Experimental study of highly turbulent isothermal opposed-jet flows

Gianfilippo Coppola\textsuperscript{a)} and Alessandro Gomez\textsuperscript{b)}

Department of Mechanical Engineering, Yale Center for Combustion Studies, Yale University, New Haven, Connecticut 06520-8286, USA

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Opposed-jet flows have been shown to provide a valuable means to study a variety of combustion problems, but have been limited to either laminar or modestly turbulent conditions. With the ultimate goal of developing a burner for laboratory flames reaching turbulence regimes of relevance to practical systems, we characterized highly turbulent, strained, isothermal, opposed-jet flows using particle image velocimetry (PIV). The bulk strain rate was kept at 1250 s\(^{-1}\) and specially designed and properly positioned turbulence generation plates in the incoming streams boosted the turbulence intensity to well above 20\%, under conditions that are amenable to flame stabilization. The data were analyzed with proper orthogonal decomposition (POD) and a novel statistical analysis conditioned to the instantaneous position of the stagnation surface. Both POD and the conditional analysis were found to be valuable tools allowing for the separation of the truly turbulent fluctuations from potential artifacts introduced by relatively low-frequency, large-scale instabilities that would otherwise partly mask the turbulence. These instabilities cause the stagnation surface to wobble with both an axial oscillation and a precession motion about the system axis of symmetry. Once these artifacts are removed, the longitudinal integral length scales are found to decrease as one approaches the stagnation line, as a consequence of the strained flow field, with the corresponding outer scale turbulent Reynolds number following a similar trend. The Taylor scale Reynolds number is found to be roughly constant throughout the flow field at about 200, with a value virtually independent of the data analysis technique. The novel conditional statistics allowed for the identification of highly convoluted stagnation lines and, in some cases, of strong three-dimensional effects, that can be screened, as they typically yield more than one stagnation line in the flow field. The ability to lock on the instantaneous stagnation line, at the intersection of the stagnation surface with the PIV measurement plane, is particularly useful in the combustion context, since the flame is aerodynamically stabilized in the vicinity of the stagnation surface. Estimates of the ratio of the mean residence time (inverse strain rate) to the vortex turnover time yield values greater than unity. The conditional mean velocity gradient suggests that, in contrast to the existing literature, the highest gradients are around the system centerline, which would result in a higher probability of flame extinction in that region under chemically reacting conditions. The compactness of the domain and the short mean residence time render the system well suited to direct numerical simulation, more so than conventional jet flames. © 2010 American Institute of Physics.

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I. INTRODUCTION

The stagnation flow produced by two opposed coaxial jets under laminar (steady and unsteady) and, more recently, turbulent conditions has been extensively studied in the literature.\textsuperscript{1–10} The interest was initially motivated by the fact that it provides flame anchoring in a wall-free environment, which simplifies the analysis of fundamental combustion research. It is also relevant to modeling turbulent flames interpreted as an ensemble of laminar flamelets with a distribution of scalar dissipation rates and stoichiometric mixture fractions.\textsuperscript{11} In recent work, we demonstrated the generation of intense turbulence in jet flows with specially designed and properly positioned plates.\textsuperscript{12} We leapfrogged to demonstrating this system in counterflow flames.\textsuperscript{13} We also proposed the turbulent counterflow flame as a benchmark for testing and validating computational models in regimes of practical combustion systems. After legitimizing the effort from a combustion viewpoint, we now step back and examine systematically the isothermal stagnation flow field under intense turbulence.

Understanding the isothermal flow field is a prerequisite to detailed studies of the reactive flow counterpart. It is also the limit field of a flame close to extinction. All the studies on this configuration, either numerical or experimental, have been carried out at low or modest Reynolds numbers.\textsuperscript{1,2,4,14–20} The reason for this is that flames extinguish when the characteristic convective-diffusive time is of the same order as a characteristic chemical time, that is, when the Damkholer number is of unity order.\textsuperscript{21} The convective-diffusive time is the inverse of the scalar dissipa-
tion rate and is related to the strain rate, while the chemical time is affected by the fuel-oxidizer composition and is related to a simplified one-step chemistry that captures the flame heat release. Therefore, for a given fuel-oxidizer composition, there will be a strain rate which will bring the flame to extinction. The extinction strain rate limits the achievable Reynolds numbers, by setting an upper limit to flow rates for a given geometry. This limit and the Reynolds number can be increased by burning in oxygen-rich environment. Even so, boosting of the turbulent Reynolds number by an effective turbulence generation scheme remains inevitable.

The most common approach to turbulence generation is to use a perforated plate downstream of a contracting nozzle. This approach allowed Kostiuk et al.\textsuperscript{2} to achieve a $\sim 12\%$ turbulence intensity, an integral scale $L \sim 2.25$ mm, and a turbulent Reynolds number based on the integral scale, $Re_L = 165$, as measured close to the jet exit. This work remains the most comprehensive description of isothermal flows of this nature, albeit at the reported modest Reynolds numbers. These authors also were the first to report a significant increase in the rms velocity as the flow stagnates. Mastorakas et al.\textsuperscript{22} discussed low-frequency flame motion in a combustion study in a similar configuration, which resulted in higher rms temperature values, as revealed by high-pass filtering of the data. The presence of large-scale intermittence of the mixing layer in the axial direction was also recognized by Geyer et al.\textsuperscript{23} without commenting on the origin of this instability. The study of Lindstedt et al.\textsuperscript{4} on opposed isothermal flows using both particle image velocimetry (PIV) data and modeling makes no comments on this effect.

Of relevance to the present discussion and not widely recognized is the fact that the opposed-jet configuration is a hydrodynamic unstable configuration even at high $Re$ and strain rates, as it is characterized by a pitchfork bifurcation with two stable solutions with the stagnation line positioned close to either nozzles and one unstable solution, with the stagnation line half way between the nozzles.\textsuperscript{5,7,24,25} A probable cause of the instability, similarly to sudden-expansion flows,\textsuperscript{26–29} is the presence of strong recirculation bubbles and their dynamic coupling either in the vicinity of the turbulence generation plates within the nozzle or at the exit of the jets. One may question whether this instability is distinct from true turbulence, in which case it would artificially masks the turbulent features of these flows, as detected, for example, by rms measurements. As a result, a robust statistical analysis of the data is necessary to assess this point.

The goal of the present contribution is twofold. First, we develop an appropriate data analysis to unveil turbulence characteristics in the vicinity of the stagnation plane, using proper orthogonal decomposition (POD) and some novel conditional statistics. The reason for focusing on the fluid dynamics in the vicinity of this plane is that it plays a critical role to the flame anchoring, which inevitably occurs within a very short distance (less than 1 mm) from it. Therefore, tracking the motion and the topology of this plane will also be useful in subsequent combustion studies. Second, we analyze key features of isothermal flows in this configuration under conditions of intense turbulence $[O(1000) \text{Re}_c]$, unlike all previous studies in the literature to date.

II. EXPERIMENTAL SETUP

The experimental apparatus (Fig. 1) is composed of two identical nozzles, aligned along their axis of symmetry and facing each other at a distance of $1.5D_{out}$. Each nozzle is surrounded by a larger one providing a coflowing stream to shield the main flow from the surrounding environment. $N_2$ ($O_2$) enters the bottom (top) nozzle and then flow through two honeycombs (not shown), positioned 38 mm apart. Each honeycomb is 19 mm thick with a 0.8 mm cell size. The uniform flow leaving this flow-conditioning section, with the last honeycomb positioned $\sim 38$ mm upstream of the turbulence generating plate, is then forced through this plate\textsuperscript{12} and eventually exits through a $12.7$ mm diameter ($D_{out}$) opening at the nozzle exit section. The selection and position of the plate ($\sim 42$ mm from the nozzle exit section) is aimed at maximizing the turbulent intensity, with typical values of $u'/U$ at 0.22 (as measured by hot-wire anemometry) and the uniformity of the flow at the nozzle exit. Different gases are used as the system, along with the gas supply, is designed for fundamental research in turbulent combustion. The flow rate is controlled by digital flow controllers (Teledyne-Hastings 300D series) providing a $\pm 0.5\%$ readout accuracy and a $\pm 0.2\%$ full scale one. The PIV system is operated in the cross-correlation mode using a multipass cross-correlation algorithm with interrogation window shifting and deformation to maximize resolution and minimize vector drop out. The system optical magnification is about 1:1, which results in a field of view of about $15 \text{mm} \times 15 \text{mm}$. The final window size is 32 by 32 pixels, equivalent to a physical size of $0.24 \times 0.24$ mm which, with 50% overlap, produced a vector every $\sim 0.12$ mm. Further details of the PIV setup are in Ref. 13.

The turbulence generator plate, when present, is placed at its optimal distance from the nozzle exit\textsuperscript{12} in each nozzle.
Several experiments are conducted: a flow baseline case, in the absence of plates in either stream (NN case), the top-bottom (TB) case with identical plates in both nozzles, and the TN and NB cases with only one plate present in the top nozzle and bottom one, respectively. The last three cases are of all of relevance to combustion applications. Unless explicitly stated otherwise, most of the presented data pertain to the TB case. The bulk strain rate, $SR = U_b/(H/2)$ is set to $\sim 1250$ s$^{-1}$, equivalent to a bulk flow velocity on the oxygen side $U_b \sim 11.9$ m/s and to an engineering Reynolds number, $Re \sim 10,000$. The turbulent integral and Taylor length scales, $L$ and $\lambda$, are measured 0.5 mm above the bottom nozzle exit section by hot-wire anemometry and are found to be, respectively, $\sim 4.5$ and $\sim 0.6$ mm, resulting in $Re_L \sim 960$ and $Re_\lambda \sim 190$, on the basis of the integral and Taylor scale, respectively. The corresponding values extracted from the PIV data will be discussed in Sec. III.

All quantities of interest are derived by analyzing large sets of double exposure images (785 for each of the NB, TN and TB cases; 471 for the NN case). The derivatives were estimated using Richardson’s finite difference scheme without smoothing the gradients of the measured field.$^{30}$

### III. RESULTS AND DISCUSSION

Figure 2 shows the phase plot of the instantaneous velocity components, u and v in three cases: NN with no turbulence generation plates, TN with one plate in the top burner, and TB with the plates on both sides. The figure shows all the $(u,v)$ states accessed by the system at a few points along the centerline, measured from the midpoint between the two burners: $z = \pm 3$ mm and $z = 0$ mm. In the NN case [Fig. 2(a)], as expected, all the points fall close to each other in a very narrow domain, indicating that at each location, there are only small velocity fluctuations. Still, the stagnation plane, $z=0$ mm, at the centerline is accessing both positive and negative values, implying that the $z=0$ mm point finds itself on either side of the stagnation plane. Case TN [Fig. 2(b)] shows a cloud of data points with a “centroid” shifted toward the side with the plate, where the largest spread is observed. Focusing on the u spread, we observe that now even the point at $z=\pm 3$ mm is accessing both positive and negative values, with a larger spread on the side of the turbulence generator, indicating a possibly large shift of the stagnation plane. In the TB case [Fig. 2(c)], this situation is repeated, but now the spread is symmetric. These results are in qualitative agreement with previous experiments,$^{23}$ but here the oscillations are significantly more pronounced. The very small velocity fluctuations in the NN configuration suggest that the stagnation plane fluctuations may also be a consequence of small fluctuations in the gas supply. In the other two cases, on the other hand, stagnation plane oscillations are certainly caused by the large-scale structures generated by the plate.

A concern arises that these oscillation may be associated with a stagnation flow intrinsic instability.$^{7,8,24}$ The “bouncing around” of the stagnation plane due to these instabilities may affect the statistics and specifically the velocity fluctuations even though it may not be strictly classified as part of the turbulent motion. Indeed, visual observation confirmed the presence of low-frequency [O(100 Hz)] or less] bouncing up and down and tilting of the stagnation plane. This phenomenon could help explain, at least in part, the disagreement between Reynolds stress models and experiments reported in Refs. 4 and 19. In fact, in the classical unconditional statistics, time averaging at fixed spatial locations in a laboratory frame of reference implies averaging “different” flows because of the different positions of the instantaneous stagnation plane. Reynolds-averaged Navier-
Stokes (RANS) models are not capable by design to deal with such situations and alternative approaches have to be pursued for numerical simulation and/or experimental data analysis, possibly by extending the simulation domain to the turbulence generation plates.

A statistical data analysis that takes into account the system large-scale/low-frequency dynamics and separates it from the turbulence fluctuations is needed to provide insight into the systems physics and suggest appropriate modeling approaches. In the ensuing discussion, we present three different tools to analyze the flow field: (a) conventional or nonconditional statistics, to compare the data with those in literature, by taking a fixed coordinate system anchored at the midpoint between the two burners, that is relevant to the steady mean flow field features; (b) POD that may help filter the large-scale instabilities unrelated to turbulence; and (c) a method to condition the statistics to the instantaneous stagnation plane, or, more properly, line in two-dimensional measurements.

A. Nonconditional statistics

A cylindrical coordinate system \((r, z)\) is used with origin at the intersection between the mean stagnation plane and the system geometric centerline. Positive \(z\) indicates locations between the stagnation plane and the top nozzle (Fig. 1). The PIV snapshots were time-averaged to extract mean and rms flow variables. The available number of instantaneous snapshots, for each case, produced well converged mean and acceptable fluctuating components. The statistical uncertainty was estimated assuming a general distribution for the samples and 95% confidence interval. Error propagation rules were applied to estimate the uncertainties in the derived quantities and found the maximum error in the region of interest (near centerline) at \(±0.35\text{ m/s}\) and \(±0.21\text{ m/s}\) for the mean axial and radial velocity components, \(±0.3\text{ m/s}\) and \(±0.19\text{ m/s}\) for the axial and radial rms velocity, and \(±570\text{ s}^{-1}\) and \(±430\text{ s}^{-1}\) for the mean gradients \(dU/dz\) and \(dV/dr\).

Figure 3(a) shows the profiles of the mean axial and radial velocity components along the radial coordinate at three different axial locations with respect to the mean stagnation plane. Measurements were performed with and without the turbulence generation plate, but no significant differences were observed in the mean measurements. Consequently only one set of measurements is presented for the top burner in the TB case, since the bottom burner produces virtually identical data. Trends are conforming to expectations. Further from the stagnation plane, near the burner exit, the axial velocity profiles exhibit the characteristic M-shape \((z=−7.0\text{ mm})\), with values close to the nozzle wall overshooting the rather flat profile in the vicinity of the axis of symmetry. When velocity measurements are performed on a single nozzle in a free jet configuration, no overshoot is detected.\(^{12}\) The finite separation between the burners is responsible for this change in the inlet velocity profile. This finding can be explained qualitatively from Bernoulli’s principle.\(^{32}\) On the axis of symmetry at the mean stagnation surface, the mean axial velocity is zero and the mean static pressure attains its maximum. As a consequence, the streamlines in the proximity of the axis are subject to a larger adverse mean pressure gradient as compared to those near the nozzle walls. To make quantitative comparisons between computational and experimental results, it is clearly indispensable to have measurements of the nozzle velocity profiles.

The degree of uniformity increases as the stagnation plane is approached and the overshoot disappears for the axial component. The radial component is linear with the radial coordinate through most of the domain, until mixing with shroud gas becomes significant. The slope also increases as the stagnation plane is approached, indicating an increase in the strain rate, which is consistent with the behavior of opposed-jet flows with finite separation between the two streams. Figure 3(b) shows the axial profiles along the centerline of the rms values of both velocity components. The profiles are in qualitative agreement with previous measurements,\(^{2,4}\) with a strong peak at the stagnation plane for \(u'\) and a less pronounced one for \(v'\). This peak will have to be probed further by other data analysis techniques to ensure that it is indeed a feature of the turbulence, as opposed to some instability artifact.
is responsible for large-scale oscillations of the stagnation line as a whole. Figures 5(c) and 5(d) show a sequence of the oscillation of the stagnation plane captured by reconstructing the flow field with only mode 0 and mode 1. The stagnation line fluctuates from a mean position at approximately $z=2$ mm (c) to $z=-2$ mm (d). The reconstruction of the flow field with only mode 2 added to mode 0 shows that this mode is responsible for the stagnation line tilting about its center [Figs. 5(e) and 5(f)]. These snapshots were chosen among the most extreme cases to underline the effect of the mode structure. By considering the system axial symmetry, a precession motion about the mean flow centerline seems to be a plausible description of this type of instability superimposed on the mean turbulent flow. As a result, the stagnation line motion can be tentatively decomposed in an axial oscillation, a precession motion about the mean centerline and the turbulent contribution.

A phase plot (Fig. 6), in which the axes represent the POD coefficient relative to modes 1 and 2, can be used to describe the relationship between these two modes. If the two modes of oscillation (axial and processional) were in phase, the scatter plot would present a circular shape, while a distortion to circular orbits of different radii would indicate a phase difference. The plot in Fig. 6 indicates that the two modes of oscillation are not correlated, therefore suggesting that they may be representative of different flow dynamics. As an additional bonus from the POD treatment, we can analyze the large-scale oscillation frequency separating the axial oscillations (reconstructing the flow with only mode 0 and mode 1) from the tilting motion (reconstructing the flow with only mode 0 and mode 2). Visual observations suggest that the oscillations are a low-frequency phenomenon, with characteristic frequency on the order of 100 Hz, as revealed by high-speed movies. The power spectrum as measured by hot-wire, 0.5 mm above the bottom nozzle exit section was reported not to show peculiar peaks at any frequency. This may suggest that the oscillations are chaotic in nature, in agreement with previous findings, and it seems plausible to assume that they may be triggered by the dynamics of the recirculation zones in the vicinity of the turbulence generation plate also in view of the results in Fig. 2.

C. Stagnation line conditional statistics (SLCS)

We now describe a third and novel method to analyze the database on conditional statistics with respect to the position of the instantaneous stagnation surface. Such a surface plays a key role in the flow field especially from a flame stabilization perspective, since under burning conditions and at large strain rates, the flame will inevitably anchor in its immediate vicinity ($<1$ mm). An analysis of critical points was used to identify the instantaneous stagnation line, that is, the intersect of the stagnation surface with the two-dimensional PIV plane, relevant critical points (saddle, vortices, etc.) and three-dimensional effects. Instantaneous and conditional mean strain rates at the instantaneous stagnation line were also calculated since they are of direct relevance to flame extinction. The estimated statistical uncertainties are at $\pm 0.23$ and $\pm 0.24$ m/s for the mean axial and
radial velocities, ±0.2 and ±0.27 m/s for the axial and radial rms velocities, and ±550 and ±430 s⁻¹ for the conditional mean gradients dU/dz and dVₛ/ds. After introducing the method and showing samples of instantaneous flow field, we present averages and rms results from this perspective and conclude with a comparison of the relative merits of this technique with respect to POD.

1. Stagnation line tracking and critical point analysis

The SLCS analysis is based on the identification and tracking of the instantaneous stagnation line in each PIV snapshot. The instantaneous stagnation line is, as its time-averaged counterpart, a flow separatrix, i.e., a line that divides the flow in two regions, the top nozzle flow region, and the bottom nozzle one. The approach implemented in this work exploits the concepts of flow topology and critical points analysis. A critical point is defined as a point in the flow where the velocity is zero in an appropriate frame of reference. The slope of the streamline, at the point, is indeterminate⁴⁸ and the critical point can be classified by the nature of the eigenvalues of the local velocity gradient tensor⁴⁸,⁴⁹ Critical points are classified in nodes, foci, saddles and bifurcation lines or asymptotic trajectories,⁴⁸ which, if closed, are called limits cycles. While the full 3D information is needed to describe the flow topology,⁴⁸ PIV measurements only provide information in a 2D plane, which

FIG. 5. Vector representations of modes 1 (a) and 2 (b) for case TB. Snapshots of the vector field reconstructed with only modes 0 and 1 (c and d). Snapshots of the vector field reconstructed with only modes 0 and 2 (e and f). TB case. Vector fields are undersampled for readability.
could lead to misinterpretation. Nevertheless, as demonstrated by many authors, sectional streamlines can provide useful insight into the flow structure. Details of the tracking algorithm to identify the stagnation line are in the Appendix.

As an example, we present an instantaneous snapshot in Fig. 7. The stagnation line is represented by the thick gray line. The saddle points are represented by the black circles and the nodes/foci (vortices) by the white circles. If one is able to find all the saddle points belonging to the stagnation line, then by deploying the associated sectional streamlines in the stretching direction, it is possible to identify the stagnation line. In reality, the stagnation line is also defined by other critical points such as nodes and/or foci (white circles in Fig. 7), representing sectional streamlines end points in this specific circumstance (e.g., sinks) and their effect is to convolve the stagnation line. These critical points can only be sinks (points where sectional streamlines end) since, by definition, if a source (point where sectional streamlines depart from) were present, it would necessarily be surrounded (in our case) by two stagnation lines, as verified in what follows. The algorithm was tested for a random sample of snapshots of different complexities by computing a large number of sectional streamlines with origin at the top and/or bottom edges of the PIV domain and verifying that the identified streamline was indeed a flow separatrix. The algorithm performed very well, producing the required stagnation lines in 783 out of 785 snapshots for the TB case, demonstrating its robustness. We also notice that the saddle points can be classified in primary and secondary points: a primary point is defined as a point whose sectional streamlines, deployed along the eigendirection associated with the positive eigenvalues, belong to the stagnation line and/or whose sectional streamlines, integrated backward in time along the direction associated to the negative eigenvalue, end on opposite sides (top and bottom) of the flow field. A secondary point is any point not satisfying this criterion. Our algorithm shows that the stagnation lines, defined as the shortest path between two extreme points, are also the lines joining the primary stagnation points.

The algorithm produced multiple paths (stagnation lines) in some instances, as shown in Fig. 8, which, at first sight, may seem unphysical. We verified all these “unusual” cases and found that they correspond typically to conditions with strong three-dimensional effects, implying a flow condition in which the flow is at large angles with respect to the PIV plane, and resulting in a number of nodes/foci, which dramatically alter the flow in limited regions of the domain, as shown in Fig. 8. The two-dimensional cut through the flow domain, resulting from the 2D PIV technique, prevents this situation to be properly described. This type of situation was verified in fewer than 15% of the 785 available snapshots for case TB, that were excluded from the statistics.

The difficulty in accounting for these cases lies in our lack of understanding of the impact of such flow condition on the flow and, for burning situations, flame dynamics. Nevertheless, to verify that the discarded snapshots do not affect the statistics, the average value (between the two intersection points at the two stagnation lines) of the quantity

FIG. 6. Phase plot of the POD coefficients, $a_1(t)$ and $a_2(t)$, relative to mode 1 (abscissa) and mode 2 (ordinate), respectively. TB case.

FIG. 7. (Color online) Snapshot of the velocity field, represented by the velocity vectors. Superimposed are the flow sectional streamlines originating from the top nozzle and from the bottom nozzle (thin lines). The thick gray line is stagnation line, the white circles are nodes/foci, and the black circles are stagnation points.

FIG. 8. (Color online) Instantaneous sectional streamlines superimposed to vector field in the presence of strong three-dimensional effects. Superimposed are the flow sectional streamlines originating from the top nozzle and from the bottom nozzle (thin lines). The thick gray line is stagnation line, the white circles are nodes/foci, and the black circles are stagnation points.
of interest was also considered, and no noticeable change was noticed in results. It should be pointed out that, in many instances, the fluid element “entering” the PIV domain may leave from only one side, generating a more local stagnation line surrounding the region of flow injection and ejection. Our algorithm is capable of capturing all these situations by exploiting graph theory. In fact, it suffices to find the shortest path using the starting and end points from the same side or a standard algorithm to find cycles. All these cases have not been removed from the statistics, since they produce a single instantaneous stagnation line and the presence of a 3D region in the flow would not produce results different from what could be achieved with a standard technique, such as POD or nonconditional statistics.

In some respect, the capability of clearly identifying 3D effects may be an important strength of the algorithm, as we now have a simple way to characterize them and study their impact on the flame physics, without the need to resort to the much more involved 3D-PIV. Of relevance are also cases with vortices embedded in the stagnation line, resulting in a roll up of the stagnation line itself (Fig. 8), which, along with the three-dimensional effects, raises the general issue of how to define a highly convoluted stagnation line as flow separatix or flow limiting line.

Once the stagnation line is identified, it is possible to track it, analyze its motion, and perform statistics relative to the stagnation line. Figure 9 shows the histograms of the intersection of the instantaneous stagnation line and the mean centerline for the same three cases: NN, TN, and TB. The width of the histogram grows progressively in the three cases, with a $\pm 0.5$ mm excursion in the NN case, growing to about $\pm 5.0$ and $\pm 6.5$ mm in the TN and TB cases, respectively. Case TN exhibits also some skewness with respect to the symmetric TB case as a result of the asymmetry of the two burners and the penetration depth of the turbulence fluctuations in the opposite half of the flow field. One may draw the conclusion that the motion is triggered by an instantaneous momentum imbalance, likely caused by the interaction of random large-scale eddies with the stagnation surface. In reality, since the integral scale is on the order of a few millimeters (see below), while the radial extension of the plane is on the order of the nozzle diameter, i.e., $\sim 12.0$ mm, an instantaneous momentum imbalance would just deform the plane, possibly creating v-shaped stagnation lines.

One has to take into account the fact that, as discussed earlier, the opposed-jet configuration is a hydrodynamic unstable configuration even at such high Reynolds numbers and strain rates. Our experiments without coflow show that under the same experimental conditions, but without turbulence generators, the stagnation surface gets steadily attached to either nozzle outlet, which happens also under laminar conditions. Introducing the coflow brings the stagnation surface back at about the mid-distance between the nozzles, and small oscillations can still be observed (Fig. 9). When one or both turbulence generators are in place, the amplitude of the oscillations increases dramatically [Fig. 9, see also Figs. 2(b) and 2(c)]. This finding suggests that the coflow brings some stability to the system by pushing radially outward the recirculation bubbles, and thus “weakening” the cause of the instability. The main flow and coflow mass sources are decoupled, but the radial jet generated by the opposed-jets impingement acts as a coupling element in the system. One can argue that the instantaneous perturbations caused by the incoming eddies alter the direction of the radial jet, which is temporarily shifted toward one or the other nozzle, forcing the coflow impingement surface away from its neutral position. This effect is transient, producing a temporary oscillation of the stagnation surface away from the “stable” position, soon overcome by the stabilizing effect of the main flow and the coflow.

2. Conditional statistics coordinate system

We can now define a curvilinear coordinate system $s$ along the instantaneous stagnation line [Fig. 10(a)], with origin at the left of the domain. In the new coordinate system, the velocity at the stagnation surface has only a component along $s$, $V_s$, corresponding to the local tangent to the streamline and zero component along the local normal. All the data are then presented in terms of the components $(u,v)$ of $V_s$ as a function of the radial component of $s$, $r$, to facilitate comparison with the nonconditional counterpart. In line with the conventional nonconditional approach, we measured the quantities of interest along the mean stagnation streamline [Fig. 10(a)], but placing the origin of the axial coordinate at the intersection of such a streamline with the instantaneous stagnation line [Fig. 10(a)]. The analysis is again “translated” in terms of axial coordinate $z$. With this choice of origin, the axial distance $z$ can no longer be seen as coincident with the geometric midpoint between the nozzles. In fact, as the origin moves with the instantaneous stagnation line [Fig. 10(b)], it moves closer to either nozzle, and it is easy to see that under the extreme case of the instantaneous stagnation line shifted very close to, say, the top nozzle, one can extract data samples only from the opposite side of the line, going toward the bottom nozzle, for a distance equal to the nozzle separation. Under this conditions, the newly defined axial coordinate $z$ can extend, in principle, from $z=-19$ mm to $z=19$ mm, producing an effective nozzle separation of

![Figure 9](image_url)
The amplitude of the stagnation line motion depends on the presence of the turbulence generators, increasing from NN to TN to TB, implying that the effective nozzle separation would be increasing accordingly. The mean stagnation streamline is almost parallel to the geometric centerline, therefore, the projection of the velocity along this line is a close approximation to the axial velocity, and in the ensuing discussion, we will make no distinction between the two.

D. Comparison among the three data processing

We now make a direct comparison among three sets of rms data: the data from the nonconditional statistics, those filtered using the POD technique by removing modes 0, 1, and 2, as per discussion of Fig. 5, and the data from the conditional statistics with respect to the stagnation line. We will start by revisiting the rms profiles along the mean stagnation streamline that had been shown only for the nonconditional statistics in Fig. 3(b). Mean stagnation streamline and stagnation line should not be confused: the first on average is roughly coincident with the burner axis and the second is the intersection of the stagnation plane with the laser sheet used in PIV.

Figure 11 shows the comparison of the rms axial component along the mean stagnation streamline (burner axis). The removal of mode 1 and, to a much lesser extent mode 2, induces a dramatic drop in \( u' \). The \( v' \) profile, on the other hand, is not greatly affected, but shows a pronounced peak around \( z \sim -0.9 \). It exhibits a short region of decay, before increasing again at \( z \sim \pm 5 \). This profile, up to the point of maximum before the dip, is in good agreement with the literature. The conditional statistics offers a more extreme picture, to some extent, than the one showed by the POD, with a more pronounced dip in \( u' \) and peak in \( v' \). The velocity rms along the mean stagnation line conforms to the picture of stagnation flow against a plate. In fact, the axial fluctuations \( u' \) tend to decrease approaching the \( z=0 \) point (representative of the stagnation point), while the radial fluctuations \( v' \) tend peak at the same point. Unlike the stagnation point flow onto a flat surface, here the \( u' \) values do not fall to zero because the instantaneous stagnation line is highly convoluted and intersects the mean stagnation streamline at angles other than 90°, introducing nonzero \( u \) and \( v \) entries in the statistics. This finding may help explaining the difference between POD-filtered data and conditional statistics. With the present choice of coordinate system for the conditional statistics, the flow can probably be described by a stagnation point flow over a continuously wobbling surface. This is further supported by the \( v' \) fluctuating component, which is in fairly good agreement with the nonconditional one, away from \( z=0 \). The large difference in the levels...
of $u'$ between conditional and POD data with respect to non-conditional data underline the large contribution of the large-scale/low-frequency oscillation of the stagnation line to the statistics. The stagnation line instability is therefore responsible for masking the truly turbulent effects by artificially increasing the turbulent fluctuations. The turbulence-induced stagnation line convolution is responsible for the differences between conditional and POD profiles. Also, moving away from the center, $z=0$, one can see that the differences in $u'$ levels tend to decrease, suggesting that the oscillation does not affect the entire domain.

Figure 12(a) shows the rms axial and radial velocity profiles evaluated by the nonconditional statistics at a few axial locations as a function of the radial coordinate. The presence of the turbulence generator produces the desired effect of boosting dramatically the turbulence levels as compared to values on the order of 0.2 m/s in the absence of the plates. The profiles show good radial homogeneity near the centerline. The radial uniformity for $u'$ tends to deteriorate as the stagnation plane is approached, with the growth rate of the axial fluctuations decreasing at the centerline as compared to larger radial positions, probably as a consequence of a mixing layer instability. Whereas the $v'$ are reasonably uniform regardless of the distance from the mean stagnation plane, $u'$ increases as the stagnation plane is approached regardless of radial location. Turning to the POD-filtered data [Fig. 12(b)], we note that subtracting mode 1 and mode 2 has a modest effect on $v'$ and fairly dramatic one on $u'$, that now becomes much more homogeneous throughout the flow field. Generally, in view of the modest difference between the filtered data of $u'$ and $v'$, the level of isotropy of the field is also good. This is yet further proof that the POD filtering approach manages to remove the artifacts of the opposed-jet instability that artificially boost the velocity fluctuations beyond those caused by turbulence. To avoid excessive cluttering of the plot, we show the comparison between nonconditional data and the SLCS ones in a separate graph [Fig. 12(c)]. The comparison is performed just along the stagnation line. When compared to the nonconditional counterpart, we see definitely lower absolute values of $u'$, $n$ underlining the artifact of the large-scale fluctuations and turbulent jets interaction (stagnation line convolution) on the velocity rms estimates, and the effectiveness of also the SLCS approach to separate them. Also in this case, the large-scale fluctuations have a much more pronounced effect on $u'$ rather than $v'$, suggesting that the fluctuations result mostly in the "bouncing" up and down of the stagnation line. This observation suggests that the RANS models are unable to predict the correct fluctuations profiles in the opposed-jet configuration because of their intrinsic inability to model an unsteady field. Generally, the conditional statistics data are in reasonable agreement with the POD-filtered ones [Fig. 12(b)] with some discrepancy in $u'$ around $r=0$. This is because the $z=0$ origin for the conditional statistics is on the instantaneous stagnation line, therefore following it in its motion, while the $z=0$ origin in the POD statistics is fixed with respect to the laboratory frame of reference and at each instant can be on either side of the instantaneous stagnation line.

Turning our attention to the turbulent length scales, we now follow the evolution of length scales in the straining field and relate them to the measured gradients. Integral scales can be estimated by integration of the two-point one-time correlation coefficients, $R_{ij}$, by full-field approaches, such as 2D spectral analysis or 2D autocorrelation estimate,\textsuperscript{46} or by spatial filtering techniques,\textsuperscript{34} possibly coupled to vortex detection schemes to estimate the vortex size. The analysis of the two-point one-time correlation coefficients is widely accepted and allows for comparison with the literature. Therefore, it will be used for the three data analysis methodologies. The 2D spectral analysis or autocor-
relation requires a region where the length scales involved are statistically homogeneous, and the present flow field is not the ideal target, in view of the axial inhomogeneity. The two-point correlation coefficients were estimated along the axial distance, starting from the edge of the PIV domain closer to the nozzle exit section (z = −8) to the other side. At every point, integration was taken from the point of interest to the point where the coefficient decreases to a value of 0.1, toward the stagnation line. Although unusual, this value was chosen in place of the first zero crossing, to account for the lack of zero crossing near the stagnation line, possibly as a consequence of the larger measurement error, and at the same time to have an estimate comparable to the previous measurements.47

The longitudinal integral length scale and transversal inner length scale are identified respectively as \( L_u \) and \( \lambda_v \) [Figs. 13(a) and 13(b)]. Near the nozzle exit section, \( z = −8 \), the nonconditional velocity longitudinal integral length scale, \( L_u \approx 5.0 \) mm, while the equivalent length scales estimated by the POD filtered velocity and by the SLCS data analysis are both \( \approx 3.0 \) mm. This suggests that the nonconditional length scale is overestimated, since the two-point correlation is affected by the stagnation line large-scale/low-frequency instabilities, notwithstanding the fact that there is an inherent underestimation in the calculation since the correlation coefficient does not drop to zero because of the limited domain size.47 Furthermore, evidence in literature47 suggests that the PIV domain size to integral length scale ratio is an indicator of length scale accuracy and a ratio in the range of 2–8 is suggested for accurate estimates, a criterion that is met in the present work, more so for the conditional and POD filtered estimates. Notice that the conditional integral scales do not change significantly if estimated from the original or POD-filtered fields (not shown), confirming the consistency of the conditional statistics. Plotting the integral scale along the burner centerline [Fig. 13(a)] through the mean stagnation point shows that the nonconditional \( L_u \) first increases to a maximum at \( z = ±5 \) mm and then decreases approaching the mean stagnation line at \( z = 0 \). The initial increase may be an artifact of the large-scale oscillations, as it is not present in either the POD filtered or the conditional estimates. The decrease is in line with the idea that the large-scale structures are deformed by the straining field, as they are convected by the flow, in fair agreement with previous results.2 In reality a different picture appears from the analysis of the conditional or POD filtered data, where there is a clear decay of the longitudinal integral length scale approaching the stagnation line at \( z = 0 \), and the decay is more pronounced in the case of the conditional length scale, reaching a value of \( \approx 0.5 \) mm, than for the POD-filtered length scale, reaching a value of \( \approx 1.5 \) mm. The transversal integral scale (not shown) decreases from about 1.6 mm at \( z = −8 \) to about 1.1 mm at \( z = 0 \), to a good approximation, independently of the data filtering and conditioning and can be considered approximately constant for practical purpose. The conditional or POD filtered results suggest that structures elongated in the axial direction downstream the nozzle exit section deform into quasicircular structures at the stagnation line, as a consequence of the straining field, in agreement with the observed near stagnation line vortices (Figs. 7 and 8). The differences between conditional and POD-filtered longitudinal length scale profiles are a consequence of the stagnation line convolution, unaccounted for in the POD-filtered statistics. The inner or Taylor scale [Fig. 13(b)] is estimated from the velocity rms and the velocity gradients rms.48 The spatial derivatives are computed using the second order accurate Richardson scheme.30 The transverse Taylor scales are relatively constant and about 0.7–0.8 mm in all the cases.

The nonconditional estimates again show some differences that can be associated with the large-scale fluctuations affecting the estimate and increasing considerably toward the stagnation line. The integral length scale behavior results in a large overestimate of the associated turbulent Reynolds number [Fig. 13(c)] for the nonconditional data near the nozzle mouths, at about 1100, versus a consistent estimate of about 600 in the other cases. Furthermore, in agreement with literature, the nonconditional statistics shows that the \( \text{Re}(L_u) \) increases toward the mean stagnation line, while both POD

FIG. 13. Comparison of the three data analyses for the axial profiles for the nonconditional (plain line), POD (dotted line), and conditional (dashed line) estimates of the longitudinal integral scale (a), transversal Taylor scale (b), integral scale based Reynolds number (c), and Taylor scale based Reynolds number (d).
filtered and conditional estimates show a more or less pronounced decrease, implying that the actual turbulent field experienced by a flame near the stagnation line is not necessarily well described by estimates at the nozzle exit section. The Taylor scale Reynolds number \( \frac{d}{H} \) is instead quite unaffected and on the order of 200 regardless of the data analysis. As a result, it is a more robust indicator of level of turbulence as compared to the outer scale Reynolds number.

Next, we consider the strain rate that, as discussed earlier, plays a critical role in combustion applications with respect to flame extinction. The axial velocity gradient in the \( z \)-direction and the radial velocity gradient in the \( r \)-direction, \( \partial u / \partial z \) and \( \partial v / \partial r \), are plotted as a function of the radial coordinate and the axial one in Figs. 14 and 15, respectively. In Fig. 14 the measurements are performed along the stagnation line, whereas in Fig. 15 they are performed along the stagnation streamline. In both figures, the plots on the top row result from the nonconditional statistics, those on the center row from the POD-filtered approach and those at the bottom from the SCSL approach. Consistently with mass conservation, the gradient of the axial velocity component, that is coincident with the strain rate, is approximately twice the radial component gradient near the mean stagnation line. The nonconditional mean gradient profiles [Fig. 14(a)] are relatively flat, but with a considerable spread that is probably reflective of the large measurement uncertainty. Mode 1 and mode 2 contribute modestly to the straining field since the data [Fig. 14(b)] are similar to those for the nonconditional statistics [Fig. 14(a)]. We notice here that in these cases the measurement uncertainty is of the same order as the variations. If we now turn to the conditional statistics, using the curvilinear coordinate system as previously defined, we can estimate the instantaneous velocity gradient experienced by a fluid element at the stagnation line as \( \partial v_s / \partial s \), where \( v_s \) is the instantaneous velocity along \( s \). Figure 14(c) shows the mean and rms values of such velocity gradient, providing a different picture from the conventional or POD statistics. The mean velocity gradient is higher at the center of the domain than at its periphery, and the velocity gradient standard deviation is again constant, suggesting that higher instantaneous velocity gradients will be experienced closer to the centerline, consistently with the fact that the extinction holes are found mostly in the center of the domain under burning conditions. The velocity gradient rms absolute values are also much lower than the nonconditional ones (not shown), reflecting the effect of turbulence structures interacting at or with the stagnation line, purged of the large-scale fluctua-

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**Fig. 14.** Profiles of axial and radial velocity gradients along the stagnation line. (a) Nonconditional statistics. (b) POD filtered statistics. (c) Conditional mean and rms velocity gradient.

**Fig. 15.** Profiles of axial and radial velocity gradients along the stagnation streamline. (a) Nonconditional statistics. (b) POD filtered statistics. (c) Conditional mean and rms velocity gradient.
tions. In addition, the lower data scattering is probably indicative of the robustness of the data analysis.

Turning to the profiles of the same variables as a function of the axial coordinate [Fig. 15(a)], we notice that there is little difference between the unconditional and POD- filtered profiles, and the bell shaped profiles along z peak around the mean stagnation point, suggesting that large strain rates are more frequent when the stagnation line is around that point. The conditional statistics allow us to estimate the instantaneous velocity gradient along the stagnation streamline, $du/dr$, which is expected to be the same as the unconditional one. In reality, the statistics are substantially different, since with the present choice of origin, the statistical quantities are calculated with respect to the instantaneous stagnation line. The conditional mean and rms [Fig. 15(c)] present a sharp peak at $z=0$, with values for the mean reaching almost 5000 s$^{-1}$, quite different from the previous results. The sharp peak and the very high value imply that the sampling is being performed in a region (around the stagnation line) characterized by higher velocity gradients than the rest of the field. Overall, the strain/stretch rates peaks are at the stagnation line (Fig. 14) and are centered on the mean stagnation streamline (Fig. 15), suggesting that in this region there is a higher probability of extinction.

We can now estimate the ratio, $\tau_r/\tau_e$, between the mean residence time $\tau_r$ and the eddy turnover time $\tau_e$. The mean residence time, $\tau_r \sim 1/S_b \sim 1.0$ ms, where $S_b \sim dV/dr \sim 1000$ s$^{-1}$ (at $r,z=0$), as we have seen, independently of the filtering procedure. The eddy turnover time, $\tau_e \sim L/k^{0.5}$ may depend on the filtering procedure, since the integral length scale $L$ and the turbulent kinetic energy $k$ are greatly affected by the removal of the large-scale oscillations, as seen previously. Using the POD filtered data, we estimated $k^{0.5} \sim 3.4$ m/s and $L \sim 2.8$ mm, therefore $\tau_e \sim 0.8$ ms, and the ratio $\tau_r/\tau_e \sim 1.2$. We conclude that the present configuration has, to some extent, sidestepped the limitation of young turbulence and, at the same time, that rapid distortion theory arguments do not apply to scales on the order of the integral length scale because the requirement $\tau_r/\tau_e \ll 1$ is not satisfied. Furthermore, in view of the millisecond residence time and the compactness of the spatial domain, with the flow field resulting in a disk with vertical dimensions of approximately 1.9 cm and transverse dimensions of less than 10 cm, the system should be better-suited for fully resolved direct numerical simulation studies as compared to turbulent jets at comparable turbulence level.

High-speed movies of particle-seeded flows revealed that the large-scale low-frequency instabilities with the ensuing oscillation and precession of the gas stagnation surface (or line) occur with characteristic times on the order of tens of milliseconds or larger, that is, much longer than the eddy turnover time, regardless of whether such a time is computed from the filtered or unfiltered data. As a result, these instabilities are unquestionably distinct from turbulence and their masking effects need to be screened out to assess the turbulence features of these flows, by using the procedures outlined in this article in the preceding sections.

It is interesting, at this point, to consider the effect of the turbulent flow field on the structure of the local instantaneous stagnation points defining the instantaneous stagnation line. The statistics are now conditional to the local instantaneous stagnation point [Fig. 16(a)], i.e., every such point is localized in each PIV snapshot, and the velocity is measured along its characteristic directions, up to some distance [Fig. 16(b)]. All the collected velocity measurements are then statistically analyzed. The stagnation points are of interest because they are closely related to turbulence pair separation and scalar mixing, both relevant to combustion and, specifically, to extinction/reignition problems. To this end, we define a local curvilinear coordinate system [Fig. 16(b)] with origin on the instantaneous stagnation point and axes $(s,n)$, tangent, respectively, to the sectional streamlines related to the eigendirection of the specific stagnation point. The velocity along $s$ and the velocity along $n$ will be described in terms of their Cartesian components $u$ and $v$, while the velocity gradients are, respectively, $dV_s/ds$ and $dV_n/dn$.

The relevant quantities are then time-averaged at fixed curvilinear distance from the origin. This coordinate has no
simple or intuitive relationship with the standard coordinate system $(r, z)$, although, statistically speaking, the $s$ coordinate will roughly align with $r$ and the $n$ coordinate with $z$. The velocity rms levels reflect the presence of the turbulence generators (Fig. 17). The zero value of the fluctuations is consistent with the definition of critical points as points of zero velocity. We also notice that under these conditions, the stagnation points present isotropic conditions around the origin, up to a few millimeters, along $s$. The velocity gradients along $s$ and $n$ (Fig. 18) show sharp peaks in the mean and sharp dips in the rms at $z=0$. The mean velocity gradients (Fig. 18) are on the order of $\sim 5500$ and $\sim 10000 \, \text{s}^{-1}$, respectively, for $\frac{dV_s}{ds}$ and $\frac{dV_n}{dn}$. Such high mean values suggest that the stagnation points are points where extinction is likely to happen.

**E. Relative merits of POD versus SLCS**

The newly developed algorithm has some advantages as compared to POD, as a very natural and intuitive approach being based on critical point analysis. In contrast to POD, which requires a number of fields to guarantee converged statistics, our approach works on instantaneous velocity field and is able to capture instantaneous events. An interesting, and possibly important, feature is the capability to identify and screen 3D effects. The main drawback with respect to POD is the higher computational cost. Furthermore, POD has a wider range of applicability.

The ability to lock on the instantaneous stagnation surface represents a highly valuable tool for studying turbulent counterflow flames, as the stagnation surface is an approximate indicator of the flame position in diffusion and partially premixed flames. Instantaneous strain rates, conditioned on the position of the stagnation surface, give access to data, which could otherwise be obtained only by more costly and difficult techniques, as by combining planar laser induced fluorescence and PIV measurements. The new algorithm can also be regarded as complementary to POD analysis, since it can be applied to a POD-filtered field.

**IV. CONCLUSIONS**

A systematic experimental study was conducted on highly strained, isothermal, opposed-jet flows under intense turbulence using PIV. The bulk strain rate was kept at $1250 \, \text{s}^{-1}$ and specially designed and properly positioned turbulence generation plates in the incoming streams boosted the turbulence intensity to $22\%$, under conditions that are suitable for flame stabilization. The turbulent flow field was analyzed by three different data reduction techniques: non-conditional, POD, and stagnation line conditional statistics (SLCS) analyses. The conditional analysis is the result of a novel approach relying on the identification and tracking of the instantaneous stagnation line (the two-dimensional counterpart of a stagnation surface in three dimensions), with respect to which the statistical analysis is conditioned. It is labeled SLCS. The ability to lock on the instantaneous stagnation line, at the intersection of the stagnation surface with the PIV measurement plane, is particularly useful in the com-
bustion context, since the flame is aerodynamically stabilized in the vicinity of the stagnation surface. Principal conclusions follow.

- The POD analysis shows the existence of two modes of oscillation, an axial oscillation, and a precession motion about the system axis of symmetry. High-speed movies confirmed that the existence of large-scale low-frequency instabilities are chaotic in nature. However, they are distinct from turbulence, as revealed by a comparison of their characteristic time with the integral turbulent timescale, the first being at least one order of magnitude larger than the latter. The ensuing motion of the instantaneous stagnation line may mask the truly turbulent fluctuations and needs to be properly filtered.

- The SLCS algorithm identifies highly convoluted stagnation lines and even roll up in the presence of vortices. It can also identify and screen three-dimensional effects that may lead to more than one stagnation line in a two-dimensional velocity field.

- The Taylor scale Reynolds number remains relatively constant from nozzle to nozzle at a value of approximately 200 and shows also good consistency among the different data reduction techniques, suggesting that it is a more reliable measure of turbulence than the Reynolds number based on the integral scale.

- The use of POD and SLCS revealed that the POD-filtered integral length scale decreases as one approaches the stagnation line, as a consequence of the straining field, with the corresponding turbulent Reynolds number following a similar trend.

- The SLCS suggests that the highest mean velocity gradients are around the system centerline, which results in a higher probability of flame extinction in that region under chemically reacting conditions.

- Estimates of the mean residence time compared to the vortex turnover time yielded values greater than unity suggest that the turbulence is reasonably well developed in the present flows.

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APPENDIX: SLCS ALGORITHM

The SLCS algorithm to identify the instantaneous stagnation line follows the following main steps.

- **Finding critical points.** The time-averaged flow field is subtracted from the instantaneous ones before identifying the zero velocity points. In our case, because of the nature of the flow, we could localize the critical points by looking directly for the zero velocity condition in each snapshot.

- **Identifying type of critical points.** At each of these points we estimate the velocity gradient tensor \( \nabla \mathbf{u} \). The velocity tensor invariants \((p, q, \lambda)\), as defined by Zhou et al.,\(^{39}\) define the critical point topology: saddle (also stagnation) point for \( q < 0 \); node for \( \Delta > 0 \) and \( q > 0 \); focus for \( \Delta < 0 \) and \( p \neq 0 \); center for \( p=0 \) and \( q > 0 \). Also, a node/focus is attracting if \( p < 0 \) and repelling if \( p > 0 \).\(^{39}\)

- **Obtaining sectional streamlines end points.** Critical point eigenvalues and eigenvectors are estimated to identify the saddle points’ principal directions. We then calculate the characteristic sectional streamlines for each stagnation point and store the corresponding end points.

- **Building adjacency matrix.** The stagnation line can be seen as the line starting at one edge of the PIV domain (e.g., left) and reaching the opposite edge (e.g., right) and connecting one or more stagnation points (and nodes/foci/limit cycles). If one now computes the end points of the sectional streamlines associated with the positive eigenvalues of each stagnation point and identifies them by integer numbers, the system can be described as a graph,\(^{50}\) where the end points are the nodes and the sectional streamlines are the edges. The graph is represented by the adjacency matrix. At this point, the problem becomes that of finding the path from two points on opposite ends of the PIV domain. To this end, we used the end point identifiers to build the adjacency matrix\(^{50}\) of the graph.

- **Applying Dijkstra algorithm.** To find the shortest path between any two nodes, we used a Matlab routine\(^{51}\) based on the Dijkstra algorithm.\(^{50}\) All the sectional streamlines end points (associated with the stagnation points) belonging to the left and right edge of the PIV domain were taken as starting and end points of the paths, respectively.

- **Identifying instantaneous stagnation line.** The previous step can return multiple paths between any two points (left and right). We assumed the shortest path to be representative of a possible stagnation line. The assumption is confirmed by the results. A stagnation flow by definition is characterized by at least one saddle/stagnation point. In a turbulent counterflow configuration, the stagnation line can be quite convoluted and more than one saddle point can be defining it.

- **Identifying three-dimensional flow regions.** The previous step can still return multiple stagnation lines. This situation requires a flow source within the PIV plane, which can be caused by strong three-dimensional effects. To automatically deal with such situations, we exploited the fact that stagnation lines cannot intersect each other, therefore, of all the stagnation line end points, we consider only the two points with the lowest and highest values of the axial coordinate. We then
impose that the starting point with high axial coordinate can only connect to the end point with the equivalent high coordinate, to avoid “crossing.”


50 D. A. Marcus, Graph Theory (Mathematical Association of America, Washington, DC, 2008).