Classification and Kinematic Equivalents of Contact Types for Fingertip-Based Robot Hand Manipulation

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In the context of robot manipulation, Salisbury's taxonomy is the common standard used to define the types of contact interactions that can occur between the robot and a contacted object; the basic concept behind such classification is the modeling of contacts as kinematic pairs. In this paper, we extend this notion by modeling the effects of a robot contacting a body as kinematic chains. The introduced kinematic-chain-based contact model is based on an extension of the Bruyninckx–Hunt approach of surface–surface contact. A general classification of nonfrictional and frictional contact types suitable for both manipulation analyses and robot hand design is then proposed, showing that all standard contact categories used in robotic manipulation are special cases of the suggested generalization. New contact models, such as ball, tubular, planar translation, and frictional adaptive finger contacts, are defined and characterized. An example of manipulation analysis that lays out the relevance and practicality of the proposed classification is detailed. [DOI: 10.1115/1.4032865]

1 Introduction

The modeling of contacts between fingers and objects has been a recurring theme in robot manipulation [1]; in fact, contact mechanics is a fairly standard topic not only in multibody abstractions but also in continuum mechanics (see, for instance, Ref. [2]). In the context of robot hands, the types of contact interactions which can occur between a grasped object and a fingertip are usually classified in nine categories, namely: *(i) no contact, (ii) point contact without friction, (iii) line contact without friction, (iv) point contact with friction, (v) planar contact without friction, (vi) line–line contact without friction, (vii) soft finger (or compliant surface contact), (viii) line(-line) contact with friction, and (ix) planar contact with friction*. The description and kinematic equivalents of all these contact types are presented in Table 1. A *kinematic equivalent* simply corresponds to a single kinematic constraint—a subset of the continuous group of displacements—that represents the constrained motion between two contacting bodies. A complete list of subgroups of displacements can be found in Ref. [3].

The above contact taxonomy was introduced by Salisbury in his groundbreaking work on the kinematics and force analysis of mechanical articulated hands [4, Chap. 2], with the exception of the line–line contact without friction entry that was later presented by Tischler et al. in Ref. [5]. Since the contact types *(iii), (v), (vi), (viii), and (ix)* are not suitable for arbitrary objects—they depend on the assumptions on the curvature properties of the surface of the object, the contact models considered in robot manipulation are often reduced to *frictionless point contact*[2], *(ii)*, *point contact with friction (iv)*, and *soft finger*[3] *(vii)*. This is the standard in the specialized literature on robot manipulation [6,7].

The basic concept behind the Salisbury's contact taxonomy is the modeling of contacts as kinematic pairs. This assumption is clearly stated by Salisbury and Roth in Ref. [8]: "In considering the effects of a finger contacting a body it will be useful to model each contact as a kinematic pair." However, this fundamental supposition has been frequently omitted in important discussions about contact modeling in robot manipulation—e.g. [7, Chaps. 27 and 28], with some relevant exceptions [1 (p. 86)], [9], probably because it was not explicitly indicated in Salisbury's Ph.D. thesis [4]. The identification of such assumption is relevant for three reasons: first, to clarify the kinematic origin of the standard contact models used in grasp analysis; second, to clarify that closed kinematic chains (e.g., spatial linkages, parallel platforms) have been employed to model within-hand prehensile manipulation since the dawn of the research field; and third, to open the door to a more general axiom that yields a richer and more insightful classification of contact types for the analysis of fingertip-based robot hand manipulation and the design of robot hands. This work focuses on the development of this last aspect.

In this paper, we extend the standard modeling of contacts as kinematic pairs used in dexterous manipulation by modeling the effects of a fingertip contacting a body as kinematic chains. The introduced contact model is based on an extension of the Bruyninckx–Hunt approach of surface–surface contact [10,11]. Thanks to such extension, a general classification of nonfrictional and frictional contact types suitable for both robot hand manipulation analyses and robot hand design is proposed. It is shown that all standard contact categories used in robotic manipulation, namely, Salisbury's taxonomy [4] along with the line–line contact without friction presented in Ref. [5], is obtained as special cases of the proposed generalization. Additionally, new contact models, such as ball, tubular, planar translation, and frictional adaptive finger contacts, are characterized and their kinematic equivalents are defined via Hervé's group-theoretic approach [12].

The suggested kinematic-chain-based contact model—the extended Bruyninckx–Hunt approach—is advantageous for classifying, defining, and characterizing contact types for robot manipulation because, in contrast to kinematic-pair-based contact models, it considers the curvature of contacting bodies to determine unequivocally the location and direction of the resulting degrees-of-freedom. This allows differentiating contact conditions that under traditional practices are considered same. Additionally, contrary to the original Bruyninckx–Hunt model, the proposed contact model considers not only passive revolute joints but also resistant passive joints (i.e., passive joints able to resist moments till some value). This feature generalizes the characterization of friction used in modeling approaches based on kinematic pairs, simplifies contact specialization via changes in the assumed

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[2]The point contact without friction, as originally proposed by Salisbury, actually assumes a curvature model of the grasped object (see Table 1).
[3]Soft finger is the historical name used for the contact model that idealizes a point contact that deforms to have a contact area large enough to resist moments about the contact normal.

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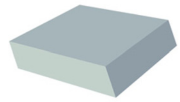
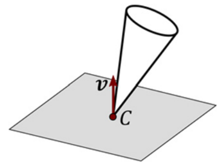
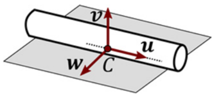
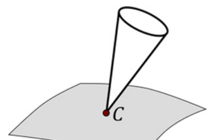
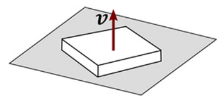
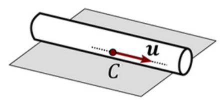
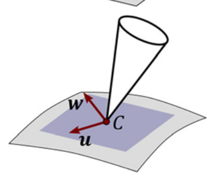
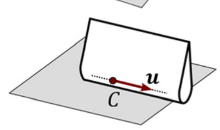
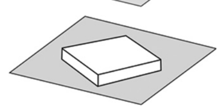
Table 1 Standard classification of contact types in robot manipulation

Contact type	Description	Kinematic equivalent	
(i) No contact	Free object in $\mathbb{E}^3$ (six degrees-of-freedom)		$\{\mathbf{D}\}$
(ii) Point contact without friction	A point on the fingertip is constrained to move on a frictionless plane surface of the grasped object (five degrees-of-freedom)		$\{\mathbf{S}(C)\}\cdot\{\mathbf{P}(\boldsymbol{v})\}$
(iii) Line contact without friction	A line in the fingertip is constrained to move on a frictionless plane surface of the grasped object (four degrees-of-freedom)		$\{\mathbf{C}(C,\boldsymbol{u})\}\cdot\{\mathbf{R}(C,\boldsymbol{v})\}\cdot\{\mathbf{T}(\boldsymbol{w})\}$
(iv) Point contact with friction	A point on the fingertip is constrained to move on a point of the grasped object (three degrees-of-freedom)		$\{\mathbf{S}(C)\}$
(v) Planar contact without friction	A plane surface of the fingertip is constrained to move on a frictionless plane surface of the grasped object (three degrees-of-freedom)		$\{\mathbf{G}(\boldsymbol{v})\}$
(vi) Line–line contact without friction	A line in the fingertip is constrained to coalesce with a line in the grasped object (two degrees-of-freedom)		$\{\mathbf{C}(C,\boldsymbol{u})\}$
(vii) Soft finger	A point on the fingertip is constrained to move on a point of the grasped object without rotation about the axis determined by the contact point and the unit normal vector of the contact tangent plane (two degrees-of-freedom)		$\{\mathbf{R}(C,\boldsymbol{u})\}\cdot\{\mathbf{R}(C,\boldsymbol{w})\}$, $\boldsymbol{u}$ and $\boldsymbol{w}$ define the contact tangent plane
(viii) Line(-line) contact with friction	A line in the fingertip is constrained to coalesce with a line in the grasped object without sliding (one degree-of-freedom)		$\{\mathbf{R}(C,\boldsymbol{u})\}$
(ix) Planar contact with friction	A plane surface of the fingertip is constrained to lie on a plane surface of the grasped object without relative motion (zero degree-of-freedom)		$\{\mathbf{I}\}$

motion constraints, and facilitates the computation of kinematic equivalents in complex contact types.

The rest of this paper is organized as follows. Section 2 introduces the Hunt's and Bruyninckx's kinematic-chain-based contact models and presents their corresponding kinematic equivalents. Section 3 details the proposed extension of the Bruyninckx–Hunt approach as well as the suggested classification of nonfrictional and frictional contact types. Section 4 discusses an example of fingertip-based robot hand manipulation that stands out the relevance and practicality of the introduced classification in manipulation studies and robot hand design. We finally conclude and present lines of future work in Section 5.

2 Kinematic Equivalents of Contact Models

2.1 Hunt's Kinematic Equivalent. Kinematic pairs are a basic constituent of any mechanism; hence, the natural generalization of Salisbury's axiom for the development of contact types is to model each contact as a kinematic chain. This assumption was presumably first proposed by Hunt in Ref. [10, p. 334] when discussing kinematic models for the connection of two rigid bodies touching at a single point. Hunt's model corresponds to three serially connected passive joints that provide exactly five degrees-of-freedom. Specifically, this kinematic model is built as follows (Fig. 1 (left)): take any two points, say A and B, on the line defined by the point of contact C and the unit normal vector between the contacting bodies whose boundaries can be represented, in the neighborhood of C, by the surfaces Φ_A and Φ_B. Then, perpendicularly connect in series perfectly aligned Hooke couplings (universal joints) at points A and B with a revolute joint at the point of contact in the direction of the common normal line between the bodies and rigidly connect the universal joints to Φ_A and Φ_B as shown in Fig. 1 (left). The resulting mechanism is a universal-revolute-universal chain of four links that provides five independent degrees-of-freedom of motion between the touching bodies.

Applying serial kinematic reduction to the described kinematic chain to obtain a single equivalent kinematic constraint between the bodies, we get, according to the notation of Fig. 1 (left),

$$\{\mathbf{R}(A,\boldsymbol{u})\}\cdot\{\mathbf{R}(A,\boldsymbol{w})\}\cdot\{\mathbf{R}(C,\boldsymbol{v})\}\cdot\{\mathbf{R}(B,\boldsymbol{w})\}\cdot\{\mathbf{R}(B,\boldsymbol{u})\}. \quad (1)$$

Now, since $\{\mathbf{R}(C,\boldsymbol{v})\}=\{\mathbf{R}(A,\boldsymbol{v})\}$–$(C,\boldsymbol{v})$ and $(A,\boldsymbol{v})$ define the same axis, $\{\mathbf{R}(C,\boldsymbol{v})\}=\{\mathbf{R}(B,\boldsymbol{v})\}$, $\boldsymbol{u},\boldsymbol{v}$, and $\boldsymbol{w}$ are linearly

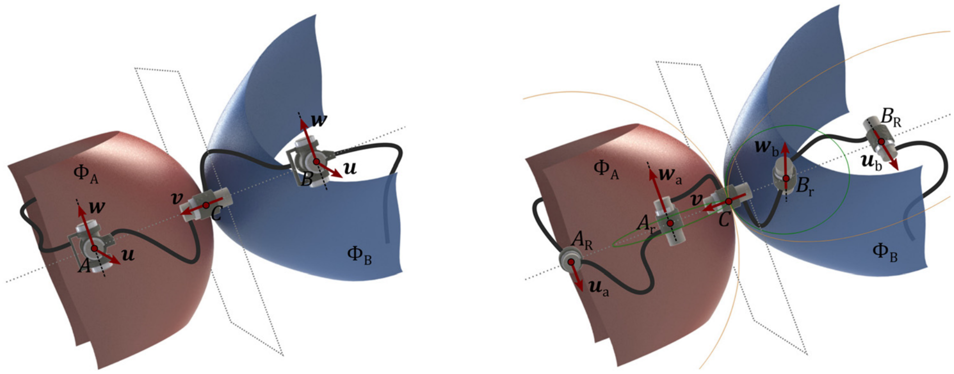


Fig. 1 Left: Hunt's kinematic-chain-based model of point contact without friction and its kinematic equivalent. Right: the Bruyninckx's kinematic-chain-based model of point contact without friction—called herein the Bruyninckx–Hunt model—and its kinematic equivalent.

independent vectors, and A and B are arbitrary points along the common normal line, Eq. (1) can be rewritten as

$$\begin{aligned}\{\mathbf{S}(A)\}\cdot\{\mathbf{S}_2(B)\} = \{\mathbf{S}(C)\}\cdot\{\mathbf{S}_2(B)\} = \{\mathbf{S}_2(A)\}\cdot\{\mathbf{S}(B)\}\\ = \{\mathbf{S}_2(A)\}\cdot\{\mathbf{S}(C)\}\end{aligned} \tag{2}$$

with $\{\mathbf{S}(O)\} = \{\mathbf{R}(O,\boldsymbol{i})\}\cdot\{\mathbf{R}(O,\boldsymbol{j})\}\cdot\{\mathbf{R}(O,\boldsymbol{k})\}$—by the property of closure of groups—provided that $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are linearly independent vectors [13] and $\{\mathbf{S}_2(O)\} = \{\mathbf{R}(O,\boldsymbol{u})\cdot\{\mathbf{R}(O,\boldsymbol{v})\}$ a submanifold included in $\{\mathbf{S}(O)\}$, defined as the composition of two different subgroups of rotations whose axes meet at a single point [14,15].

Equation (2) corresponds to the kinematic equivalent (i.e., single kinematic constraint relating two contacting bodies) of the Hunt's kinematic-chain-based model of point contact without friction. Note that the selected directions of the vectors $\boldsymbol{u}$ and $\boldsymbol{w}$ are indeed irrelevant, a consequence of the use of Hooke couplings in the model regardless of the geometry of the touching objects. Hunt's model seems to be a more general approach than that of Salisbury (see Table 2). However, since in the Hunt's model the direction of vectors $\boldsymbol{u}$ and $\boldsymbol{w}$ does not affect the description of the motion between the bodies and the location of points A and B is not explicit, the whole Salisbury's taxonomy cannot be deduced from it. Most relevant, no new contact types can be obtained from such approach. Despite all these drawbacks, the Hunt's model is still matter of research [16]. A similar kinematic-chain-based model of point contact was introduced by Montana in Ref. [9].

2.2 Bruyninckx's Kinematic Equivalent. Independently from Hunt's work and in the context of compliant robot motion, Bruyninckx et al. [11] proposed a kinematic-chain-based model of point contact that logically resolves all the weaknesses of the Hunt's approach; it can be indeed seen as an extension of such perspective. The Bruyninckx's model, which we call the generalized kinematic-chain-based model of point contact, or the Bruyninckx–Hunt model, consists of five serially connected passive revolute joints whose location and direction are unequivocal. The model is described next.

According to the notation of Fig. 1 (right), given two bodies touching at a single point C with Φ_A and Φ_B being smooth surfaces representing their boundaries in the neighborhood of the contact point (i.e., Φ_A and Φ_B has the same tangent plane at C), let

- A_r and A_R (correspondingly B_r and B_R) be the centers of maximum and minimum curvature of Φ_A (Φ_B), respectively,
- $\boldsymbol{u}_\mathrm{a}$ and $\boldsymbol{w}_\mathrm{a}$ (correspondingly $\boldsymbol{u}_\mathrm{b}$ and $\boldsymbol{w}_\mathrm{b}$) be the directions of maximum and minimum curvature of Φ_A (Φ_B), respectively, and
- $\boldsymbol{v}$ a unit normal vector defining the common contact tangent plane.

A convenient introduction to the curvature theory of surfaces can be found in, e.g., Ref. [2, Chap. 2]. From elementary differential geometry, it is known that $\boldsymbol{u}_\mathrm{i}$, $\boldsymbol{w}_i$, and $\boldsymbol{v}$ ($i = \mathbf{a}, \mathbf{b}$) are orthogonal with directions uniquely defined unless the surface is locally a sphere or a plane[5].

The generalized kinematic-chain-based model of point contact without friction is then built by serially connecting revolute joints at points A_r, A_R, B_r, B_R, and C in the direction determined by vectors $\boldsymbol{u_a}$, $\boldsymbol{w_a}$, $\boldsymbol{u_b}$, $\boldsymbol{w_b}$, and $\boldsymbol{v}$, respectively, and rigidly connecting the revolute joints at A_R and B_R to Φ_A and Φ_B as shown in Fig. 1 (right). The resulting mechanism is a five-revolute chain of six links that provides five independent degrees-of-freedom of motion between the touching objects. Now, by applying serial kinematic reduction to this kinematic chain to obtain a single equivalent kinematic constraint between the bodies, we get

$$\{\mathbf{R}(A_R,\boldsymbol{u_a})\}\cdot\{\mathbf{R}(A_r,\boldsymbol{w_a})\}\cdot\{\mathbf{R}(C,\boldsymbol{v})\}\cdot\{\mathbf{R}(B_r,\boldsymbol{w_b})\}\cdot\{\mathbf{R}(B_R,\boldsymbol{u_b})\} \tag{3}$$

The above equation corresponds to the kinematic equivalent of the Bruyninckx–Hunt model.

3 Classification of Contact Types for Fingertip-Based Manipulation

The Bruyninckx–Hunt model of point contact depends on the local curvature properties of the touching bodies, represented by the smooth surfaces Φ_A and Φ_B; thus, the model is indeed an infinitesimal characterization of the contact where, according to the notation of Fig. 1 (right), $r_a = \|\mathbf{A}_r\mathbf{C}\|$ and $R_a = \|\mathbf{A}_R\mathbf{C}\|$ (correspondingly $r_b = \|\mathbf{B}_r\mathbf{C}\|$ and $R_b = \|\mathbf{B}_R\mathbf{C}\|$) correspond to the radii of maximum and minimum curvatures of Φ_A (Φ_B), respectively, with $\mathbf{AB}$ the vector pointing from A to B and $\|\mathbf{AB}\|$ the Euclidean norm of $\mathbf{AB}$; and $\mathcal{K}_{\Phi_i} = \frac{1}{r_i\ R_i}$, $(i = a(A), b(B))$ is the Gaussian curvature of Φ_i in the vicinity of the contact point. r_a and R_a are indeed oriented distances, defined as positive in the direction of the inward normal of the tangent plane at point C.

Despite the explicit local definition of the Bruyninckx–Hunt model, useful novel finite kinematic equivalents of contact types can be defined since suppositions about the Gaussian curvature of the fingertip (and the object) can be made. Additionally, nonfrictional and frictional cases can be considered by replacing the three

[5]For these particular cases, there exist an infinite number of directions for the vectors u and w.

Table 2 Proposed classification of contact types

			Special cases	
Contact type		Kinematic equivalent (nonfrictional/frictional)	Particular geometry[a]	Limit instances[b]
Elliptic contact	$\mathcal{K}_{\Phi_A} > 0$	$\{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\}$ $(\eta_{B_r} = \eta_{B_R} = \eta_C = 0)$	Elliptic-ball contact, elliptic-cylinder contact, elliptic-plane contact, nonfrictional ball contact	Point-ball contact, point-cylinder contact, point-plane contact (or (ii) point contact without friction), ball-plane contact
		$\{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\}$ $(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C = 0)$	Ball contact	(iv) Point contact with friction, (vii) soft finger $(\eta_C > 0)$
Cylindrical contact	$\mathcal{K}_{\Phi_A} = 0, R_a \to \infty$	$\{\mathbf{C}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\}$ $(\eta_{B_r} = \eta_{B_R} = \eta_C = 0)$	Cylindrical-ball contact, cylindrical–cylinder contact, cylindrical-plane contact	Line-ball contact, line-cylinder contact, line-plane contact (or (iii) line contact without friction), frictional line-plane $(\eta_{B_r} > 0, \eta_C > 0)$ (or (vi) line–line contact without friction), fully frictional line-plane $(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C > 0)$ (or (viii) line(-line) contact with friction)
		$\{\mathbf{C}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\}$ $(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C = 0)$	—	Tubular contact $(\eta_C > 0)$
Flat contact	$\mathcal{K}_{\Phi_A} = 0, r_a = R_a \to \infty$	$\{\mathbf{G}(\boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\}$ $(\eta_{B_r} = \eta_{B_R} = \eta_C = 0)$	Flat-ball contact, flat-cylinder contact, flat-plane contact (or (v) planar contact without friction)	Fully frictional flat-plane $(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C > 0)$ (or (ix) planar contact with friction)
		$\{\mathbf{G}(\boldsymbol{v})\}$ $(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C = 0)$	—	Planar translation contact $(\eta_C > 0)$
Adaptive finger	$\mathcal{K}_{\Phi_A} = \mathcal{K}_{\Phi_B}, r_a = r_b$ and $R_a = R_b$	$\{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\}(\eta_{B_r} = \eta_{B_R} = \eta_C = 0)$	Adaptive ball contact, adaptive cylindrical contact, adaptive plane contact	Multiple cases
		$\{\mathbf{R}(C, \boldsymbol{v})\}(\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C = 0)$	—	Fully frictional adaptive finger $(\eta_C > 0)$

[a]Particular geometry cases correspond to instances with particular values of the model variables $\mathcal{K}_{\Phi_A}$ and $\mathcal{K}_{\Phi_B}$.
[b]Limit instances cases correspond to instances with extreme values of $\mathcal{K}_{\Phi_A}$ and/or relaxed conditions on η_{B_r}, η_{B_R}, and η_C of particular geometry cases; then limit instances of nonfrictional equivalents may be frictional.

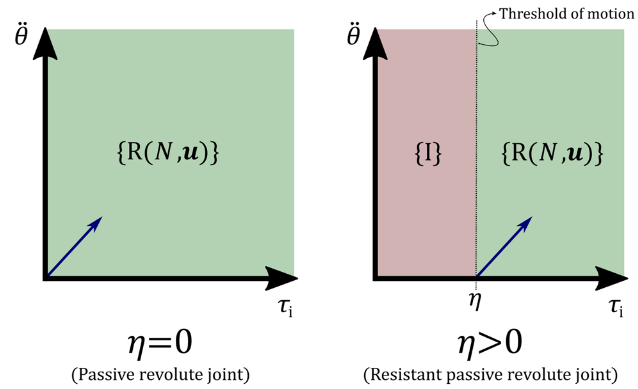


Fig. 2 A resistant passive revolute joint (right) is able to resist moments till some value η before entering in motion. In a passive revolute joint (left) $\eta = 0$. Then, by assuming that any moment induced on the resistant passive revolute joint is not greater than η, the joint can be considered as locked.

passive revolute joints of one of the surfaces—including the revolute joint at the contact point—by resistant passive joints, that is, passive joints able to resist moments till some value η before entering in motion (in a passive joint $\eta = 0$). Therefore, from the kinematic viewpoint, assuming that any moment τ_i induced on a resistant passive revolute joint is not greater than η, the joint can be considered as locked; in other words, the kinematic constraint between the involved links reduces to a rigid connection (i.e., $\{\mathbf{I}\}$ (see Fig. 2)[6]). Under these considerations, the contact is said to be unable to resist moments about the axis of the revolute joint if $\eta = 0$. Otherwise, the contact is said to be able to resist moments about the involved revolute axis.

In classical kinematic-pair-based contact models (i.e., Salisbury's taxonomy plus line–line contact without friction), it is assumed that, in cases where friction is said to be active, frictional forces larger than the forces they must resist are created. This frictional assumption constrains the displacement between bodies along or about specific axes of a common reference frame. In the extended Bruyninckx–Hunt contact model herein proposed, this notion is generalized via the introduced concept of resistant passive joints and their corresponding threshold moments.

In what follows, we present a general classification of contact types for robot manipulation based on the proposed kinematic-chain-based contact model (i.e., Bruyninckx–Hunt model with resistant passive joints). It will be shown how the Salisbury's taxonomy along with line–line contact without friction are obtained from special instances of some of these more general types and how new useful contact types are deduced. In all cases, Φ_A represents the boundary of the fingertip and Φ_B that of the grasped object; the resistant revolute joints that model the friction effects on the contact motion are located in the grasped body, explicitly at points B_r, B_R, and C with directions $\boldsymbol{u}_\mathbf{b}$, $\boldsymbol{w}_\mathbf{b}$, and $\boldsymbol{v}$ and threshold moments η_{B_r}, η_{B_R}, and η_C, respectively.

3.1 Elliptic Contact. The elliptic contact is characterized by a positive Gaussian curvature of the fingertip, that is, $\mathcal{K}_{\Phi_A} = \frac{1}{r_a\ R_a} > 0$. In the frictionless case, it is assumed that $\eta_{B_r} = \eta_{B_R} = \eta_C = 0$ and the kinematic equivalent correspond to that of Eq. (3). In the frictional case, it is supposed that the contact is able to resist moments but not about the contact normal, that is, $\eta_{B_r} > 0, \eta_{B_R} > 0$, and $\eta_C = 0$. Then, the kinematic equivalent reduces to

$$\{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \tag{4}$$

[6]We have opted for the more general term resistant joint rather than frictional joint to avoid implicit assumptions about the nature of the power loss and the shape/geometry of the revolute pair.

since $\{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\}$ and $\{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\}$ are replaced by $\{\mathbf{I}\}$. Next, some special cases of the nonfrictional and frictional elliptic contact are discussed; these are divided in two categories: (1) particular geometry, which corresponds to instances with particular values of the model variables $\mathcal{K}_{\Phi_A}$ and $\mathcal{K}_{\Phi_B}$, and (2) limit instances, which corresponds to extreme values of $\mathcal{K}_{\Phi_A}$ and/or relaxed conditions on η_{B_r}, η_{B_R}, and η_C of particular geometry cases. This also applies for the other contact models discussed in this section.

(1) *Particular geometry (elliptic contact)*

In the case of the frictional elliptic contact, the curvature of the grasped object does not affect the kinematic equivalent. However, in the nonfrictional model, this is not the case since the subgroups $\{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\}$ and $\{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\}$ are in the equation. Then, the nonfrictional elliptic contact can be specialized by considering specific shapes of the grasped object (i.e., explicit values of $\mathcal{K}_{\Phi_B}$), for instance, if in the vicinity of the contact point the grasped object is a plane, then $\mathcal{K}_{\Phi_B} = 0$ ($r_b \to \infty$ and $R_b \to \infty$) and the kinematic equivalent is

$$\begin{aligned}&\{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u}_\mathbf{b})\} \cdot \{\mathbf{T}(\boldsymbol{w}_\mathbf{b})\}\\ &\quad = \{\mathbf{R}(A_R, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{G}(\boldsymbol{v})\}\cdot\end{aligned} \tag{5}$$

Similar analyses to the elliptic-plane contact can be done for the cases in which the grasped object corresponds to a ball-like shape (elliptic-ball contact) or a cylinderlike shape (elliptic–cylinder contact).

An interesting particularization in the frictional case results when in the fingertip is assumed that $r_a = R_a > 0$. Then, $A_R = A_r = A$. In this frictional ball contact, the kinematic equivalent is

$$\{\mathbf{R}(A, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(A, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} = \{\boldsymbol{S}(A)\} \tag{6}$$

since $\{\mathbf{R}(C, \boldsymbol{v})\} = \{\mathbf{R}(A, \boldsymbol{v})\}$ and for any couple of selected vectors $\boldsymbol{u}_\mathbf{a}$ and $\boldsymbol{w}_\mathbf{a}$; $\boldsymbol{u}_\mathbf{a}$, $\boldsymbol{w}_\mathbf{a}$, and $\boldsymbol{v}$ are linearly independent. Similarly, the nonfrictional ball contact can be defined. For this case, the kinematic equivalent is

$$\{\mathbf{S}(A)\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w}_\mathbf{b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u}_\mathbf{b})\} \tag{7}$$

In both cases, if $r_\boldsymbol{a} = R_\boldsymbol{a} < 0$, the kinematic behavior is equivalent, but physically the shape of the fingertip implies the contact is interior.

(2) *Limit instances (elliptic contact)*

Limit instances of the elliptic contact result from reducing the shape of the fingertip to a single point, that is, $r_\boldsymbol{a} = R_\boldsymbol{a} = 0$ and $\mathcal{K}_{\Phi_A} \to \infty$. In this extreme case, the kinematic equivalent of the frictional ball contact reduces to

$$\{\mathbf{S}(C)\} \tag{8}$$

since $A = C$; thus obtaining the classical point contact with friction ((iv) in Table 1). Since $\{\boldsymbol{S}(C)\}$ can be represented as $\{\boldsymbol{R}(C, \boldsymbol{u})\} \cdot \{\boldsymbol{R}(C, \boldsymbol{w})\} \cdot \{\boldsymbol{R}(C, \boldsymbol{v})\}$, where $\boldsymbol{u}$ and $\boldsymbol{w}$ are any linearly independent vectors that define the contact tangent plane; then, if we assume that the point contact is able to resist moments about the contact normal—axis $(C, \boldsymbol{v})$, that is, to relax the condition $\eta_C = 0$, then $\{\boldsymbol{R}(C, \boldsymbol{v})\}$ becomes $\{\boldsymbol{I}\}$ and the kinematic equivalent is

$$\{\mathbf{R}(C, \boldsymbol{u})\} \cdot \{\mathbf{R}(C, \boldsymbol{w})\} \tag{9}$$

which corresponds to the soft finger model ((ix) in Table 1).

If we apply the point assumption to the elliptic-plane contact, since the subgroup $\{\boldsymbol{G}(\boldsymbol{v})\}$ can be written as $\{\boldsymbol{R}(C, \boldsymbol{v})\} \cdot \{\boldsymbol{P}(\boldsymbol{v})\}$, we get

$$= \{\mathbf{R}(C, \boldsymbol{u}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{w}_\mathbf{a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{P}(\boldsymbol{v})\} = \{\boldsymbol{S}(C)\} \cdot \{\mathbf{P}(\boldsymbol{v})\} \tag{10}$$

known as the point contact without friction model ((ii) in Table 1). Again, for the elliptic-ball contact and the elliptic–cylinder contact, limit instances can be similarly defined.

3.2 Cylindrical Contact. The cylindrical contact corresponds to a fingertip with zero Gaussian curvature, that is, $\mathcal{K}_{\Phi_A} = \frac{1}{r_a R_a} = 0$, with $R_a \to \infty$ and r_a finite. In the frictionless case (i.e., $\eta_{B_r} = \eta_{B_R} = \eta_C = 0$), the kinematic equivalent is

$$\{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w_b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u_b})\} = \{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w_b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u_b})\} \tag{11}$$

since a rotation $\{\mathbf{R}(O, \boldsymbol{u})\}$ center at infinity respect to a point M is equivalent to a translation in the direction of the cross product between $\boldsymbol{u}$ and $\mathbf{MO}/\|\mathbf{MO}\|$, that is, $\{\mathbf{R}(A_R, \boldsymbol{u_a})\} = \{\mathbf{T}(\boldsymbol{w_a})\}$ if $R_a \to \infty$. Additionally, the subgroup $\{\mathbf{C}(A, \boldsymbol{v})\}$ can be written as $\{\mathbf{T}(\boldsymbol{v})\} \cdot \{\mathbf{R}(A, \boldsymbol{v})\}$.

In a frictional cylindrical contact, it is supposed that the contact is able to resist moments but not about the contact normal, that is, $\eta_{B_r} > 0, \eta_{B_R} > 0$, and $\eta_C = 0$. Applying these conditions to Eq. (11), the resulting kinematic equivalent is

$$\{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \tag{12}$$

(1) *Particular geometry (cylindrical contact)*

Similarly to the case of the elliptic contact, the nonfrictional cylindrical contact can be specialized by considering specific shapes of the grasped object. Then, for instance, if in the vicinity of the contact point the grasped object is a plane—$\mathcal{K}_{\Phi_B} = 0$ ($r_b \to \infty$ and $R_b \to \infty$), the resulting kinematic equivalent is

$$\{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_b})\} \cdot \{\mathbf{T}(\boldsymbol{w_b})\} = \{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{G}(\boldsymbol{v})\} \tag{13}$$

However, note that, by the property of closure, $\{\mathbf{C}(A_r, \boldsymbol{w_a})\} = \{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w_a})\} = \{\mathbf{R}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{w_a})\}$, $\{\mathbf{G}(\boldsymbol{v})\}$ can be written as $\{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_a})\}$, and $\{\mathbf{T}(\boldsymbol{w_a})\} = \{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{w_a})\}$. Therefore,

$$\begin{aligned}\{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{G}(\boldsymbol{v})\} &= \{\mathbf{R}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{w_a})\} \cdot \\ \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_a})\} &= \{\mathbf{R}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \\ \{\mathbf{T}(\boldsymbol{u_a})\} &= \{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_a})\}\end{aligned} \tag{14}$$

Equation (14) is the kinematic equivalent of the cylindrical-plane contact. Likewise, if it is assumed that in the grasped object $r_b = R_b > 0$. Then, $B_R = B_r = B$ and we get

$$\{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B, \boldsymbol{w_b})\} \cdot \{\mathbf{R}(B, \boldsymbol{u_b})\} = \{\mathbf{C}(A_r, \boldsymbol{w_a})\} \cdot \{\mathbf{S}(B)\} \tag{15}$$

since $\{\mathbf{R}(C, \boldsymbol{v})\} = \{\mathbf{R}(B, \boldsymbol{v})\}$ and for any couple of selected vectors $\boldsymbol{u_b}$ and $\boldsymbol{w_b}$; $\boldsymbol{u_b}$, $\boldsymbol{w_b}$, and $\boldsymbol{v}$ are linearly independent. The above equation is the kinematic equivalent of the cylindrical-ball contact. A similar deduction can be elaborated for the circumstance in which the grasped object corresponds to a cylinderlike shape (cylindrical–cylinder contact). Moreover, given the assumptions of the frictional cylindrical contact (Eq. (12)), no particularizations of interest can be deduced from it and itself can be considered as a particular case.

(2) *Limit instances (cylindrical contact)*

Note that if in the cylindrical-plane contact model, the fingertip boundary is reduced to a line, that is, $r_a = 0$, then $A_r = C$. Applying this condition in Eq. (14) yields

$$\{\mathbf{C}(C, \boldsymbol{w_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_a})\} \tag{16}$$

that corresponds to the kinematic equivalent of the line-plane contact, known as line contact without friction ((iii) in Table 1). Similarly, the kinematic equivalents of the line-ball contact model and the line-cylinder contact model can be easily obtained. Now, if we assume that the line-plane contact is able to resist moments about the contact normal and about the axis $(B_r, \boldsymbol{w_b})$, that is, to relax the condition $\eta_{B_r} = \eta_C = 0$, then the kinematic equivalent reduces to

$$\{\mathbf{C}(C, \boldsymbol{w_a})\} \tag{17}$$

the frictional line-plane contact model, known as line–line contact without friction in the standard classification of contacts ((vi) in Table 1). Moreover, if we accept that the line-plane contact is also able to resist moments about the axis $(B_R, \boldsymbol{u_b})$—$\eta_{B_R} > 0$, the translational motion of the subgroup $\{\boldsymbol{C}(C, \boldsymbol{w_a})\}$ is restricted. Thus,

$$\{\mathbf{R}(C, \boldsymbol{w_a})\} \tag{18}$$

is the kinematic equivalent of the fully frictional line-plane contact model or line(-line) contact with friction ((viii) in Table 1).

In the case of the frictional cylindrical contact, if it is allowed to resist moments about the contact normal ($\eta_C > 0$), then $\{\mathbf{R}(C, \boldsymbol{v})\}$ becomes $\{\mathbf{I}\}$ and the kinematic equivalent simplifies to

$$\{\mathbf{C}(A_r, \boldsymbol{w_a})\} \tag{19}$$

that we call the tubular contact model (fully frictional cylindrical contact). The frictional line-plane contact model (Eq. (17)) and the tubular contact model seem analogous, but the second does not assume any curvature properties of the grasped object.

3.3 Flat Contact. The flat contact corresponds to a fingertip with zero Gaussian curvature, that is, $\mathcal{K}_{\Phi_A} = 1/r_a R_a = 0$, with $r_a \to \infty$ and $R_a \to \infty$. Then, $\{\mathbf{R}(\boldsymbol{A_R}, \boldsymbol{u_a})\} = \{\mathbf{T}(\boldsymbol{w_a})\}$ and $\{\mathbf{R}(\boldsymbol{A_r}, \boldsymbol{w_a})\} = \{\mathbf{T}(\boldsymbol{u_a})\}$. In the frictionless case (i.e., $\eta_{B_r} = \eta_{B_R} = \eta_C = 0$), the kinematic equivalent is

$$\{\mathbf{T}(\boldsymbol{w_a})\} \cdot \{\mathbf{T}(\boldsymbol{u_a})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w_b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u_b})\} = \{\mathbf{G}(\boldsymbol{v})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w_b})\} \cdot \{\mathbf{R}(B_R, \boldsymbol{u_b})\} \tag{20}$$

Therefore, for the frictional case—$\eta_{B_r} > 0, \eta_{B_R} > 0$, and $\eta_C = 0$, the resulting kinematic equivalent is

$$\{\mathbf{G}(\boldsymbol{v})\} \tag{21}$$

The obtained displacement of frictional flat contact does not depend on the curvature properties of the grasped object.

(1) *Particular geometry (flat contact)*

Similarly to the previous contact types, the nonfrictional flat contact can be particularized by assuming some geometry of the grasped object. Thus, for instance, if $\mathcal{K}_{\Phi_B} = 0$ ($r_b \to \infty$ and $R_b \to \infty$), then $\{\mathbf{R}(B_R, \boldsymbol{u_b})\} = \{\mathbf{T}(\boldsymbol{w_b})\}$ and $\{\mathbf{R}(B_r, \boldsymbol{w_b})\} = \{\mathbf{T}(\boldsymbol{u_b})\}$ and we have

$$\{\mathbf{G}(\boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_b})\} \cdot \{\mathbf{T}(\boldsymbol{w_b})\} = \{\mathbf{G}(\boldsymbol{v})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \cdot \{\mathbf{T}(\boldsymbol{u_b})\} \cdot \{\mathbf{T}(\boldsymbol{w_b})\} = \{\mathbf{G}(\boldsymbol{v})\} \cdot \{\mathbf{G}(\boldsymbol{v})\} = \{\mathbf{G}(\boldsymbol{v})\} \tag{22}$$

the kinematic equivalent of the flat-plane contact model, known as planar contact without friction ((v) in Table 1).

(2) *Limit instances (flat contact)*

The geometry of the fingertip in the frictional flat contact (Eq. (21)) is already particularized. If in addition we assume that the contact is able to resist moments about the contact normal ($\eta_C > 0$), the kinematic equivalent is

$$\{\mathbf{P}(\boldsymbol{v})\} \tag{23}$$

since the subgroup $\{\mathbf{G}(\boldsymbol{v})\}$ can be written as $\{\mathbf{P}(\boldsymbol{v})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\}$. This planar translation contact model can be seen as an alternative to the soft finger model. Moreover, note that the kinematic equivalent of the fully frictional case of the flat-plane contact model ($\eta_{B_r} > 0, \eta_{B_R} > 0, \eta_C > 0$) reduces to the identity displacement $\{\mathbf{I}\}$—this contact is indeed the planar contact with friction model ((ix) in Table 1).

3.4 Adaptive Finger. In general, contact types are considered suitable for arbitrary objects if the model does not depend on the assumptions on the curvature properties of the surface of the object (e.g., ball contact, point contact without friction, tubular contact). However, for manipulation analysis purposes, it can be useful to define a contact type in which the boundary of the fingertip, Φ_A, matches that of the grasped object, Φ_B. That is, $\mathcal{K}_{\Phi_A} = \mathcal{K}_{\Phi_B}$ with $r_{\boldsymbol{a}} = r_{\boldsymbol{b}}$ and $R_{\boldsymbol{a}} = R_{\boldsymbol{b}}$. Since this contact mimics the behavior of a compliant fingertip, we call it: adaptive finger.

Under the above assumption, we have $A_r = B_r$, $A_R = B_R$, $\boldsymbol{u}_{\mathbf{a}} = \boldsymbol{u}_{\mathbf{b}}$, and $\boldsymbol{w}_{\mathbf{a}} = \boldsymbol{w}_{\mathbf{b}}$. Then, in the frictionless case ($\eta_{B_r} = \eta_{B_R} = \eta_C = 0$), the kinematic equivalent of the adaptive finger contact reduces to

$$\{\mathbf{R}(A_R, \boldsymbol{u}_{\mathbf{a}})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_{\mathbf{a}})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} = \{\mathbf{R}(B_R, \boldsymbol{u}_{\mathbf{b}})\} \cdot \{\mathbf{R}(B_r, \boldsymbol{w}_{\mathbf{b}})\} \cdot \{\mathbf{R}(C, \boldsymbol{v})\} \tag{24}$$

In the standard frictional supposition, it is said that the contact is able to resist moments but not about the contact normal, that is, $\eta_{B_r} > 0, \eta_{B_R} > 0$, and $\eta_C = 0$. Thus, the kinematic equivalent of the frictional adaptive finger is

$$\{\mathbf{R}(C, \boldsymbol{v})\} \tag{25}$$

(1) *Particular geometry (adaptive finger)*

Since $\mathcal{K}_{\Phi_A} = \mathcal{K}_{\Phi_B}$, the adaptive finger model can be considered by itself a case of particular geometry. However, if explicit curvature properties of the grasped object are additionally assumed, special cases of the nonfrictional adaptive finger can be defined in a similar fashion as it was performed in the previous contact models. The kinematic equivalents of these particular contact types, namely, adaptive ball contact, adaptive cylindrical contact, and adaptive plane contact, can be straightforwardly obtained by applying the conditions of $\mathcal{K}_{\Phi_B}$ to Eq. (24).

(2) *Limit instances (adaptive finger)*

By relaxing the conditions on η_{B_r}, η_{B_R}, and η_C of the frictionless adaptive finger and its particular cases, multiple contact models can be deduced; from these options we present the case in which the frictionless adaptive finger contact is allowed to resist moments about the contact normal ($\eta_C > 0$). Then, we get

$$\{\mathbf{R}(A_R, \boldsymbol{u}_{\mathbf{a}})\} \cdot \{\mathbf{R}(A_r, \boldsymbol{w}_{\mathbf{a}})\} \tag{26}$$

as kinematic equivalent. A similar equivalent can be obtained from the frictional elliptic contact model (Eq. (4)) by also assuming resistance to moments about the contact normal; but, in such a case, no assumptions on the curvature of the grasped object are made. Finally, in the case of the fully frictional adaptive finger, the kinematic equivalent is the identity displacement $\{\mathbf{I}\}$. This contact type is certainly similar to the Cutkosky's very soft finger model discussed in Ref. [17].

The above classification of contact types for robot manipulation is summarized in Table 1. In all cases the degrees-of-freedom of the contact models can be easily obtained by summing the degrees-of-freedom of the subgroups of displacements that define their kinematic equivalent. A hyperbolic contact could also be defined (characterized by $\mathcal{K}_{\Phi_A} = 1/r_a R_a < 0$); however, from a finite kinematic viewpoint, the behavior of the contact is equivalent to that of the elliptic model, thus an explicit exposition of this contact is redundant. The instantaneous kinematics analysis of the introduced contact types should follow straightforwardly from the methods presented in Refs. [11,18,19], but this is certainly an aspect that deserves further research.

4 Example

Precision manipulation analysis is a finite (gross) kinematic manipulation technique to characterize the capabilities of a given robot hand architecture to reposition with its fingertips a grasped object within the hand without breaking or changing contact. The method allows determining the composition of the displacement manifold of the object relative to the palm of the robot hand as well as defining which of these possible displacements can actually be controlled by the hand actuators without depending on external factors to the hand (e.g., forces). The technique basically consists of five steps, namely: (1) selection of the contact model for each fingertip, (2) definition of the resulting closed kinematic chain of the hand-object system (HOS), (3) construction of the HOS's graph of kinematic constraints, (4) reduction of the graph to a single edge, and (5) determination of the controllable movements. The interested reader may refer to Ref. [20] for details of this method. Next, we discuss an example of precision manipulation analysis that stands out the relevance of the introduced contact models. This analyzes the capabilities of a simple three-fingered robot hand grasping a special object under different

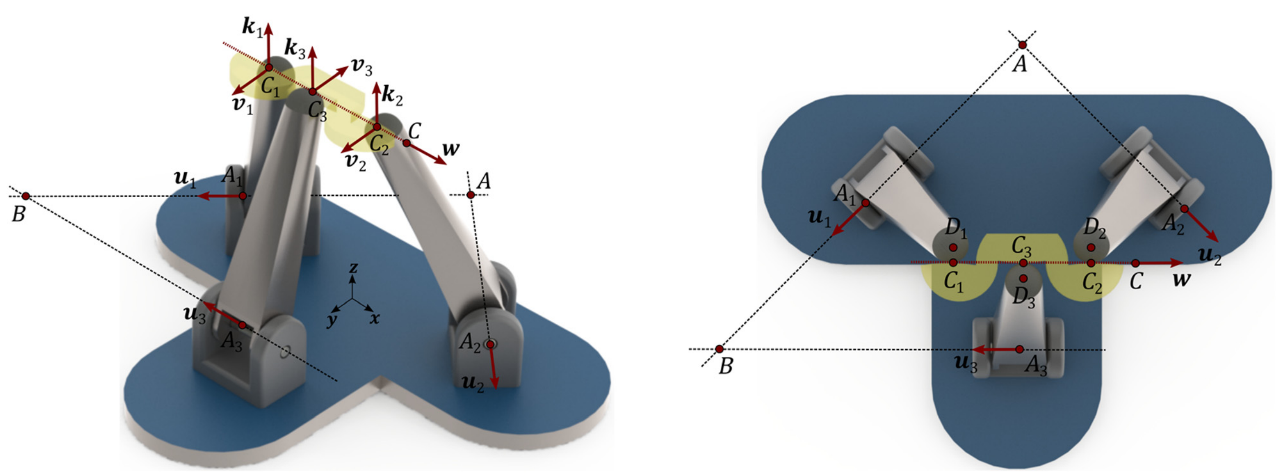
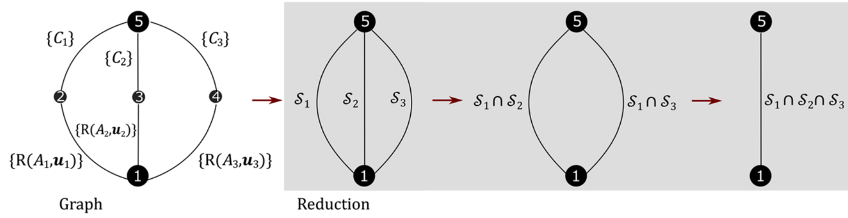


Fig. 3 A 3F-3R robot hand grasping a special object (s-shaped body) with the notation used for its fingertip-based within-hand manipulation analysis using different contact models, namely: *point contact with friction, soft finger, ball contact, and frictional adaptive finger* (see Table 3)

Table 3 Precision manipulation analysis of the 3F-3R hand grasping the s-shaped object

Step	Point contact with friction	Soft finger	Ball contact	Frictional adaptive finger
1	$\{\mathbf{S}(C_i)\}$	$\{\mathbf{R}(C_i,\boldsymbol{k}_i)\}\cdot\{\mathbf{R}(C_i,\boldsymbol{w})\}$	$\{\mathbf{S}(D_i)\}$	$\{\mathbf{R}(C_i,\boldsymbol{v}_i)\}$
2	Closed Kinematic Chain (CKC) composed of five links with three revolute-spherical serial limbs that connect the base of the robot hand to the grasped object ($M=0$)	CKC composed of five links with three revolute-universal serial limbs that connect the base of the robot hand to the grasped object ($M=0$)	CKC composed of eight links with three revolute-universal-revolute serial limbs that connect the base of the robot hand to the grasped object ($M=0$)	CKC composed of five links with three revolute-revolute serial limbs that connect the base of the robot hand to the grasped object ($M=0$)
3	 $\{C_i\}$ as in step 1			
4	$\mathcal{S}_1=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{S}(C_1)\}$ $=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ac}_1})\}\cdot\{\mathbf{S}(C_1)\}$ $=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ac}_1})\}\cdot\{\mathbf{S}(C_1)\}$ $=\{\mathbf{R}(B,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bc}_1})\}\cdot\{\mathbf{S}(C_1)\}$ $=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bc}_1})\}\cdot\{\mathbf{S}(C_1)\}$, O_1 is the pole of $A_1C_1C_2A_2$, O_2 is the pole of $A_1C_1C_3A_3$. $\mathcal{S}_2=\{\mathbf{R}(A,\boldsymbol{u}_2)\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ac}_2})\}\cdot\{\mathbf{S}(C_2)\}$ $=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ac}_2})\}\cdot\{\mathbf{S}(C_2)\}$. $\mathcal{S}_3=\{\mathbf{R}(B,\boldsymbol{u}_3)\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bc}_3})\}\cdot\{\mathbf{S}(C_3)\}$ $=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bc}_3})\}\cdot\{\mathbf{S}(C_3)\}$. $\mathcal{S}_1\cap\mathcal{S}_2=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(C_1,\widehat{\boldsymbol{c}_1\boldsymbol{c}_2})\}$ $=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(C,\boldsymbol{w})\}$. $\mathcal{S}_1\cap\mathcal{S}_3=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(C,\boldsymbol{w})\}$. $\mathcal{S}_1\cap\mathcal{S}_2\cap\mathcal{S}_3=\{\mathbf{R}(C,\boldsymbol{w})\}$.	$\mathcal{S}_1=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{k}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{w})\}$ $=\{\mathbf{R}(B,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{k}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{w})\}$. $\mathcal{S}_2=\{\mathbf{R}(A,\boldsymbol{u}_2)\}\cdot\{\mathbf{R}(C_2,\boldsymbol{k}_2)\}\cdot\{\mathbf{R}(C_2,\boldsymbol{w})\}$. $\mathcal{S}_3=\{\mathbf{R}(B,\boldsymbol{u}_3)\}\cdot\{\mathbf{R}(C_3,\boldsymbol{k}_3)\}\cdot\{\mathbf{R}(C_3,\boldsymbol{w})\}$. $\mathcal{S}_1\cap\mathcal{S}_2=\{\mathbf{R}(C,\boldsymbol{w})\}$. $\mathcal{S}_1\cap\mathcal{S}_3=\{\mathbf{R}(C,\boldsymbol{w})\}$. $\mathcal{S}_1\cap\mathcal{S}_2\cap\mathcal{S}_3=\{\mathbf{R}(C,\boldsymbol{w})\}$.	$\mathcal{S}_1=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{S}(D_1)\}$ $=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ad}_1})\}\cdot\{\mathbf{S}(D_1)\}$ $=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ad}_1})\}\cdot\{\mathbf{S}(D_1)\}$ $=\{\mathbf{R}(B,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bd}_1})\}\cdot\{\mathbf{S}(D_1)\}$ $=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bd}_1})\}\cdot\{\mathbf{S}(D_1)\}$, O_1 is the pole of $A_1D_1D_2A_2$, O_2 is the pole of $A_1D_1D_3A_3$. $\mathcal{S}_2=\{\mathbf{R}(A,\boldsymbol{u}_2)\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ad}_2})\}\cdot\{\mathbf{S}(D_2)\}$ $=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(A,\widehat{\boldsymbol{ad}_2})\}\cdot\{\mathbf{S}(D_2)\}$. $\mathcal{S}_3=\{\mathbf{R}(B,\boldsymbol{u}_3)\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bd}_3})\}\cdot\{\mathbf{S}(D_3)\}$ $=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(B,\widehat{\boldsymbol{bd}_3})\}\cdot\{\mathbf{S}(D_3)\}$. $\mathcal{S}_1\cap\mathcal{S}_2=\{\mathbf{R}(A,\widehat{\boldsymbol{ao}_1})\}\cdot\{\mathbf{R}(D_1,\widehat{\boldsymbol{d}_1\boldsymbol{d}_2})\}$. $\mathcal{S}_1\cap\mathcal{S}_3=\{\mathbf{R}(B,\widehat{\boldsymbol{bo}_2})\}\cdot\{\mathbf{R}(D_1,\widehat{\boldsymbol{d}_1\boldsymbol{d}_3})\}$. $\mathcal{S}_1\cap\mathcal{S}_2\cap\mathcal{S}_3=\{\mathbf{I}\}$.	$\mathcal{S}_1=\{\mathbf{R}(A,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{v}_1)\}$ $=\{\mathbf{R}(B,\boldsymbol{u}_1)\}\cdot\{\mathbf{R}(C_1,\boldsymbol{v}_1)\}$. $\mathcal{S}_2=\{\mathbf{R}(A,\boldsymbol{u}_2)\}\cdot\{\mathbf{R}(C_2,\boldsymbol{v}_2)\}$. $\mathcal{S}_3=\{\mathbf{R}(B,\boldsymbol{u}_3)\}\cdot\{\mathbf{R}(C_3,\boldsymbol{v}_3)\}$. $\mathcal{S}_1\cap\mathcal{S}_2=\{\mathbf{I}\}$. $\mathcal{S}_1\cap\mathcal{S}_3=\{\mathbf{I}\}$. $\mathcal{S}_1\cap\mathcal{S}_2\cap\mathcal{S}_3=\{\mathbf{I}\}$.
5	$\{\mathbf{R}(C,\boldsymbol{w})\}$ Regardless of the actuation scheme	$\{\mathbf{R}(C,\boldsymbol{w})\}$ Regardless of the actuation scheme	$\{\mathbf{I}\}$ Regardless of the actuation scheme	$\{\mathbf{I}\}$ Regardless of the actuation scheme

assumptions of contact types, namely: *point contact with friction, soft finger, ball contact, and frictional adaptive finger*.

Figure 3 presents a 3F-3R hand grasping with its fingertips an s-shaped body. This hand is composed of three identical fingers with only one link; two of them arranged in semi-opposed configuration with the third finger acting as an opposable thumb. In each finger, the link is connected to the palm through a revolute joint, which determines the motion plane of the finger. This finger/palm layout is similar to those proposed in the context of the CMU Simple Hands project [21]. Intuitively, it is clear that, regardless of the assumed contact model, the fingertip-based within-hand manipulation capabilities of the 3F-3R hand are very limited. Indeed, it is expected that an object grasped by all the fingertips of the hand cannot be relocated without breaking contact, that is, the kinematic constraint relating the palm and the grasp object reduces to the identity displacement $\{\mathbf{I}\}$. This circumstance can be used to study the effects and relevance of the contact assumptions in manipulation applications.

Table 3 presents the finite precision manipulation analysis of the 3F-3R hand grasping, as presented in Fig. 3, an s-shaped object in the cases where the contacts between the three fingertips and the object are modeled as point contact with friction, soft finger, ball contact, and frictional adaptive finger. The results show that under the assumptions of point contact with friction and soft finger, the grasped object has an uncontrollable feasible displacement (regardless of the friction conditions of the contacts) that corresponds to the rotation about the axis defined by the three contact points ($\{\mathbf{R}(C,\boldsymbol{w})\}$ in Table 3). However, in the cases of ball contact and frictional adaptive finger, the resulting displacement is certainly the identity (under the frictional assumptions of each contact).

The above results do not imply that ball contact and frictional adaptive finger are better models than point contact with friction and soft finger for manipulation studies. What they show is that, depending on the finger/palm layout and the manipulation application, a wrong selection of contact models can yield to erroneous estimates of the possible motions of a grasped object, an aspect completely relevant to robot hand design. This, besides showing the importance of having more contact models than the standard limit instances (i.e., Salisbury's taxonomy and line–line contact without friction), makes evident that some contact assumptions are more robust than others in ill-conditioned cases (e.g., grasping an object in three aligned contact points).

5 Conclusion

In this paper, the effects of a fingertip contacting a body in robot hand manipulation are modeled as kinematic chains; the suggested contact model is based on an extension of the Bruyninckx–Hunt approach of surface–surface contact that uses the concept of resistant passive joints located in the object being manipulated. From this extended model, a general classification of nonfrictional and frictional contact types is developed in which all standard contact categories used in robotic manipulation, namely, the Salisbury's taxonomy plus line–line contact without friction, appear as special cases, and new contact models are defined and characterized with their kinematic equivalents determined via Herve's group-theoretic approach. An example of fingertip-based within-hand robot manipulation showing the impact of the assumed contact types in manipulation analyses as well as the relevance of having more contact models to perform, for instance, better predictions on the feasible and controllable movements of a grasped object is discussed. The main purpose of the proposed classification, rather than being exhaustive, is to present a more general structure for the organization of contact models for robotic manipulation and to serve of guidance for the design of robot hands, in particular fingers and fingertips. Given the generality of the concepts used, the classification and contact types introduced can be applied to other contact conditions arising in robotics, as those appearing in robot locomotion for instance. Some lines of future work include the exploration of the suggested contact classification and extended Bruyninckx–Hunt model in kinesthetic and dynamics analyses of dexterous manipulation and the experimental quantification of the threshold moments appearing in the definition of the presented contact types.

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Nomenclature

$\{\mathbf{C}(N,u)\}$ = cylindrical motion about the axis determined by the unit vector u and point N (two degrees-of-freedom)

$\{\mathbf{G}(v)\}$ = planar gliding motion on the plane determined by the unit normal vector v (three degrees-of-freedom)

$\{\mathbf{I}\}$ = the identity displacement. Rigid connection between bodies, no relative motion (zero degrees-of-freedom)

$\{\mathbf{P}(v)\}$ = planar translation on the plane determined by the unit normal vector v (two degrees-of-freedom)

$\{\mathbf{R}(N,u)\}$ = rotation about the axis determined by the unit vector u and point N (one degree-of-freedom)

$\{\mathbf{S}(N)\}$ = spherical rotation about a point N (three degrees-of-freedom)

$\{\mathbf{T}(v)\}$ = translation parallel to the unit vector v (one degree-of-freedom)

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