetry and hence only the second term, called as the diamagnetic term, contributes to the energy. From the experimentally measured energy shift one can thus determine the lateral extent of the localized state wavefunction, \( < x^2 + y^2 > \).

Figure 4.15 show the forward bias I(V) characteristics in magnetic field parallel to current ranging from 0 to 9 Tesla. The traces are offset by a constant current value for clarity. Note that there is a diamagnetic movement of all steps to higher bias with the magnetic field parallel to the current. This movement can be better observed in figure 4.16 which
Figure 4.15: Current-voltage characteristics in a dilution fridge with mixing chamber temperature of 35 mK, in magnetic field (0–9 Tesla in 0.094 Tesla increments) parallel to the current for the forward bias direction of device 2014-A (8). Traces are offset by a constant current value for clarity.

plots the bias location of the current step versus the magnetic field up to 11 T. The figure shows some large spikes which are due to the “fine structure” movement in a magnetic field which will be discussed in section 4.5. A plot of the bias location versus the square of the field shows a linear behavior (not shown) as is expected if it is diamagnetic. At 11 Tesla, the diamagnetic shift is observed to be 1.4 mV. Using $\alpha = 0.48$ for this device in forward bias, one gets the diamagnetic shift to be equal to 0.67 meV. This and equation 4.10 gives the lateral extent of the localized state wavefunction, $\sqrt{<x^2 + y^2>}$, to be 42 Å. This is smaller than 100 Å which is the radius of the ground state of a hydrogenic impurity in bulk GaAs semiconductor.

The situation is more complicated in the case of the field being perpendicular to the current direction. Assuming the field to be $\mathbf{B} = B_y \hat{j}$, the Hamiltonian in this case can be written as,

$$H = \frac{1}{2m^*} \mathbf{p}^2 + V(z) + V_{imp}(\mathbf{r}) + \frac{1}{2m^*} eB_y (zp_x - xp_z) + \frac{1}{8m^*} e^2 B_y^2 (x^2 + z^2),$$  \hspace{1cm} (4.11)$$

Due to the strong quantum well confinement along the $z$ direction, one expects the dia-
magnetic term, $\frac{e^2 B^2}{8m^*} < x^2 + z^2 >$ in this case to be even smaller. The situation can be further complicated if the impurity is not centrally located in the well and then even the linear term could contribute to the change in the energy of the state and it could even be negative. Figure 4.17 shows the I(V) characteristics in perpendicular magnetic field, in forward bias at a mixing chamber temperature of 35 mK. The shift in the threshold voltage of the current steps is very small (< 0.2 mV) and is negative. Moreover the steps are also observed to split into two. This splitting is attributed to the lifting of the spin degeneracy and will be discussed in detail in chapter 5.

Apart from the diamagnetic shifts, suppression of the plateau current magnitude is observed for both bias directions in magnetic field oriented perpendicular to the current (figure 4.18). In forward bias the current gets suppressed by a factor slightly larger than 2 and in reverse bias by a factor slightly smaller than 2 when the field is perpendicular to the current. In parallel field, the current is approximately the same throughout the field range except for some fluctuations. These fluctuations are attributed to the “fine structure” on
the current plateaus and its movement in a magnetic field which will be discussed in the next section. I will discuss the cause of this current suppression in more detail later.

4.5 “Fine structure”

In this section I will discuss the “fine structure” which was mentioned earlier. Figure 4.5 shows the presence of an oscillatory structure on the current plateaus which I refer to as the “fine structure”. This structure is different for different current steps and for different devices but is reproducible for a given step even after thermal cycling of the device to room temperature. The structure is observed on devices with different barrier thicknesses implying that the relative strength of this structure to the current step magnitude is independent of the barrier thickness. This indicates that the oscillatory structure is intimately related to the single electron tunneling processes and is not due to some other additive effect. I will first present the observations showing the effects of thermal broadening and a magnetic field on this structure and then discuss the origin of this structure.

Figure 4.17: Current-voltage characteristics in magnetic field (0-9 Tesla in 0.094 Tesla steps) perpendicular to the current for device 2014-A (8) in forward bias direction.
Figure 4.18: Maximum current values for the first current step of device 2014-A (8) in forward bias as a function of perpendicular and parallel magnetic field.

4.5.1 Temperature independence of the “fine structure”

As the temperature of the device is increased, the Fermi level in the emitter broadens. This leads to the broadening of the current steps which was discussed in section 4.3. The “fine structure” however exhibits no dependence upon temperature, especially for those regions located far away from the plateau edges, as can be seen from figure 4.11 or figure 4.13. This indicates that the structure is not due to some other states in the quantum well itself, as current associated with tunneling through those states would exhibit Fermi broadening. Therefore, the “fine structure” is attributed to effects in the emitter contact. Electrons in the emitter states nearest to the Fermi level will tunnel into the impurity state first. States below the Fermi level in the emitter then contribute to the tunneling as the bias is increased. The narrow impurity state thus acts as a “spectroscopic probe” to investigate the emitter states below the Fermi level. This is qualitatively shown in figure 4.19. The current through this “probe” depends upon the local electron density of states in the emitter. The current vs. voltage characteristics thus directly translates to density of states vs. energy.
Figure 4.19: Schematics of the tunneling process through a localized state in the quantum well illustrating how the state acts as a “spectroscopic probe” to investigate the electronic states in the emitter below the Fermi level. The 3d bulk density of states in the emitter has a smooth, $\sqrt{E}$ dependence on energy upon which the fluctuations in local density of states are superimposed (solid lines). In a magnetic field Landau bands are formed (dashed line) which are also modified due to the local density fluctuations.
characteristics. The occupancy of the states at an energy $E$, below the Fermi energy $E_F$, are not affected by the temperature increase if the temperature is smaller than the energy difference $(E_F - E)$. If the emitter is a simple 3d emitter, then the density of states should show a simple, smooth, monotonic $\sqrt{E}$ dependence on the energy. The observation of the oscillatory structure on the current plateaus thus indicates that there are fluctuations in the local density of states in the emitter as a function of energy as depicted in figure 4.19.

4.5.2 Magnetic field dependence of the “fine structure”

Once again one can use the magnetic field to investigate the nature of the transport properties. Figure 4.20 shows the evolution of the I(V) characteristics in a parallel magnetic field ranging from 6 T to 9 T. It is apparent that the field dependence of the fine structure is different from that of the impurity energy states (step edge behavior).

To better view the data, fan diagrams are generated which plot the plateau edge and fine structure peak voltage positions versus magnetic field. Figures 4.21 and 4.22 show fan diagrams for the forward bias in parallel field ($B \parallel I$) and perpendicular field ($B \perp I$),
Figure 4.21: Fan diagram showing peak voltage location versus magnetic field parallel to the current for the forward bias direction.

respectively. Two lines due to the two current plateau edges are clearly visible which show only small movement with magnetic field. Between plateau edges, the fine structure peaks show distinct, systematic movement to lower voltage as the field is increased. This movement is most prominent at fields greater than 2 T. The rate of this movement is approximately 5 meV/T for $B \parallel I$ and 3.5 meV/T for $B \perp I$. Similar movement is observed in reverse bias.

One can understand this effect qualitatively in our model of the fluctuations of the local density of states in the emitter. A high magnetic field leads to the formation of Landau bands in the emitter. As the field is increased the energy of these bands increases. A fluctuation in the local density of states thus overrides the Landau bands and moves to higher energy as the field is increased. This is schematically shown by the dashed line in figure 4.19.
Figure 4.22: Fan diagram showing peak voltage location versus magnetic field perpendicular to the current for the forward bias direction.

4.5.3 Analysis of the “fine structure”

In previous work by Su and Goldman[24,67], the fine structure was observed to quench in magnetic field. This was viewed as evidence against the fine structure being attributed to a discrete density of states in the emitter. This is not the case observed for the samples studied in this work. Even at fields of up to 11 Tesla the fine structure is still strong and clearly present. In strong perpendicular field, the fine structure is observed to quench at the same rate as the current steps. Both are suppressed due to a decrease in the emitter-dot transition probability in perpendicular field, and this is the expected result if the fine structure is due to the density of states in the emitter.

Recently Schmidt et al.[56, 58] have investigated a similar fine structure in transport through impurity states in an asymmetric quantum well. The asymmetric quantum well facilitated the analysis by allowing a better “forward-reverse” asymmetry. They investigated the characteristics in field up to 17 T and could observe the fine structure movement rate with field to change with the Landau index of the band as the field is changed. This conclusively attributes the “fine structure” to the fluctuations in the emitter local density
of states. They attribute the fluctuations to interplay of Landau quantization and quantum interference of scattered electron waves at the length scale of the mean free path in the disordered bulk semiconductor.

4.6 Summary

This chapter gives a detailed introduction to the experimental system under investigation. I first described the fabrication and growth details of the samples and then presented the data for the various devices. We observe new features in the $I(V)$ characteristics of the familiar resonant tunneling diodes which were attributed to single electron tunneling through localized states in the quantum wells. A detailed statistical analysis of the observed number of features was presented which allowed the attribution of these features to single or pairs of donor impurities in the quantum well. Then I focused on the details of the nature of these localized states. An investigation of the thermal broadening of these states displayed a non-trivial effect of the spin degeneracy of these localized states even at zero magnetic field. An analysis of this effect allowed the determination of the electron tunneling rates through the two potential barriers of the device individually. I then discussed the effects of a magnetic field on these states. We observe a diamagnetic shift in the field parallel to current case and a current suppression when the field is perpendicular to the current. Finally I demonstrated using the states as spectroscopic probes to investigate the physical properties of the the emitter contacts. The narrow, localized states serve as important probes to investigate the fluctuations in the local density of states of the disordered bulk semiconductor contact.
Chapter 5

Spin splitting of single localized states

As mentioned in the introductory chapter 1, research is now focusing on the basic physical properties of discrete states. The impurity system discussed so far has certain unique features which facilitate the measurement of some important properties of the zero dimensional system. One major property is the spin degeneracy of the localized states and the lifting of this degeneracy in a magnetic field. The technique developed here has the ability to probe single states and thus observe the Zeeman splitting and the two spin states of single, localized, 0d states. This observation leads to the accurate determination of the spin $g$ factor (the Landé $g$ factor) of the conduction band electrons in the GaAs quantum well. It provides an important test of the band theory of confined semiconductor systems.

In section 5.1 the physical significance of the spin $g$ factor is discussed. In the next section, 5.2, the $\mathbf{k} \cdot \mathbf{p}$ perturbation theory is reviewed to quantitatively understand why the $g^*$ factor in bulk GaAs is different from its free space value ($g^* = -0.44$ in bulk GaAs as opposed to $\approx +2$ in vacuum). In section 5.3 the effect of electron confinement is considered which causes the spin $g^*$ factor of electrons in the quantum well to change dramatically from its bulk value and also become asymmetric as the quantum well width decreases[28, 30, 62, 63]. In section 5.4 the observations are presented. The current steps in the I(V) characteristics at low temperatures are observed to split into two in an applied magnetic
field. The voltage separation of the two fragments of this split step gives the spin energy and thus the spin $g^*$ factor. An understanding of the current through the two state spin system enables the determination of the electron tunneling rates through the two potential barriers of the quantum well device independently as discussed in section 5.5. Final section summarizes this chapter.

5.1 Introduction

The spin quantum states of elementary particles is a purely quantum mechanical phenomena with no classical analogue. The Hamiltonian of an electron in a magnetic field has an additional term,

$$H_{\text{spin}} = g_0 \mu_B \mathbf{S} \cdot \mathbf{B}, \tag{5.1}$$

where $\mathbf{S}$ is the spin of the electron, $\mathbf{B}$ is the magnetic field, $\mu_B = e \hbar / m_0$ is the Bohr magneton and $g_0$ is a constant called the spin $g$ factor of electrons. Dirac’s equation tells us that the spin of an electron is $\pm 1/2$ and $g_0$ is 2 in free space. Further experimental and theoretical research shows that this is only approximately correct and is given by a perturbative calculation in $\alpha$ in quantum electrodynamics (qed) as,

$$g_0 = 2 \left( 1 + \frac{\alpha}{2\pi} + \cdots \right), \tag{5.2}$$

where $\alpha$ is the fine structure constant. Thus the $g$ factor of electrons is a fundamental physical constant of the universe whose value depends only upon other physical constants. $g_0$ is one physical quantity whose value is accurately known and agrees with the predicted value up to many significant digits. In fact the accurate experimental determination of $g_0$ and its theoretical prediction by quantum electrodynamics (qed) has been one of the major success stories of modern day physics where theory and experiments have provided useful insight and a healthy competition for each other.

In a crystal, an electron experiences a periodic atomic potential. This potential leads to the formation of allowed energy bands with certain special symmetry points in the reciprocal space, corresponding to local extrema in the energy. An understanding of the dispersion
relations which describe the energy of the electrons around these extrema is necessary in order to understand the various opto-electronic properties of the semiconductor. There are a variety of theoretical models and approaches towards getting these dispersion relations like the tight binding model. One important technique in getting these dispersion relations has been a perturbative approach which gives these relations in terms of what is denoted as the effective mass \((m^*)\) and the effective spin \(g\) factor \((g^*)\) for the electron. An accurate experimental determination of these factors is thus important in understanding the band theory and the properties of the crystal.

In confined, low-dimensional semiconductor systems the effective spin \(g^*\) factor of conduction band electrons is of interest for various reasons. As discussed above, accurate measurements can provide a test of band theory of the confined system on par with that given by effective mass \(m^*\). It is an important parameter for interpretation of integer and fractional quantum Hall effects and it is also important in phenomena involving electron-nuclear spin coupling such as optical detection of nuclear magnetic resonance.

In the following section 5.2 first a simple case of a spin independent non-degenerate band is considered to illustrate the perturbative analysis. This analysis is then modified to tackle the case of the spin of the electron and the \(g^*\) factor. In the next section 5.3 the effects of quantum confinement on the effective \(g^*\) factor are considered.

### 5.2 \(k \cdot p\) perturbation theory in bulk GaAs

#### 5.2.1 For a single spin band in zero magnetic field

In a bulk crystal the one electron Schrödinger equation which has to be solved is (from Bastard’s book[2]),

\[
\left[ \frac{\mathbf{p}^2}{2m^*} + V(\mathbf{r}) + \frac{\hbar}{4m_0^2} (\hat{\sigma} \times \nabla V) \cdot \mathbf{p} \right] \psi(\mathbf{r}) = \epsilon \psi(\mathbf{r}),
\]

where \(m_0\) is the free electron mass, \(\hat{\sigma}\) are the spin sigma matrices and \(V(\mathbf{r})\) is the crystalline potential. The later includes some electron-electron interaction and is periodic with
periodicity of the underlying Bravais lattice.

\[ V(\mathbf{r} + \sum_i n_i \mathbf{a}_i) = V(\mathbf{r}), \quad (5.4) \]

where \( n_i \) are integers and \( \mathbf{a}_i \) are three basis vectors on the Bravais lattice. The solutions of the Schrödinger equation can be written in the Bloch form as,

\[ \psi_{nk}(\mathbf{r}) = N u_{nk} \exp(i \mathbf{k} \cdot \mathbf{r}), \quad (5.5) \]

where \( N \) is a normalization constant, \( \mathbf{k} \) is a crystal wave vector, which is restricted to the Brillouin zone of the reciprocal lattice and \( u_{nk}(\mathbf{r}) \) are periodic functions of \( \mathbf{r} \) with the periodicity of the lattice,

\[ u_{nk}(\mathbf{r} + \sum_i n_i \mathbf{a}_i) = u_{nk}(\mathbf{r}). \quad (5.6) \]

In GaAs the \( \mathbf{k} = 0 \) (\( \Gamma \) point) is a special symmetry point which has the lowest energy in the conduction band. In order to get the electronic dispersion relations for small \( \mathbf{k} \) points near the \( \Gamma \) point a perturbative analysis can be carried out. The periodic part of the Bloch functions \( u_{nk} \) are solutions of,

\[ \left[ \frac{\hbar^2}{2m^*} + V(\mathbf{r}) + \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar}{m_0} (\hat{\mathbf{s}} \times \nabla V) \right] u_{nk} = \epsilon_{nk} u_{nk}, \quad (5.7) \]

This can be formally written as,

\[ [H(\mathbf{k} = 0) + W(\mathbf{k})] u_{nk} = \epsilon_{nk} u_{nk}, \quad (5.8) \]

where \( H(\mathbf{k} = 0) \) is the crystal Hamiltonian at the high symmetry \( \Gamma \) point (\( \mathbf{k} = 0 \)) with eigenfunction and eigenenergies given by,

\[ H(\mathbf{k} = 0) u_{n0} = \epsilon_{n0} u_{n0}. \quad (5.9) \]

\( u_{nk} \) can be expanded in terms of \( u_{n0} \) as,

\[ u_{nk} = \sum_m c_m(\mathbf{k}) u_{m0}. \quad (5.10) \]
The standard perturbation analysis then gives the second order perturbative energy as,

\[ \epsilon_{nk} = \epsilon_{n0} + \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar^2}{m_0^2} \sum_{m \neq n} \frac{|\pi_{nm} \cdot k|^2}{\epsilon_{n0} - \epsilon_{m0}} \]  

(5.11)

where the vector \( \pi \) is defined as,

\[ \pi = p + \frac{\hbar}{4m_0} (\tilde{\sigma} \times \nabla V). \]  

(5.12)

This can be schematically written as,

\[ \epsilon_{nk} = \epsilon_{n0} + \frac{\hbar^2}{2} \sum_{\alpha, \beta} k_{\alpha} \frac{1}{m_n} k_{\beta}, \]  

(5.13)

where the effective mass tensor is defined to be,

\[ \frac{1}{m_n^{\alpha \beta}} = \frac{1}{m_0} \delta_{\alpha \beta} + \frac{2}{m_0^2} \sum_{m \neq n} \frac{\pi_{nm}^{\alpha} \pi_{nm}^{\beta}}{\epsilon_{n0} - \epsilon_{m0}}. \]  

(5.14)

### 5.2.2 For a spin degenerate system in a magnetic field

In order to determine the effective \( g^* \) factor of electrons in the conduction band of GaAs this treatment has to be generalized to a perturbation theory for two fold (spin) degenerate bands and include magnetic field. A convenient starting place is the Luttinger and Kohn[37] effective mass Hamiltonian in the presence of a magnetic field as described by Laura Roth[52],

\[ H_{ij} = \frac{P^2}{2m_0} \delta_{ij} + \frac{g_0 \mu_B}{2} \tilde{\sigma}_{ij} \cdot B + \frac{1}{m_0^2} \sum_{n \neq i,j} \frac{P \cdot \pi_{in} \pi_{nj} \cdot P}{E_0 - E_n}. \]  

(5.15)

Here \( P = p + eA \) where \( p \) is the momentum operator and \( A \) is the vector potential both acting upon envelope functions for the two degenerate conduction band edge over which the indices \( i \) and \( j \) run. The second term in the above equation (equation 5.15) is the electron spin interaction with the magnetic field \( B \) with \( \mu_B = e \hbar / m_0 \) being the Bohr magneton, \( g_0 \) being the free electron spin \( g \) factor \( (g_0 = 2) \) and \( s_{ij} = \sigma_{ij} / 2 \) is the matrix element of the electron spin between the band edge wave function. The index \( n \) runs over all the bands.
except the conduction band \((n \neq i,j)\). \(E_0\) is the conduction band energy and \(E_n\) is the energy of the \(n\)th band. In the third term \(\pi\) is the modified momentum operator,

\[
\pi = p + \frac{\hbar}{4m_0} \vec{\sigma} \times \vec{\nabla} V, \tag{5.16}
\]

and the matrix elements are between the respective band edge wavefunction. The diadic in the third term can be written in its symmetric and antisymmetric forms and noting that,

\[
P \times P = \frac{\hbar}{i} B, \tag{5.17}
\]

the third term in equation 5.15 becomes,

\[
\frac{1}{m_0^2} \sum_n \frac{P \cdot \{\pi_{in}, \pi_{nj}\} \cdot P}{E_0 - E_n} + \frac{\hbar}{2im_0^2} B \cdot \sum_{n \neq i,j} \frac{\pi_{in} \times \pi_{nj}}{E_0 - E_n}, \tag{5.18}
\]

where the curly bracket indicates the symmetric product. This \(2 \times 2\) matrix can be represented as,

\[
H_{ij} = \frac{1}{2}P \cdot [m^*]^{-1} \cdot P \delta_{ij} + \frac{\mu_B}{2} \sigma_{ij} \cdot g^* \cdot B, \tag{5.19}
\]

where \([m^*]^{-1}\) is a real symmetric dyadic given by,

\[
[m^*]^{-1} = \frac{1}{m_0} 1 + \frac{2}{m_0^2} \sum_n \frac{\{\pi_{in}, \pi_{ni}\}}{E_0 - E_n}, \tag{5.20}
\]

and \(g^*\) is a real dyadic given by,

\[
\sigma_{ij} \cdot g^* = g_0 \delta_{ij} + \frac{2}{im_0} \sum_n \frac{\pi_{in} \times \pi_{nj}}{E_0 - E_n}. \tag{5.21}
\]

For cubic semiconductors in bulk it can be shown that the conduction band \(m^*\) and \(g^*\) are isotropic. Hence for \(i=j=\uparrow\),

\[
g^\ast = g_\uparrow^\ast = g_0 + \frac{2}{im_0} \sum_n \frac{\pi_{n\uparrow} \pi_{n\uparrow} - \pi_{n\uparrow} \pi_{n\uparrow}}{E_0 - E_n}. \tag{5.22}
\]

It can also be shown that the contribution of the second term in equation 5.16 for \(\pi\) is negligible\([32,51]\). Thus,

\[
(m^*)^{-1} = m_0^{-1} + \frac{1}{m_0} \sum_n \frac{|<S \uparrow |p_x |n>|^2}{E_0 - E_n}, \tag{5.23}
\]

\[
g^\ast = g_0 + \frac{2}{im_0} \sum_n \frac{|<S \uparrow |p_x |n>|^2 - |<S \uparrow |p_y |n>|^2 - |<S \uparrow |p_z |n>|^2}{E_0 - E_n}. 
\]

Figure 5.1: Schematic of the band structure near $k = 0$ showing the conduction band and the valence bands.

As is usual in perturbation theory, significant contribution comes from the bands nearest to the conduction band. In a simple model called as the three band model only contribution from the valence bands is calculated as shown in figure 5.1[27]. The Bloch functions for the bands depicted above are characterized by their total angular momentum and are given by G. Bastard [2] to be,

$$\begin{align*}
|1/2, 1/2> &= i |S \uparrow>
|3/2, 1/2> &= -\sqrt{2}/3 |Z \uparrow> + \sqrt{1}/6 |(X + iY) \downarrow>
|3/2, 3/2> &= \sqrt{1}/2 |(X + iY) \uparrow>
|1/2, 1/2> &= \sqrt{1}/3 |Z \uparrow> + \sqrt{1}/3 |(X + iY) \downarrow>
|1/2, -1/2> &= i |S \downarrow>
|3/2, -1/2> &= -\sqrt{2}/3 |Z \downarrow> - \sqrt{1}/6 |(X - iY) \uparrow>
|3/2, -3/2> &= \sqrt{1}/2 |(X - iY) \downarrow>
|1/2, -1/2> &= \sqrt{1}/3 |Z \downarrow> - \sqrt{1}/3 |(X - iY) \uparrow> .
\end{align*}$$ (5.24)
Using these and their respective energies as shown in figure 5.1 equation 5.24 gives,

\[
(m^*)^{-1} = m_0^{-1} + \frac{P^2}{3m_0}(\frac{2}{E_{\text{gap}}} + \frac{1}{E_{\text{gap}} + \Delta_0}),
\]

\[
g^* = g_0 + \frac{P^2}{3}(\frac{1}{E_{\text{gap}}} - \frac{1}{E_{\text{gap}} + \Delta_0}),
\]

where \(P^2 = \frac{2}{m_0}|<S\uparrow|p_x|\Gamma_0^\uparrow>|^2 = 28.9\) eV, is the interband matrix element, \(E_{\text{gap}} = 1.52\) eV is the band gap and \(\Delta_0 = 0.34\) eV is the split off valence band separation as shown in figure 5.1. Using these known values of the above quantities for GaAs one gets \(m^*/m_0 = 0.052\) and \(g^* = -0.32\). The experimentally determined values are 0.067 and -0.44 respectively. Including the contribution of other bands in the calculation, in particular the bands right above the conduction band, gives the correct values as is shown by Hermann and Weisbuch[27].

5.3 Effect of quantum well confinement on the effective spin 
\(g^*\) factor of conduction band electrons.

There are three major physical effects that electrons in a confined quantum well experience which modify its effective \(g^*\) factor. I will discuss these effects separately and then give the results of a self consistent \(k \cdot p\) theory of Ivchenko and Kiselev [30] which accounts for all of them.

As the quantum well width decreases, the electron wavefunction penetrates more and more in the Al\(_x\)Ga\(_{1-x}\)As barriers on either side of the well. Electrons in this barrier material have a different effective \(g^*\) factor which depends upon the concentration \(x\) (\(g^* = +0.5\) for \(x = 0.35\)). Thus the effective \(g^*\) factor of electrons in the well can be written as a linear combination as,

\[
g^* = g_w^* <\Theta_w> + g_b^* <\Theta_b>,
\]

where \(g_w^*\) and \(g_b^*\) are the effective \(g^*\) factors in the well and the barrier material respectively and \(<\Theta_w>\) and \(<\Theta_b>\) are the probabilities of finding the electron in the well and the
barrier respectively.

Another major effect is the dependence of the effective spin $g^*$ factor upon the electron energy. Due to quantum confinement, the lowest energy an electron can have in the well is not the conduction band edge energy but the quantum eigenstate energy. One can consider the electron in the well to be similar to an electron in bulk GaAs with the same energy as the quantum eigenstate energy. The $k \cdot p$ analysis described in section 5.2 works best for small $k$ values. As the well width decreases the electron energy increases and other effects like the band non-parabolicity become important. The Zeeman interactive Hamiltonian in such a case is given by[30],

$$H_B = \frac{1}{2} \hbar B \left[ (g^* + h_1 k^2) \sigma B + h_2 (Bk) (\sigma k) + h_3 \sum_i k_i^2 \sigma_i B \right],$$  \hspace{1cm} (5.27)

where $h_1$, $h_2$, and $h_3$ are coefficients obtained using $k \cdot p$ perturbation theory to higher orders. In a three band model $h_3$ vanishes and $h_2$ is much smaller than $h_1$. Thus the additional contribution to the $g^*$ factor given by

$$\Delta g^* = h_{1w} k^2 < \Theta_w > - h_{1b} \kappa^2 < \Theta_b >.$$  \hspace{1cm} (5.28)

Here $k = (2m_w E_{el}^0 / \hbar^2)^{1/2}$ and $\kappa = (2m_w (V_0 - E_{el}^0) / \hbar^2)^{1/2}$ are the wave vectors of the electron in the well and the barrier corresponding to the quantum eigenstate energy ($E_{el}^0$) where $V_0$ is the barrier height.

So far only the effect of confinement on the conduction band electrons is considered. The equations 5.26 and 5.28 give corrections to the $g^*$ factor in what is called as the one band approximation. These effects do not lead to any asymmetry in the $g^*$ factor. An asymmetry is introduced by the splitting of the heavy hole and light hole valence bands in the size quantized quantum well region. In such a case $g^*_z = g^*_x \neq g^*_{1z} = g^*_{1x} = g^*_{1y}$. This can be demonstrated for the simplest structure consisting of infinite barriers by modifying the $k \cdot p$ perturbation theory slightly to obtain,

$$g^*_1 - g^*_\parallel = P^2 \left( \frac{1}{E_{gap} + E_{el}^0 + E_{hh}^0} - \frac{1}{E_{gap} + E_{el}^0 + E_{lh}^0} \right),$$  \hspace{1cm} (5.29)
Figure 5.2: Theoretical predictions of the behavior of the effective spin $g^*$ factor for an Al$_{0.35}$Ga$_{0.65}$As / GaAs / Al$_{0.35}$Ga$_{0.65}$As quantum well as a function of the well width ($L$) as discussed by Ivchenko and Kiselev[30]. The dotted line is the result of a one band calculation which ignores the asymmetry.

where $E^0_{el}$ is the size quantized energy of the electrons, $E^0_{lh}$ is the size quantized energy of the light holes and $E^0_{hh}$ is the size quantized energy of the heavy holes.

It turns out that all these three effects contribute to the net effective $g^*$ factor of electrons in the quantum well. The results of a self consistent calculation in the Kane model by Ivchenko and Kiselev[30] is given in figure 5.2. These calculations are for an Al$_{0.35}$Ga$_{0.65}$As / GaAs / Al$_{0.35}$Ga$_{0.65}$As quantum well structure.

5.4 Observation of Zeeman splitting and measurement of spin $g^*$ factor of electrons

As noted in chapter 4 the I(V) characteristics of a quantum well devices under study show current steps in the prethreshold region which are attributed to single electron tunneling through separate impurity states in the quantum well. This chapter is about the edges of these steps. As the temperature is lowered the edges become sharp. In a magnetic field they
Figure 5.3: I(V) characteristics of the first current step edge of the 85 Å barrier device ($T_{\text{mix}} = 35$ mK) in reverse bias at 0T and at 11 T under both magnetic field orientations (field parallel to current and field perpendicular to current). The three curves are given some arbitrary offset along the x-axis for clarity.

are observed to split. Figure 5.3 shows an expanded view of one such current step with and without a magnetic field when the sample is in a dilution fridge with the base temperature of the fridge ($T_{\text{mix}}$) being 35 mK. At zero field, the ground state of the impurity is spin degenerate leading to a single current step. Upon lifting of the degeneracy at finite field, a splitting of the current step is observed. Note that the splitting is not the same for the two different field orientations. This splitting is observed in all devices, and for all the current steps in a given device, investigated at low temperatures. Figure 5.4 shows the I(V) for the same device at higher bias showing six different current steps. Each of those steps are observed to split in a magnetic field. I report the detail analysis of the spin $g^*$ factor for two different devices with different barrier thicknesses (85 Å and 65 Å). For the 85 Å device (2014-A (8)), the splitting was investigated in both forward and reverse bias and for both the magnetic field orientations (parallel and perpendicular to current). The 65 Å device (2015-C (32)) only in the field perpendicular to current orientation in forward bias was
Figure 5.4: I(V) characteristics of the 85 Å barrier device at $T_{mix} = 35$ mK showing six different current steps in zero field (dashed line) and with a field of 9 T oriented perpendicular to current (solid line).

Figure 5.5 shows the evolution of the I(V) characteristics upon increasing magnetic field. Note that the spin splitting increases as the field is swept from 0 T (bottom) to 9 T (top).

Figure 5.6 shows the voltage separation between the corresponding two spin-split conductance peaks which increases linearly with the magnetic field strength as expected. Due to the finite widths of the conductance peaks, it is not possible to resolve the splitting for magnetic fields less than 5.5 T. The best line fits to the data (figure 5.6) closely intersects $\Delta V = 0$ at $B = 0$ and has a slope $g^* \mu_B/\alpha$, where $\mu_B$ is the Bohr magneton, $g^*$ is the effective spin $g$ factor of the impurity and $\alpha$ is the voltage to energy conversion factor[14, 15,22,61,67]. The observed spin splitting is different for the two field orientations. $g^*_\perp$ and $g^*_\parallel$ denote the effective $g$ factors for the field perpendicular and parallel to the current flow direction respectively.

To determine the spin $g^*$ factor it is necessary to know the voltage to energy conversion factor, $\alpha$. $\alpha$ is accurately determined by studying the thermal broadening of the current.
Figure 5.5: I(V) characteristics ($T_{mix} = 35$ mK) of the first current step edge in forward bias of the $85$ Å barrier device in increasing magnetic fields, (field perpendicular to current), from 0 T (bottom) to 9 T (top). The successive field curves are offset a constant amount along the y-axis for clarity.

Figure 5.6: The experimental spin splitting versus magnetic field for the first current step of the $85$ Å barrier device for the different bias and magnetic field orientations at $T_{mix} = 35$ mK. The solid lines are linear fits to the data.
step as discussed in chapter 4. The fits are done for data taken at different temperatures, from 0.5 K to 5 K, and \( \alpha \) is determined for each. The average value of \( \alpha \) along with the root mean square (rms) error is reported in table 5.4 The fits are done for the region \( V \leq V_{ch} \) when the current is small and not affected by the occupancy of the impurity state.

From these values of \( \alpha \) and the measured slopes of the fittings in figure 5.6, the absolute magnitude of the spin \( g_\perp^* \) and \( g_\parallel^* \) values of the impurities can be calculated. Table 5.4 lists the measured \( g^* \) values for the various devices and current steps. Note that the slopes of the two lines in figure 5.6 for the first step in forward and reverse bias are different but so are the corresponding \( \alpha \) values giving us the same \( g^* \) values. This asymmetry in the slopes is another example of the “forward-reverse” asymmetry discussed in chapter 4 (section 4.2.1) which is attributed to slight difference in the barrier thicknesses of the devices on 2014-A-A. It is however quite satisfying to note that the \( g^* \) value does not depend upon this asymmetry and is measured to be the same in both forward and reverse bias. For the device with 85 Å barrier the \( g_\perp^* \) factor is determined for two separate impurity steps in both forward and reverse bias directions. \( \alpha \) value in this device is not determined for the second current step independently. I assume that the \( \alpha \) value for the second step to be the same as that of the

<table>
<thead>
<tr>
<th>device name and current step</th>
<th>barrier thickness</th>
<th>( \alpha ) +ve bias</th>
<th>( \alpha ) -ve bias</th>
<th>( g_\perp^* ) +ve bias</th>
<th>( g_\perp^* ) -ve bias</th>
<th>( g_\parallel^* ) +ve bias</th>
<th>( g_\parallel^* ) -ve bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014-A (08) first step</td>
<td>85 Å</td>
<td>0.48 ± 0.02</td>
<td>0.42 ± 0.02</td>
<td>0.28 ± 0.02</td>
<td>0.28 ± 0.02</td>
<td>0.14 ± 0.04</td>
<td>0.13 ± 0.04</td>
</tr>
<tr>
<td>2014-A (08) second step</td>
<td>85 Å</td>
<td>-</td>
<td>-</td>
<td>0.28 ± 0.02</td>
<td>0.27 ± 0.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2015-C (32) first step</td>
<td>65 Å</td>
<td>0.40 ± 0.02</td>
<td>-</td>
<td>0.27 ± 0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.1: \( \alpha \) and \( g^* \) values as calculated for the various devices and current steps.
Figure 5.7: Comparison of the measured $g^*$ factors with the theoretical predictions of Ivchenko and Kiselev[30] assuming that the sign of the measured quantities is positive.

As discussed earlier, $g^*$ is theoretically predicted to be a strong function of the quantum well width and changes from the bulk value of $-0.44$ to greater than $+0.4$ for well widths less than $30$ Å and also becomes asymmetric[30]. Figure 5.7 shows the data with respect to the above theoretical predictions of Ivchenko[30]. The measurements are consistent with this prediction and with recent experimental results[28,62,63]. It is not possible to determine the sign of the $g^*$ factors and I have assumed here that it is positive. Note that the predictions are made for quantum wells with $\text{Al}_x\text{Ga}_{1-x}\text{As}$ barriers with $x = 0.35$ while the measured data is on quantum wells with $\text{Al}_x\text{Ga}_{1-x}\text{As}$ barriers with $x = 0.27$. I expect the effect of this difference in Al concentration to be small. Also note that strictly speaking the measured $g^*$ factor is that of an electron bound to a Coulomb potential in the quantum well and not that of free electrons in the quantum well. Also note that the current steps under investigation in this chapter have a high binding energy and hence these are attributed to
pairs of impurities in the quantum well as discussed in chapter 4. I expect the effect of this on the value of $g^*$ to be small as the impurity states are shallow states.

So far the spin splitting of the electrons in the emitter is neglected. Naively one may expect that observed splitting of the current steps is due to the difference in the spin splitting of the emitter electrons and the impurity state. This is not the case as the localized impurity state is an individual, isolated state while the emitter is a 3-d sea of electrons with a continuum of occupied electron states. All electron levels in the emitter split in an applied magnetic field but due to the continuum, at any given energy electrons of both spin orientations are available for tunneling. The spin splitting energy ($\approx 0.15$ meV at 10 T) is much smaller than the Fermi energy in the emitter ($\approx 40$ meV), and hence any spin polarization effects in the emitter would not be observed. Figure 5.8 shows thermal broadening of the first current step edge split at 9 T. Separate Fermi broadening of the two fragments of the split step indicates that in both cases electrons involved in tunneling are the Fermi electrons in the emitter. The observed experimental voltage difference is thus entirely due to the spin splitting of the impurity state only, and is not affected by the $g^*$-factor of the electrons in the emitter. If the Fermi energy in the emitter is small and
weakly localized states are formed in the emitter, the $g^*$-factor of these weakly localized states would have to be taken into account as reported by Sakai et al.[54].

Also note that electron-electron interactions will not alter the relative energy of the two spin states and thus the measured $g^*$ factor in this experiment. There are two possible mechanisms of electron-electron interactions. One is between the electrons in the well and other is between the tunneling electron and the electrons in the emitter. The first mechanism can be ignored as at the bias of interest, there is very little electron density in the well. I would like to mention that transport studies in 2-d electron gases show an enhancement in $g^*$ factor of electrons which is attributed to electron-electron interactions. But such an enhancement is not observed when single electron energy levels of the 2-d electron gas are probed as was shown by Dobens et al.[17]. In this experiment there is no 2-d electron gas in the quantum well and also single electron states are probed. The second mechanism, the Coulomb interaction between the tunneling electron and the electrons in the emitter, causes a rise in the step current near the threshold at low temperatures, which is observed (as first reported by Geim et al.[22]), but it should not affect the energy separation of the two spin states.

### 5.5 Electron tunneling rates

Analysis of the tunneling current through a system yields information about the tunneling rates through the system. This experiment provides a unique opportunity to investigate a two state tunneling system which gives more information than just a single state tunneling system. Figure 5.9, shows that the two fragments of the spin-split step at 9 T are observed not to have the same current magnitude ($I_2 - I_1 \neq I_1$). Where $I_1$ and $I_2$ mark the current values as shown. $I_1$ gives the current of the first fragment while $I_2$ is the net current of both fragments of the split step edge. This difference is more prominent in reverse bias. An understanding of this asymmetry gives us the electron tunneling rates through each of the two barriers of the double barrier heterostructure individually.

This analysis follows a discussion similar to that in section 2.4.4 of the theory chapter 2.
<table>
<thead>
<tr>
<th>Forward Bias</th>
<th>Reverse Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>$I_1$</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$I_2$</td>
</tr>
</tbody>
</table>

Figure 5.9: I(V) characteristics at $T_{mix} = 35$ mK of the first current step edge of the 85 Å barrier device in forward bias (left) and reverse bias (right) at 0 T (dashed line) and 9 T (solid line). The magnetic field is oriented perpendicular to the current direction. $I_1$ and $I_2$ mark the current values at 9 T as shown. $I_1$ gives the current of the first fragment while $I_2$ is the net current of both fragments of the split step edge.

I define $T_b$ and $T_t$ to be the tunneling rates for an electron to tunnel through the bottom and the top (referred to growth direction) potential barriers, respectively, of the double barrier heterostructure. Also define $T_{cl}$ and $T_{em}$ to be the electron tunneling rates through the collector (downstream) and the emitter (upstream) (referred to electron flow direction) potential barriers. In forward bias, electron injection is through the top barrier into the quantum well ($T_{em} \equiv T_t$, $T_{cl} \equiv T_b$), while in reverse bias, electron injection is through the bottom barrier into the quantum well ($T_{em} \equiv T_b$, $T_{cl} \equiv T_t$). I also define $p$ to be the occupation probability for an electron in a localized state in the quantum well. In the sequential tunneling picture, $p = f T_{em} / (T_{em} + T_{cl})$. Since the spin splitting energy ($\Delta E \leq 150 \mu eV$) is much smaller than the barrier potential energy ($\approx 300$ meV), I assume that the tunneling rates are the same for electrons tunneling through the spin up or the spin down states irrespective of the slight energy difference. I also assume that the emitter electrons are not spin polarized which is a good assumption since the Fermi energy ($\approx 40$ meV) is much larger than the spin splitting energy even at 10 T. In high magnetic fields and at low temperatures, when the Fermi level is sharp, (figure 5.8), it is possible to adjust
Figure 5.10: Schematic of electron tunneling through the two state system.
the bias near a given impurity to have the following two conditions as noted schematically in figure 5.10. At lower bias, \( V_1 \), only the lower energy spin state channel is active for conduction, and the current is given by,

\[
I_1 = p e T_d.
\]  

(5.30)

At a higher bias, \( V_2 \) the higher energy spin state channel is also active for conduction, and the current is given by

\[
I_2 = p' e T_d = (2p - p^2) e T_d,
\]  

(5.31)

where \( p' = (2p - p^2) \) is the probability of occupying either the lower or the higher state, but not both of them. Both states cannot be occupied at the same time due to the large Coulomb energy required for another electron to simultaneously occupy the second state. For this system the single electron Coulomb charging energy (\( U_C = e^2/2C \) where \( C \) is the effective capacitance of the double barrier device) is much larger than the spin splitting energy \( \Delta E \). The capacitance of the system can be estimated to be,

\[
C = \varepsilon_0 k \pi r_0^2 (d_t^{-1} + d_b^{-1}),
\]

where \( k \) is the dielectric constant, \( d_t \) and \( d_b \) are the top and bottom barrier thicknesses and \( r_0 \) is the radius of the lateral region affected by the localized impurity state in the quantum well. Taking \( r_0 \) as the Bohr radius of hydrogenic impurities in GaAs (\( \approx 100 \) Å) gives \( U_C \approx 9 \) meV. Although taking \( r_0 \) as the Bohr radius is only an approximation, even taking \( r_0 \) seven times larger gives \( U_C \geq \Delta E \).

In equations 5.30 and 5.31 I have assumed that the tunneling rates are the same at the two different biases \( V_1 \) and \( V_2 \) which is a good assumption since the bias difference (\( V_1 - V_2 \)) is much smaller than the barrier potential. In the extreme limits these equations indicate that for \( T_d \gg T_{em} \), \( p \approx 0 \) and \( I_2 \approx 2I_1 \) while for \( T_d \ll T_{em} \), \( p \approx 1 \) and \( I_2 \approx I_1 \). This qualitatively explains the behavior observed in figure 5.9. To get a quantitative understanding, equations 5.30 and 5.31 can be solved, using the experimentally measured \( I_1 \) and \( I_2 \), to determine \( p \).
5.5.1 Tunneling rates in a magnetic field

Magnetic field perpendicular to current

For the rest of this chapter is confined to the discussion of the results of only one device, the device with 85 Å barrier and data from the first current step only. The results of the other device (65 Å barrier) show similar trends. The data shown in figure 5.9 at 9 T (field perpendicular to current), gives $p = 0.21$ for forward bias and $p = 0.62$ for reverse bias. A high $p$ value indicates that the electron tunneling rate through the collector (downstream) barrier ($T_{cl}$) is lower than that through the emitter (upstream) barrier ($T_{em}$) causing an accumulation in the well. A higher $p$ value for reverse bias (as compared to forward bias) suggests an asymmetry in the heterostructure growth with one barrier being slightly thicker than other barrier. In forward bias the top barrier is the emitter barrier while in reverse bias the top barrier is the collector barrier. This implies that the top barrier is slightly thicker than the bottom barrier. This is consistent with the discussion in chapter 4 where I first refer to this asymmetry which I call the “forward-reverse” asymmetry and is in agreement with previous characterization[45,46].

It is possible to obtain the absolute magnitude of the electron tunneling rates through the two potential barriers and study their dependence upon the magnetic field. Figures 5.11 and 5.12 show the tunneling rates as a function of the magnetic field perpendicular to the current in forward bias and reverse bias orientations respectively. Note that $T_{em}$ is smaller than $T_{cl}$ in forward bias orientation while $T_{cl}$ is smaller than $T_{em}$ in reverse bias orientation. This is because $p$ is smaller than 0.5 in forward bias while it is larger than 0.5 in reverse bias. As mentioned before this asymmetry suggests that the top barrier of the heterostructure is slightly thicker than the bottom barrier.

Despite the asymmetry, note that in either orientation $T_{em}$ decreases with field strength while $T_{cl}$ is approximately constant. $T_{em}$ roughly decreases by a factor of two as the field increases from 6 T to 11 T in both bias orientations. The probability of occupation $p$ also decreases as the field strength increases. The oscillations in the tunneling rate are probably
Figure 5.11: Tunneling rates $T_{em}$ and $T_{cl}$ as a function of the magnetic field strength perpendicular to current for the 85Å barrier device in forward bias orientation.

Figure 5.12: Tunneling rates $T_{em}$ and $T_{cl}$ as a function of the magnetic field strength perpendicular to current for the 85 Å barrier device in reverse bias orientation.
due to the fine structure observed on the current plateaus and its movement in magnetic field[13] which is discussed in chapter 4. This fine structure does get suppressed as the field strength increases. The suppression of $T_{em}$ results in the observed plateau current suppression with magnetic field. Sakai and coworkers[53] have proposed a model to account for this current suppression. Assuming a 3d emitter I can write the electron wavefunctions as a plane waves in the $x$ and $y$ directions. In their model, current as a function of voltage and magnetic field perpendicular to current is given by,

$$J(V, B) = -\frac{2em^*}{\pi \hbar^3} \int_{k_0 - k_c}^{k_0 + k_c} \frac{|M(k_y, k_v)|^2}{k_v} dk_y. \quad (5.32)$$

The value $M(k_y, k_v)$ is determined primarily by the overlap integrals between the emitter and dot wave functions, and therefore this model is qualitatively similar to this case. The value $k_c$ depends upon applied bias, and can range from $k_F$, the Fermi wavevector, to 0. $k_v$ is the value of $k_x$ determined from energy conservation for a given value of $k_y$, $k_v^2 = k_c^2 - (k_y - k_0)^2$. In this model, $k_0$ is the separation of the dispersion relationships between the emitter and the dot, given by $eB\Delta S/\hbar$, where $\Delta S$ is the separation between the emitter and dot states.

Figures 5.11 and 5.12 and the above discussion indicates that the magnetic field affects the emitter-to-well tunneling process ($T_{em}$) more than it affects the well-to-collector tunneling process ($T_{d}$). I therefore expect more current suppression when the current is emitter barrier limited than when it is collector barrier limited, which is observed (figure 5.9). For forward bias (thicker emitter) the current at 9 T is suppressed by a factor of 41% (as compared to 0 T), while the suppression is only 20% for reverse bias (thicker collector).

Magnetic field parallel to current

Figures 5.13 and 5.14 show the tunneling rates as a function of the magnetic field parallel to the current in forward bias and reverse bias respectively. Note that again $T_{em}$ is smaller than $T_{d}$ in forward bias orientation while $T_{d}$ is smaller than $T_{em}$ in reverse bias orientation.
Figure 5.13: Tunneling rates $T_{em}$ and $T_{cl}$ as a function of the magnetic field strength parallel to current for the 85Å barrier device in forward bias orientation.

Figure 5.14: Tunneling rates $T_{em}$ and $T_{cl}$ as a function of the magnetic field strength parallel to current for the 85Å barrier device in reverse bias orientation.
There is no decrease in $T_{em}$ with field strength in this orientation. This is expected because with the field being parallel to the current (along $z$) it does not have any substantial effect on the $\psi(z)$ part of the electron wavefunction and thus does not affect the overlap between impurity wavefunction and the emitter electron states. The oscillations in the tunneling rate due to the fine structure are more prominent in this field orientation as the fine structure does not get suppressed.

### 5.6 Summary

This chapter focusses on the spin degeneracy of the localized states. In a magnetic field this degeneracy is lifted causing the current steps in the $I(V)$ characteristics to split. An accurate measurement of this splitting gives the Landé $g^*$ factor of the conduction band electrons in the quantum well. I first discuss the significance of the Landé $g$ factor in semiconductors and then outline the $k\cdot p$ perturbation theory that gives the effective $g^*$ factor in semiconductors. I then discuss the change in the $g^*$ factor due to the quantum confinement. An analysis of the tunneling current through this two-state system allows determination of the electron tunneling rates through the two potential barriers independently. I also investigate the tunneling rates as a function of the magnetic field strength. It is observed that in magnetic field perpendicular to current case the emitter barrier tunneling rate decreases as the field strength increases.
Chapter 6

Observation of Random Telegraph Signal in tunneling through single localized states.

So far in this thesis only the dc, time independent, phenomena were investigated. This chapter reports some of the time varying features observed in single electron tunneling through the impurity channels. It is observed that in some devices, at a given fixed bias near the threshold of the current steps, the current fluctuates with time. The current is observed to undergo discrete transitions between two values, a high current value and a low current value, in a random fashion. This fluctuation is termed as a Random Telegraph Signal (RTS). Similar type of signal is observed in many different experimental systems and is a major source of 1/f noise is sub micron MOSFETs. In the following section 6.1 I will first review the observations of RTS in other systems and note its characteristics. In the next section 6.2 I will present the observations and discuss the similarities and differences in the characteristics of our observations and those in other systems. In section 6.3 I propose a “two state model” which quantitatively explains the observed dependence of the RTS amplitude on bias and temperature. In section 6.4 I will discuss the plausible models which could explain the observations. Finally I will summarize this chapter.
6.1 Introduction

Recently much research interest has been generated in the study of non equilibrium transport in a variety of mesoscopic systems. Dekker et al. and Wybourne et al report discrete current or resistance switching (random telegraph signal (RTS)) in quantum point contacts at biases corresponding to the onset of higher subbands[8,76,77]. Similar switching has been observed in planar quantum dots and in quantum dot arrays[72,18]. The real cause of such switching in the different systems is not completely understood. It has been shown that discrete resistance fluctuations, (RTS), in sub-micron Silicon MOSFETs (and a few other systems) are due to changes in the charge states of single traps located near the conducting region[6,38,43,50]. But similar phenomena do not appear to explain the switching in the above mentioned systems.

In this chapter I report the observation of RTS in the system involving single electron tunneling through single, localized states. This system is different from the ones mentioned above. It is also unique in that the RTS switching current amplitude shows a dramatic increase as the temperature is decreased whereas the characteristic times of the switching events do not have any dependence upon the temperature. The study of RTS in this new experimental system may be useful in the understanding of RTS in other systems as well as in understanding the tunneling process itself.

6.2 Observation of RTS in impurity system

In order to investigate time dependent phenomena the experimental data acquisition set up was modified as has been discussed in chapter 3. The dc I(V) characteristics are obtained by using a dc amplifier to monitor the voltage across a sense resistor in series with the device while slowly ramping the voltage across the series combination of the device and the sense resistor. The ac fluctuating current is monitored by similarly ramping the voltage but using a wideband (~ 100 kHz) ac amplifier (with a low frequency cutoff less than 2 Hz) to monitor the voltage across the sense resistor. The time traces and the spectrum of the
random telegraph signal is obtained by applying a fixed bias and monitoring the fluctuating voltage across the sense resistor on an HP 3561 spectrum analyzer/oscilloscope.

Figure 6.1(top) shows the dc I(V) characteristics for a particular device at low temperatures ($T_{\text{mix}} = 35$ mK) in forward bias. This device is 2015-C (32). The barrier thicknesses in this device are 65 Å. The (32) indicates that the lateral size of this device is 32 μm and its area is 1024 μm$^2$. The current-voltage (I(V)) characteristics shows sharp current steps similar to those observed in other devices. Figure 6.1(bottom) shows the ac rms current through the device as the bias voltage across it is ramped up slowly. This is not the derivative of the dc I(V) nor does the applied bias have any alternating component. This fluctuating current is measured using an amplifier with a low frequency cut off (less than 2 Hz) which thus removes the dc component of the current as the bias across the device is ramped up slowly. Note that the ac current is observed only at biases corresponding to the onset of the dc current steps. The background current level in the ac trace of figure 6.1 is due to the noise in the electronic measurement set up. This noise level makes it difficult to investigate the ac behavior. Though I have studied only one device with these barrier thicknesses at
Figure 6.2: dc and ac characteristics of the reverse bias second current step. Note that the ac signal is not symmetrical about the dc step.

low temperatures (in a dilution fridge), such an ac signal is observed at the threshold of all observed impurity steps (total 5 steps) in this device in both forward and reverse bias. Such a signal could not be detected in other devices investigated at low temperatures (85 Å barrier devices). The dc current step magnitudes in those devices are approximately 10 times smaller than those in the 65 Å barrier devices and the present experimental set up would not have enabled the detection of the ac signal in those devices.

It is important to investigate and understand the exact bias location where the ac signal is observed relative to the dc current step. Figure 6.1 shows that for most steps the ac signal is present over the entire edge of the dc current step. The signal is most prominent for the second step in both forward and reverse bias. It is so strong that it can even be seen in the dc traces. Figure 6.1 shows that in forward bias the ac signal is almost symmetrically located relative to the dc step. In reverse bias for the second step, one observes the ac signal not to be symmetric about the dc current step (figure 6.2). The ac signal appears in the prethreshold region at the bottom of the dc current step and no ac signal is observed at the top of the dc current step.

In general it is not possible to identify the different current steps in the forward and
reverse bias and say whether they are due to the same impurity state or not. In this case however, a strong ac signal is observed only for the second dc current step in both forward and reverse bias. One can thus identify these two steps and say that they are due to the same localized state in the quantum well exhibiting themselves in forward and reverse bias orientations.

These relative bias locations of the ac and the dc signals and the difference in the forward bias and the reverse bias, is a reproducible effect. The bias locations are found to be the same after numerous thermal cycling of the device over a period of more than a year. Similar relative bias locations are observed in the presence of a magnetic field. In a magnetic field applied perpendicular to the current direction, the threshold of the dc current step is observed to move to a lower bias location. The ac signal also appears to move along with it. The dc current step edge is observed to split into two due to the lifting of the spin degeneracy of the localized state. In reverse bias one can clearly see that the ac signal is present at the bottom of both the fragments of the split current step. This indicates that the ac signal is directly related to the dc current step and hence to the single electron tunneling process.

6.2.1 RTS Characteristic Times

If one applies a constant bias across the device, at biases where the ac signal is observed, the current is observed to switch between two discrete levels. The time between the switching appears to be random and hence it is called a Random Telegraph Signal (RTS). Figures 6.3 shows the RTS switching current at biases corresponding to the first two dc current steps in forward bias. Figure 6.4 shows the RTS signal at biases corresponding to the second dc current step in reverse bias. It is convenient to define two characteristic times to quantify the switching events. The on time, \( \tau_{on} \), is defined to be the characteristic time the current stays high before switching back to low state. The off time, \( \tau_{off} \), is defined to be the characteristic time the current stays low before switching back to high state. One notes that the characteristic switching times for the two localized states in forward bias are quite
Figure 6.3: The time dependence of the current at fixed bias voltages in forward bias at $T_{mix} = 35$ mK. The top curve is at 90.00 mV corresponding to the second current step in the dc I(V) characteristics, while the bottom curve is at 85.85 mV corresponding to the first current step. The two curves are vertically offset for clarity.

Figure 6.4: The time dependence of the current at a fixed bias voltage of 93.15 mV corresponding to the second dc current step in reverse bias at $T_{mix} = 35$ mK.
different. The RTS for the first step (bias = 85.85 mV) is mostly off, $\tau_{on}$ being less than 1 mSec while $\tau_{off}$ being a few tens of mSec. On the other hand the RTS for the second step (bias = 90.00 mV) is equally on/off with the characteristic times being of the order of 15 mSec. The second step in reverse bias however has entirely different switching time characteristics than the second step in forward bias. Also note that the RTS switching amplitudes are approximately the same for the two different impurities and in forward and reverse bias. The measured ac signal, as shown in figures 6.1 and 6.2, is a root mean square (rms) signal which depends upon the amplitude as well as the characteristic time scales of the fluctuating signal. Thus one can see that the ac signal in figure 6.1 is higher for the second state than that for the first state because of the different characteristic switching time behavior and not because of any slight difference in the switching amplitudes. The other steps (not shown in figure 6.1) show characteristics similar to those of the first step. Because of the strong signal observed for the second step (figure 6.1) it is easier to study this step in more detail and for the remaining part of this chapter I will confine myself to it. Qualitatively similar behavior is observed for the other states.

If the two level switching is a perfectly random process then the on and off times should have a Poisson distribution,

$$P(t) = \frac{1}{\tau} \exp(-t / \tau),$$

(6.1)

where $\tau$ is the characteristic switching time which also happens to be the average switching time. Figures 6.5 and 6.6 shows histograms of such on and off times at a fixed forward bias voltage. Exponential fits to these histograms are also shown which give the characteristic on and off times $\tau_{on}$ and $\tau_{off}$. A good exponential fit to the histogram indicates that the switching is truly random and obeys the Poisson distribution.

Similar histograms for reverse bias at a fixed voltage of 93.15 mV corresponding to the second dc step are shown in figures 6.7 and 6.8.

Table 6.1 lists the switching times for different closely spaced bias locations corresponding to the same dc current step in forward bias. This table indicates that within the experimental error the characteristic times do not change with bias within the small bias
Figure 6.5: Histogram (symbols) of the on times at a fixed bias voltage of 89.9 mV in forward bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives $\tau_{on} = 14$ mSec.

Figure 6.6: Histogram (symbols) of the off times at a fixed bias voltage of 89.9 mV in forward bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives $\tau_{off} = 17$ mSec.
Figure 6.7: Histogram (symbols) of the on times at a fixed bias voltage of 93.15 mV in reverse bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives $\tau_{on} = 2$ mSec.

Figure 6.8: Histogram (symbols) of the off times at a fixed bias voltage of 93.15 mV in reverse bias corresponding to the threshold of the second current step in dc characteristics. The solid line is an exponential fit to this histogram which gives $\tau_{off} = 91$ mSec.
Table 6.1: Switching times for different closely spaced bias locations corresponding to the second dc current step in forward bias.

<table>
<thead>
<tr>
<th>bias mV</th>
<th>$\tau_{on}$ mSec</th>
<th>$\tau_{off}$ mSec</th>
</tr>
</thead>
<tbody>
<tr>
<td>89.9</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>90.0</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>90.1</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>90.2</td>
<td>14</td>
<td>18</td>
</tr>
</tbody>
</table>

range in which the RTS is observed. We can approximate $\tau_{on} \approx 14$ mSec and $\tau_{off} \approx 17$ mSec.

The frequency spectra of such a signal is a Lorentzian centered around zero as first discussed by S. Machlup[38].

$$S(f) = \frac{4(\Delta I)^2}{(\tau_{on} + \tau_{off})^2[(1/\tau_{on} + 1/\tau_{off})^2 + (2\pi f)^2]} \quad (6.2)$$

Figure 6.9 shows the power spectrum obtained at three different biases for the RTS corresponding to the second impurity step in forward bias. Bias voltage of 90.00 mV corresponds to the peak in the ac rms signal (figure 6.1). Bias of 89.85 mV and 90.23 mV are on either side of this peak. The spectra show the expected behavior of 20 dB per decade roll off beyond the knee frequency which is around 21 Hz. The knee frequency is defined to be the frequency at which the power is 3 db lower than the power at low frequencies. Note that the knee frequency does not change for the spectra taken at the three different bias locations but only the amplitude changes. Further I would like to note that the knee frequency obtained from the spectra (figure 6.9) is consistent with that calculated from the histogram of the time traces (table 6.1) according to equation 6.2,

$$f_{knee} = \frac{1}{2\pi} \left( \frac{1}{\tau_{on}} + \frac{1}{\tau_{off}} \right) = 20.5 Hz. \quad (6.3)$$
RTS is observed at the onset of tunneling through a localized state, \textit{i.e.} when the state is close to the Fermi level. Figure 6.9 and table 6.1 indicate that the characteristic on and off times do not change as bias is changed within the narrow region of the onset. Thus $\tau_{on}$ and $\tau_{off}$ are not dependent upon the relative position of the localized state with respect to the Fermi level. This observation is contrary to the observations in MOSFETs\cite{6,43} and in quantum point contacts\cite{76,77} where the characteristic times change dramatically as the traps (for MOSFETS) or the quantum subbands (for point contacts) are swept through the Fermi level.

Further studies indicate that the characteristic times do not depend upon the temperature either. The power spectrum taken at a given bias at different temperatures (from 0.1 K to 4.2 K) shows the same knee frequency though with decreasing amplitudes. This observation is contrary to the observation of thermally activated behavior of the characteristic time scales in MOSFETs.
The observed ac signal is strongly bias dependent as can be seen from figures 6.1 and 6.2. The signal peaks at a certain bias and decays on either side of that bias. Similar conclusion can be reached by observing either the time traces or the spectra at different bias locations. The RTS switching amplitude for a given tunneling channel is observed to depend strongly upon the bias as can be seen from figure 6.9. The spectrum at 90.00 mV forward bias (bias where the ac rms signal peaks (figure 6.1)) has maximum power and it is less at biases on either side of 90.00 mV.

The amplitude also depends strongly upon the temperature. Figure 6.10 shows the temperature dependence of the RTS switching amplitude at a fixed bias of 90.00 mV. The RTS switching amplitude drops dramatically as the temperature increases. This strong temperature and bias dependence of the switching amplitude is contrary to the observations in MOSFETs.
6.3 Two state model for RTS amplitude

In order to understand the strong dependencies of the RTS amplitude on temperature and bias I propose a model which I refer to as the “two state model”. This model is schematically explained in figure 6.11. The current through a localized state in the quantum well at bias V and temperature T depends upon the density of occupied electronic states in the emitter, at the same energy as the impurity state, which is proportional to the Fermi distribution function \( f \). \( I(V, T) = I_0 f(V - V_{th}, T) \), where \( I_0 \) is the dc current step height and \( V_{th} \) is the threshold bias at which the dc current step is observed. The current step exhibits characteristic Fermi level broadening which is observed as discussed in chapter 4[14, 15, 22]. I propose that observed RTS is due to two closely spaced (in energy) tunneling levels. These levels are related to each other and electrons can tunnel through only one of them at a give time. RTS is observed due to random switching from one tunneling channel to another. The RTS switching current can be modeled as,

\[
\Delta I(V, T) = I_0[f(V - V_b, T) - f(V - V_a, T)] = I_0[f(V - (V_{th} - \Delta V), T) - f(V - (V_{th} + \Delta V), T)].
\]

(6.4)

Here \( I_0 \) is the dc current step height and \( V_a \) and \( V_b \) are the threshold voltages of the two closely spaced levels separated by \( 2\Delta V \).

The observed dc signal would be a weighted average of the current through the two channels B and A. The weight for each channel is given by the characteristic time that channel is active. The characteristic on time, \( \tau_{on} \), corresponds to the characteristic time channel B stays active before switching to channel A and similarly for \( \tau_{off} \) which corresponds to channel A. If the characteristic on and off times are approximately equal then the observed dc signal would be the numerical average of the current through the two channels individually and the observed ac signal would be symmetrically located about the observed dc current step edge. This is the case in forward bias where \( \tau_{on} \) and \( \tau_{off} \) are approximately equal and the observed ac signal is symmetrically located in bias about the dc current step edge (figure 6.1). In reverse bias on the other hand \( \tau_{on} \ll \tau_{off} \). The observed dc current
Figure 6.11: Schematic of the “Two state model” for the observed Random Telegraph Signal. I propose the existence of two closely spaced tunneling levels and claim that the observed RTS is due to random switching between these two tunneling levels.
would be essentially equal to the current due to channel A only as the weight of channel B is negligible. The observed ac signal would thus be asymmetrically located in bias relative to the dc current step. The ac signal would be observed in the prethreshold region at the bottom of the dc step but not at the top of the step. This is precisely the case as is seen in figure 6.2. Thus this model accounts for the observed asymmetry in reverse bias too.

A plot of equation 6.4 as a function of bias at a few different temperatures is shown in figure 6.12. We can see that this figure 6.12 qualitatively explains the observed ac rms signal in figure 6.1. The switching amplitude is maximum when $V = V_{th}$ and the amplitude is lower at biases on either side of $V_{th}$. A quantitative fit of equation 6.4 to the data is not possible in this form because the ac signal as measured in figure 6.1 also depends upon the characteristic switching times while the “two state model” does not detail the switching times. However one can do a quantitative fit to the temperature dependence of the switching amplitude.

For $V = V_{th}$ equation 6.4 reduces to,

$$
\Delta I(V_{th}, T) = I_0 \tanh(\alpha \Delta V / 2kT).
$$

(6.5)
Here $\alpha$ is the voltage to energy conversion factor which is determined from the thermal broadening of the first dc current step in forward bias to be 0.4 as discussed in section 4.3[15]. $I_0$ is the dc current step height which is determined from figure 6.1, and $k$ is Boltzmann's constant. A fit of this equation to the data is shown in figure 6.10. $\Delta V$ is the only free parameter in this fit and it is determined to be 0.11 mV.

We can thus see that the “two state model” and equation 6.4 qualitatively explains the bias dependence and is in good quantitative agreement to the temperature dependence of the RTS switching amplitude in forward bias. This model also explains the asymmetry observed in reverse bias regarding the bias location of the ac signal not being symmetrically located about the dc current step. I can thus say with confidence that the “two state model” is a true representation of observation of RTS in this system and all other models that try to understand the RTS switching behavior will have to be in the domain of the “two state model”.

This model however does not say anything about the actual origin of the two closely spaced states nor about the cause of the switching or about the actual switching times.

### 6.4 Plausible models for the observation of RTS in the impurity system.

As mentioned above a model that explains the observation of RTS in this system has to be in the domain of the “two state model”. It has to explain what are the two closely spaced energy states and what causes the system to randomly switch from one to other. I categorize the possible models into two classes one “external models” and the other “internal models”. I define the class of external models to contain those models where the mechanism of switching is separate from the single electron tunneling process. Similarly the class of internal models contain models where the mechanism of switching is directly coupled to the single electron tunneling process.
6.4.1 “external models”

One possible candidate in this class is the presence of a deep level or a trap near the single electron tunneling channel. A random change in the charge state of this trap could change the energy state of the tunneling channel by a small amount in a random fashion. This is the physical mechanism behind the observation of RTS in MOSFETs. This model however does not explain many of the observations. First of all the fact that RTS is observed in both forward and reverse bias indicates that such a trap would have to be present in the well region and not either of the two barriers. The life time of any state in the well is dominated by the strong quantum well potential and hence it is unclear why such a trap state would have a life time of the order of mS when the quantum state (and the tunneling channel states) have a life time of less than nS. It is thus not clear why such a trap state in the well would behave different from any normal tunneling state.

Another problem with this model is the fact that it requires the presence of a tunneling state and a separate trap state in the close vicinity of each other which can occur only by chance. RTS is observed at the threshold of all the current steps in the device. The different steps are attributed to different localized states which could be physically separated by a large distance. Thus a different trap state is required to explain the RTS for all the different steps. Probabilistically it is not very likely to have a trap state near each of the tunneling states.

Hence one is forced to explore the possibility of an internal model.

6.4.2 “internal models”

In order to get a plausible internal model for RTS switching, one has to first try and understand the experimental impurity system in more detail. Figure 6.1 shows that the bias locations of the dc current steps are around 90 mV. The turn on voltage in this device is 170 mV and the $\alpha$ factor is 0.4. Thus these states have a binding energy of about 32 meV. These low bias current steps thus cannot be due to single impurity Coulomb potential. It is proposed (as discussed in section 2.4.2) that such high binding energy states are due to pairs
Figure 6.13: Coulomb repulsion energy of two singly charged impurities in a 44 Å quantum well, electronic energy of an electron in the potential of these two impurities and the total energy of the impurities-electron system as a function of the impurity separation, $R$.

of impurities. It was also shown in section 4.2.3 that such an assignment of the high binding energy current steps in the I(V) characteristics to pairs of impurities in the quantum well is statistically reasonable.

Figure 6.13 shows the binding energy for an electron in the potential of two impurities located in the quantum well, as a function of the impurity separation $R$. This figure also shows the Coulomb repulsion energy between the two impurities and the total energy of the combined system of two impurities and one electron. The figure shows that it is possible to achieve higher binding energy states up to 50 meV in the experimental system under investigation. One can also see that a pair of impurities with binding energy of 32 meV would have an impurity separation, $R$ approximately 250 Å.

6.4.3 A concrete “internal model”: Impurity pair relaxation and RTS

In this section I discuss a concrete “internal” model which suggests a microscopic mechanism that generates two closely spaced (in energy) tunneling channels and it also suggests why
there would be switching between them. The basic assumption in this model is that the RTS is observed only at the onset of the low bias current steps which are attributed to tunneling through electronic states formed by the Coulomb potential of a “pair” of impurities. The two impurities in the pair are separated by a distance $R$ and are rigidly held in place by the lattice forces. When an electron is not present along with the two impurities, these impurities experience the mutual Coulomb repulsion force. They thus have a tendency to move as far away from each other as the lattice distortion would allow and $R$ is the maximum possible. When an electron is present along with the two impurities, it screens the mutual Coulomb repulsion of the impurities. As can be seen from figure 6.13, the total energy of the system is actually lower for smaller $R$. There is thus a preference for the system to achieve a lower $R$ as much as the lattice would allow. The binding energy of the electron in the impurity pair system is a function of the impurity separation, $R$. For this model to be consistent with the “two state model” as discussed in sec 6.3 the electronic binding energy difference between the two states has to be $\approx 0.1 \text{ meV}$ as was experimentally determined in that section.

From figure 6.13 we can see that if the binding energy $E_b$ is around 32 meV then $R \approx 250 \text{ Å}$. Also from the slope of the electronic energy curve in that figure we can estimate that in order to achieve a binding energy difference $\Delta E_b \approx 0.1 \text{ meV}$ the change in the impurity separation $\Delta R$ will have to be $\approx 1 \text{ Å}$. This small $\Delta R$ is conceivable given that the lattice spacing in GaAs is 5.35 Å.

Let us now discuss the process of switching. I denote $R_a$ and $R_b$ ($R_a > R_b$) to be the two possible separations between the two impurities (figure 6.14). Initially there is no electron tunneling through the state and hence we expect the separation to be $R_a$ and the pair to be in state 1 as shown in figure 6.14. As the bias across the device is increased electrons start to flow through the well and the state of the pair changes rapidly and randomly form 1 to 2 and then back to 1 as the electron tunnels in and out of the well. The average tunneling time is about 0.5 nS and hence the pair maintains its state for a time of that order. When the pair is in state 2 there is a tendency for the system to relax to the smaller impurity
Figure 6.14: The energy of the impurity pair system with and without an electron as a function of the impurity separation, \( R \). When there is no electron, the energy of the system is just the Coulomb repulsion energy of the two singly charged impurities. When there is an electron present in the state, the Coulomb repulsion gets screened and the total energy of the impurities-electron system is as shown (from figure 6.13). \( R_a \) and \( R_b \) mark the two possible separations between the two impurities. 1,2,3,4 mark the four possible states of the impurity pair with or without electron and with separation \( R_a \) or \( R_b \) ((\( R_a-R_b \)) is shown to be large in this figure for clarity).
separation \( R_0 \) and thus to state 3. Since state 2 corresponds to a higher impurity separation and hence a lower electronic binding energy (figure 6.13) than state 3, the current through state 2 is lower than the current through state 3 as the current is proportional to the Fermi distribution function in the emitter. Thus switching from state 2 to state 3 corresponds to switching from the lower current state to the higher current state or from the off state to the on state. The electrons then keep tunneling in and out of the well thus the state of the pair changes randomly and rapidly from 3 to 4 and then back to 3. When the pair is in state 4 it has then a tendency to relax back to state 1 with a separation of \( R_a \). This tendency would cause the system to randomly flip back to off state. This switching from on to off and then back to on does not occur each time an electron tunnels through the system. That would be a very fast switching as opposed to the observed switching times of \( \approx 15 \) msec. The tunneling current, \( I \), is \( \approx 0.3 \) nA which implies that on an average there is 1 electron passing through the system in a time interval \( \tau_{av} \) of \( 5 \times 10^{-10} \) sec. We propose that though there is a tendency to switch states each time an electron passes through the system it takes a finite time for the system to change from state 2 to state 3 or from state 4 to state 1. Also there is a finite energy difference between these two states which has to be passed on or taken from some source. Hence the system relaxes only once in a while with a time constant \( \approx 15 \) msec, and not each time an electron tunnels through it.

**Poisson tunneling process and impurity pair relaxation**

The exact mechanism for this relaxation is still under investigation and a topic of debate. In this subsection I present one possible mechanism. This mechanism is based on the assumption that tunneling through the system is a sequential process. Electrons tunnel through the emitter into the well and then tunnel out from the well into the collector in two independent processes. From the discussion in chapter 5 we know that there is a finite occupancy \( p \), for the electron within the state in the well. This \( p \) depends upon the relative tunneling rates of the two potential barriers (emitter barrier, \( T_{em} \) and collector barrier \( T_{cl} \)) as \( p = T_{em} / (T_{em} + T_{cl}) \). We assume that each of the two tunneling processes is a Poisson
Figure 6.15: Schematic representation of the Poisson tunneling process. We can see that the probability for the tunneling event not to occur for a time $\tau_0$ exponentially decreases as the time $\tau_0$ increases.

The time interval between two successive tunneling events in a Poisson process has an exponential distribution as given for the incoming electron (emitter to well) process by,

$$P(t) = T_{em} \exp(-t \tau_{em}),$$

and similar for the outgoing electron (well to collector) process,

$$P(t) = T_{cl} \exp(-t \tau_{cl}).$$

Initially the pair has a separation $R_a$. It is proposed that every time an electron resides in the well for a long enough time ($t \geq \tau_0$), the system will relax from state 2 to state 3. Figure 6.15 depicts a Poisson tunneling process. From equation 6.7 we can see that the probability for an electron to reside in the well longer than $\tau_0$ is given by $\exp(-\tau_0 T_{cl})$. Since the average rate of electron flow is given by $1/\tau_{av}$, one expects the rate of flipping from 2 to 3 to be given by $\frac{1}{\tau_{av}} \times \exp(-\tau_0 T_{cl})$. As mentioned earlier switching from state 2 to state 3 corresponds to switching from the lower current state to the higher current state. The average time the current stays low, which is denoted as the off time, thus corresponds
to the average time the current stays in state 1 or 2 and is given by,

$$\tau_{off} = \tau_{av} \exp(\tau_0 T_d)$$  \hfill (6.8)

This calculation assumes that the supply of the electrons into the well is not limited by the emitter tunneling process. In other words it assumes that the probability of occupation, \(p\), is close to 1. If the impurity state is occupied only a fraction of the time then flipping can occur only during the fraction it is occupied. Thus if \(p\) is small then the flipping times will increase and will be given by,

$$\tau_{off} = (\tau_{av} / p) \exp(\tau_0 T_d).$$  \hfill (6.9)

Similarly one can get an expression for the on time when the current is high

$$\tau_{on} = (\tau_{av} / (1 - p)) \exp(\tau_0 T_{em}),$$  \hfill (6.10)

which corresponds to the time the system is in state 3 or 4 before it flips back to state 1. From the experimentally measured values of \(\tau_{on} \sim 15\) mS, \(\tau_{off} \sim 15\) mS and \(\tau_{av} \sim 0.5\) nS and assuming \(p = 0.5\) implying \(\tau_{em} = \tau_d = \tau_{av}/2\) we can estimate \(\tau_0\) to be approximately 4.4 nS. This suggests that the impurity pair has to maintain its state for a long enough time (compared to \(\tau_{av}\)) in order for it to flip.

These equations are derived under the assumption that the Fermi distribution function \(f\) in the emitter is close to 1. If that is not the case then the probability of occupation and the tunneling rates get modified. One can see that the incoming tunneling rate gets modified from \(T_{em}\) to \(f T_{em}\) and if \(f\) is small the inflow of electrons is small. The outgoing tunneling rate gets modified from \(T_d\) to \(T_d + (1 - f)T_{em}\) as now the electron in the well can tunnel either to the collector or back to the emitter where there are available empty states if \(f\) is small. Thus the average times are given by,

$$\tau_{off} = (\tau_{av} / f p) \exp(\tau_0 (T_d + (1 - f)T_{em})), $$  \hfill (6.11)

and

$$\tau_{on} = (\tau_{av} / (1 - fp)) \exp(\tau_0 f T_{em}).$$  \hfill (6.12)
One clearly sees an exponential dependence of these two times upon the Fermi distribution function, \( f \). This strong dependence upon \( f \) is not observed and hence this specific model of relaxation is incomplete.

**Possible changes to the Poisson tunneling model of pair relaxation**

It is clear from the discussion in the above subsection that an impurity pair relaxation based on the direct Poisson tunneling model needs to be modified. That model ignores correlation effects in tunneling due to the finite occupancy of the localized state. It is however not plausible that those effects would lead to a correction strong enough to overcome the strong exponential dependence of the times on \( f \).

The Poisson relaxation mechanism is based on the assumption that each time the state is occupied (or unoccupied) for a time longer that a specific time, \( \tau_0 \), the system relaxes. Our observations suggests that such a simple picture is probably not true. The relaxation is probably much slower than is described above. This is possible if there is a potential barrier in between the two states with pair separation \( R_a \) and \( R_b \) which also has to be overcome in order for the system to relax. At low temperatures the system possibly relaxes across the barrier, from \( R_a \) to \( R_b \) (and other way round), quantum mechanically. This quantum mechanical relaxation process would be independent of the temperature as is observed. At higher temperatures however one may expect there being thermally assisted, over the barrier relaxation. One thus expects to observe a transition from quantum relaxation to thermal relaxation as the temperature is increased. Such transitions have been observed in other systems\(^7\). To verify this conjecture it is necessary to determine the RTS switching times at temperatures higher than 4.2 K. That is however difficult to do in this system as the RTS amplitude decreases sharply as the temperature increases.

### 6.5 Summary

In this chapter we investigated some time dependent features in single electron tunneling through the impurity localized states. A two level fluctuating signal (RTS) was observed
at the threshold of tunneling through the localized states. The characteristics amplitude of this fluctuation was observed to be strongly dependent upon the bias and the temperature of the system. A “two state model” was proposed which quantitatively explains this dependence. This model however provides no insight into the actual switching mechanism. The characteristic switching times of the RTS are observed to be independent of the bias and the temperature of the system. I introduce two classes of plausible models, internal and external models and discuss the merits and problems of each. The external models are unlikely to explain the RTS due to the symmetry of the observations and the likelihood of such occurrences. I propose a concrete internal model- the pair relaxation model which explains most of the observations. A specific form of this model “pair relaxation based on direct Poisson tunneling” predicts a strong dependence of the characteristics times of RTS upon the bias. This dependence is however not observed and hence this model requires some modifications.
Chapter 7

Conclusions

With the advent of epitaxial crystal growth techniques and the advances in nanoscale patterning technology, the design and implementation of nanometer scale systems that exhibit new and interesting classical and quantum mechanical effects has become possible. One research field that has generated major physics interests has been the field of single electron transport systems. As the system size reduces, new classical (charge quantization) and quantum mechanical (energy quantization) effects become important. From a technological point of view also this field has gained significance as a possible alternative technology for smaller and faster electronic devices. The present day semiconductor technology and the scaling of the transistor is some day going to reach its physical limits and search is on for an alternate technology to go beyond that. A variety of experimental systems have been conceived and realized and now investigation is focusing on exploring the true properties of the discrete electronic states and the microscopic nature of single electron transport through these localized states.

This thesis dissertation introduces a new, clean and simple single electron tunneling system. This system consists of semiconductor donor impurities in the quantum well regions of a resonant tunneling diode. The coulomb potential of these impurities results in the formation of three dimensionally confined, discrete energy states bound to the quantum well eigenstate. This system is thus physically similar to a quantum dot system. The simplicity of this system, its 3d-0d-3d nature, facilitates the investigation of the basic properties of
the localized states and the tunneling phenomena.

The important property of the localized states upon which this thesis concentrates is the spin-degeneracy of the states. In a magnetic field this degeneracy is lifted leading to the observation of spin-splitting. This is a simple, theoretically expected behavior but is significant as spin-splitting has not been observed previously in quantum dot systems and it has become a topic of debate. In horizontal quantum dots, the situation is complicated due to the presence of a relatively large number of electrons in the dot which leads to strong electron-electron interaction\[39\]. In vertical quantum dots however, where the number of electrons in the dots is small, one does expect to observe spin splitting under proper experimental conditions. Other groups investigating similar impurity systems (Sakai et al.)\[54\], also did not observe a simple, monotonic spin-splitting of the impurity states. One major advantage that the experimental system under investigation in this thesis possesses is its having simple 3d contact electrodes with large Fermi energy. The contact electrodes in vertical quantum dots are quasi 1d while those in the devices investigated by Sakai et al.\[54\] are quasi 2d with low Fermi energy and possibly having localized emitter states. Thus the non-observation of simple spin-splitting in these other systems can now be attributed to additional complications in the contact electrodes.

This spin degeneracy has a non-trivial effect on the transport properties of the device even at zero magnetic field. The two-fold spin degeneracy does not just add a factor of two. The finite occupancy of the electron in the discrete state causes a saturation of the tunneling current. This saturation effect exhibits itself in magnetotunneling measurements at high fields and also in the thermal broadening characteristics at zero field. Introduction of a single new parameter, “the occupancy, \(p\)”, explains both the magnetotunneling and the thermal broadening observations.

The two-state spin system imparts valuable information about the properties of the semiconductor device which a single-state system cannot. An analysis of the tunneling current through the two-state system facilitates the determination of the electron tunneling rates across the two potential barriers in the device individually. An investigation of
the splitting between the two spin states in an applied magnetic field allows the accurate
determination of the spin $g^*$ factor of electrons in the confined quantum well region. An
accurate determination of $g^*$ is important for the band theory of confined regions and this
system allows the investigation of the asymmetry of $g^*$ with respect to the magnetic field
being either parallel or perpendicular to the confined quantum well region.

Finally this thesis focuses on the actual nature of the single electron tunneling process
itself. A random telegraph signal is observed associated with the threshold of tunneling
conduction through the discrete states. This signal is attributed to random fluctuations in
the energy of the localized state induced by the presence or absence of an electron in the
localized state. Further investigation of this effect is necessary to completely understand
this phenomena.

Finally I would like to say that the “devices” under investigation in this thesis are not
real devices. I do not foresee any real applications for this particular system. One of the
most important properties of real or future “single electron devices”, from the technology
perspective, is their small lateral dimensions which would facilitate high packing density.
The system studied in this thesis is of samples with large lateral dimensions. The significance
of this system however lies in the fact that it is physically similar to the vertical
quantum dot system which has the potential of being a real device system. The simplicity
of the impurity system allows one to focus on the basic physical properties of the localized
states and on the microscopic nature of tunneling itself. The observation of spin-splitting,
determination of $g^*$ factor and the tunneling rates, investigation of fluctuations in the tun-
neling process, and a probe of the true nature of the disordered bulk semiconductor contacts
are physics issues which are important not only for any single electron system but for the
field of semiconductors itself. The techniques introduced in this thesis can be generalized
to other measurements of interest such as $g^*$ measurements in different heterojunction ma-
terial systems, the determination of tunneling rates for bandgap engineered structures of
interest and the investigation of magnetic impurities.
Bibliography


