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Brief paper

Supervisory control with state-dependent dwell-time logic and constraints☆

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Abstract

The paper deals with the supervisory control of a nonlinear uncertain system in which the switching is directed by the recently introduced state-dependent dwell-time switching logic. The proposed supervisory control architecture is shown to regulate to zero the state of the system without requiring the switching to stop in finite time. A significant class of systems to which the control architecture can be applied is the class of linear systems with input saturation.

Keywords: Supervisory control; Constraints; Uncertain dynamic systems; Adaptive control; Regulation

1. Introduction

The supervisory control of *linear* uncertain systems is now very well understood (cf. e.g. Morse, 1995, 1996; Narendra & Balakrishnan, 1997; Hockerman-Frommer, Kulkarni, & Ramadge, 1998). In the case of nonlinear systems, the papers of Hespanha and Morse (1999b) and Hespanha, Liberzon, and Morse (2002) have shown that supervisory control based on scale-independent hysteresis switching logic (Hespanha & Morse, 1999a) and on a family of input-to-state (or integral input-to-state) stabilizing controllers guarantees asymptotic regulation to zero of the state of the plant. (See Angeli & Mosca (2002) for an additional study of ISS and supervisory control.) The analysis relies on the property that the hysteresis switching logic stops switching in finite time. In a realistic scenario, however, this property may not hold. This motivated the introduction in De Persis, De Santis, and Morse (2003) of a *state-dependent dwell-time* switching logic. In that paper, the asymptotic convergence of nonlinear switched systems with state-dependent dwell-time is shown in the case the switching occurs among integral input-to-state stable systems driven by input signals with "bounded energy". The analysis can be carried out even for those situations in which switching never stops.

In this paper, the results of De Persis et al. (2003) are used to design a supervisory control scheme for regulating to zero the state of a nonlinear system under suitable assumptions. The contribution can be viewed as a first step towards the design of supervisory control of uncertain processes affected by disturbances or even time-varying uncertainties. An overview of the supervisory control problem for nonlinear uncertain systems is given in the next section, and its solution is presented in Section 3, along with a brief review of some relevant results from De Persis et al. (2003). This general scheme is then applied to a very important problem in Section 4: the supervisory control of linear systems in the presence of input saturation. It is shown that—under the standard assumption of stabilizability and detectability—the proposed control scheme achieves *global* regulation of linear uncertain systems whose eigenvalues lie in the closed left-half plane.

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2. An overview of the supervisory control problem

The problem of interest is that of designing a supervisory control system (Hespanha & Morse, 1999b) which regulates to zero the state $x_{\mathbb{P}}$ of an imprecisely modeled process $\mathbb{P}$ with control input u and output y

$$\dot{x}_{\mathbb{P}} = f(x_{\mathbb{P}}, u), \qquad y = h(x_{\mathbb{P}}) \tag{1}$$

with f and h locally Lipschitz. The model of $\mathbb{P}$ is an unknown member of a family of dynamical systems of the form $\mathscr{F} = \bigcup_{p\in\mathscr{P}} \mathscr{F}_p$ where each $\mathscr{F}_p$ is a singleton consisting of a given *nominal process model* $\mathbb{N}_p$, and $\mathscr{P}$ is a finite set of indices,[1] i.e. $\mathscr{P} := \{p_1, \ldots, p_m\}$.

The supervisory control approach employs a family of candidate *controllers* $\mathbb{C} = \{\mathbb{C}_p : p \in \mathscr{P}\}$, each one designed for a nominal model in $\mathscr{F}$ so as to guarantee desired control objectives. An estimator-based supervisor generates—depending on the signals produced by an estimator—a piece-wise constant switching signal $\sigma(\cdot)$ which takes on values in $\mathscr{P}$ and whose value p determines at each time the controller $\mathbb{C}_p$ to be put in the feedback loop. More precisely, an estimator-based supervisor consists of a multi-estimator $\mathbb{E}$, a bank of monitoring signal generators $\mathbb{M}_p$, $p \in \mathscr{P}$, and a switching logic $\mathbb{S}$. The *multi-estimator* $\mathbb{E}$

$$\dot{x}_{\mathbb{E}} = E(x_{\mathbb{E}}, y, u), \qquad y_p = h_p(x_{\mathbb{E}}), \qquad p \in \mathscr{P} \tag{2}$$

is an input-to-state stable dynamical system (Sontag & Wang, 1996) with $h_p(0) = 0$ whose pth output is a signal y_p which asymptotically converges to y, provided that $\mathbb{N}_p$ is the actual process model.

A *monitoring signal generator* $\mathbb{M}_p$ is a dynamical system whose input is the pth output estimation error

$$e_p = y_p - y \tag{3}$$

and whose output μ_p is a signal which measures the size of e_p. We consider monitoring signal generators described by equations of the form

$$\dot{\mu}_p = \Xi(\mu_p, e_p), \qquad p \in \mathscr{P} \tag{4}$$

with Ξ such that (4) is input-to-state stable (ISS) with respect to the input e_p. The time history of the monitor signal μ_p can be viewed as a measure of the similarity during the time of the pth nominal model to the actual plant and drives the decision process of the switching logic $\mathbb{S}$. From time to time, $\mathbb{S}$ searches for the monitoring signal μ_p with the smallest value, set $\sigma(\cdot)$ equal to the corresponding index p and maintains $\sigma(\cdot)$ fixed at that value until a new search is completed and a new minimal value is found. This decision process is based on certainty equivalence principle and a formal justification of this strategy has been provided in Hespanha and Morse (1999b) and Hespanha et al. (2002).

[1] The case in which the unknown model of the process coincides with a member of a finite family of nominal plants is usually referred to as the "exact matching" case.

3. Supervisory control with state-dependent dwell-time switching logic

We assume for the controllers $\mathbb{C}_p$ the general form (Hespanha et al., 2002)

$$\dot{x}_{\mathbb{C}} = g_p(x_{\mathbb{C}}, x_{\mathbb{E}}, e_p), \qquad u_p = k_p(x_{\mathbb{C}}, x_{\mathbb{E}}, e_p), \tag{5}$$

with $g_p(0,0,0) = 0, k_p(0,0,0) = 0$ and g_p, k_p locally Lipschitz, which includes static and dynamic output feedbacks. Notice that such a controller is actually implementable because both $x_{\mathbb{E}}$ and e_p are available for measurements. Consider the system obtained from multi-estimator (2) in closed-loop with controller (5) and replacing y with $y_p - e_p$. Denote it as

$$\dot{x} = A_p(x, e_p), \tag{6}$$

where $x^{\mathrm{T}} = (x_{\mathbb{C}}^{\mathrm{T}}, x_{\mathbb{E}}^{\mathrm{T}})$, $x \in \mathbb{R}^n$, and assume that for each $p \in \mathscr{P}$ it is possible to design the candidate controller $\mathbb{C}_p$ in such a way that the closed-loop system (6) is integral input-to-state stable (iISS) (cf. Sontag, 1998) and, when $e_p \equiv 0$, locally exponentially stable, namely:

Assumption 1. There exist[2] class-$\mathscr{K}_\infty$ functions $\alpha(\cdot)$, $\tilde{\theta}_1(\cdot), \tilde{\theta}_2(\cdot)$, and a class-$\mathscr{K}$ function[3] $\gamma(\cdot)$ such that, for each $p \in \mathscr{P}$, the solution $x(t)$ of (6) from the initial condition $x(t_0) = x_0$ under the input $e_p(\cdot)$ exists for all $t \geqslant t_0$ and satisfies

$$\alpha(|x(t)|) \leqslant \tilde{\theta}_1(\tilde{\theta}_2(|x_0|)\mathrm{e}^{-(t-t_0)}) + \int_{t_0}^{t} \gamma(|e_p(\tau)|)\,\mathrm{d}\tau \tag{7}$$

for all $t \geqslant t_0 \geqslant 0$, all $x_0 \in \mathbb{R}^n$ and all $e_p(\cdot)$. Also, there exist class $\mathscr{K}_\infty$ functions $\alpha_1(\cdot), \alpha_2(\cdot), \alpha_3(\cdot)$, positive real numbers $a_1, a_2, a_3, \bar{s}$ and smooth functions $W_p(\cdot) : \mathbb{R}^n \to \mathbb{R}$, such that for all $x \in \mathbb{R}^n$

$$\alpha_1(|x|) \leqslant W_p(x) \leqslant \alpha_2(|x|),$$
$$\frac{\partial W_p}{\partial x} A_p(x, 0) \leqslant -\alpha_3(|x|) \tag{8}$$

and for all $s \in [0, \bar{s},]$

$$\alpha_i(s) = a_i s^2, \qquad i = 1, 2, 3. \tag{9}$$

Remark. The iISS property, expressed in Assumption 1 by relation (7), implies that system

$$\dot{x} = A_p(x, 0) \tag{10}$$

is globally asymptotically stable, i.e. if (7) holds necessarily (8) holds for some $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\alpha_3(\cdot)$, and $W_p(\cdot)$.

[2] $\mathscr{K}$ is the class of functions $[0, \infty) \to [0, \infty)$ which are zero at zero, strictly increasing and continuous, $\mathscr{K}_\infty$ is the subset of $\mathscr{K}$ consisting of all those functions that are unbounded.

[3] The function $\gamma(\cdot)$ will be sometimes referred to in the sequel as gain function.

Requiring (10) to be also locally exponentially stable implies that we can assume, without loss of generality (see e.g. Isidori, 1999, Lemma 10.1.5), the functions $\alpha_i(\cdot)$ to be quadratic in a neighborhood of the origin. The iISS property is implied by the ISS property (cf. Sontag, 1998). Hence, the methods presented in the paper can be adapted to deal with uncertain nonlinear systems which are ISS. Furthermore, as there are systems which can be made iISS but not ISS—such as those considered in Section 4—assuming iISS property to hold broadens the applicability of the method. Notice that simply assuming asymptotic stability of system (10) does not guarantee any "robustness" property of the system with respect to input $e_p(\cdot)$. Finally, observe that systems which are both iISS and locally exponentially stable include—besides those in Section 4—the class of nonlinear feedforward systems (cf. Teel, 1996; De Persis et al., 2003).

In this paper, the switching logic $\mathbb{S}$ which generates $\sigma(\cdot)$ is the *state-dependent dwell-time* switching logic introduced in De Persis et al. (2003), from now on referred to as $\mathbb{S}_{\mathrm{SD}}$. Let $\alpha(\cdot)$, $\tilde\theta_1(\cdot)$ and $\tilde\theta_2(\cdot)$ be as in (7). Define the functions $\theta_1(r) := \tilde\theta_1^{-1}(\alpha(r/3)/2)$, $\theta_2(r) := \tilde\theta_2(r)$, and[4] $\tau_\Delta(r) := \ln(\theta_2(r)/\theta_1(r))$, $r > 0$. Set $\bar r := \theta_2^{-1}(\theta_1(3\bar s))$, with $\bar s$ as in Assumption 1. Let a_1, a_2, a_3 be as in (9) and fix a constant $k \in (0, a_3/(2a_2)]$ and a dwell-time function $\tau_{\mathrm{D}}(\cdot): \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{>0}$ satisfying

$$\tau_{\mathrm{D}}(r) \geqslant \begin{cases} \tau_\Delta(r), & r \geqslant \bar r, \\ \max\{\tau_\Delta(\bar r), \frac{1}{k}\ln\frac{a_2}{a_1}\}, & r < \bar r. \end{cases} \qquad (11)$$

The functioning of the switching logic can be explained as follows.

The state-dependent dwell-time switching logic $\mathbb{S}_{\mathrm{SD}}$: Suppose that at some time t_0 $\sigma(\cdot)$ has just changed its value to p. At this time, a *timing signal* τ is reset to 0 and a variable X is set equal to $|x(t_0)|$, that is in X is "stored" the magnitude of the state of system (6) at that switching time. Compute now the *dwell-time* $\tau_{\mathrm{D}}(X)$. At the end of the switching period, when $\tau = \tau_{\mathrm{D}}(X)$, if there exists the minimal value $q \in \mathscr{P}$ such that μ_q is smaller than μ_σ, then $\sigma(\cdot)$ is set equal to q, τ is reset to zero and the entire process is repeated. Otherwise, $\sigma(\cdot)$ is kept equal to p, until a minimal value $q \in \mathscr{P}$ is found such that μ_q is smaller than μ_σ.

Remark. If in Assumption 1 relations (8) and (9) hold for all $s \in [0, \infty)$, that is for each $p \in \mathscr{P}$ system (10) is globally exponentially stable, then it is not hard to see that the dwell-time function $\tau_{\mathrm{D}}(r)$ can be taken equal to a constant for values of r ranging over the interval $[0, \bar r,]$, with $\bar r$ arbitrarily large.

[4] Note that $\tau_\Delta(r) > 0$ for all $r > 0$. Indeed, setting $t = t_0$ and replacing $|x(t_0)|$ with r in (7) we obtain that $\alpha(r) \leqslant \tilde\theta_1(\tilde\theta_2(r))$. As $\alpha(\cdot)$ is a strictly increasing function, the previous inequality also implies $\alpha(r/3)/2 < \tilde\theta_1(\tilde\theta_2(r))$. Therefore, $\tilde\theta_1^{-1}(\alpha(r/3)/2) =: \theta_1(r) < \tilde\theta_2(r) =: \theta_2(r)$, and hence $\theta_2(r)/\theta_1(r) > 1$ for all $r > 0$.

Remark. Observe that the switching signal $\sigma(\cdot)$ can never experience an infinite number of switchings in a finite interval of time. This is a consequence of the fact that without loss of generality function $\tau_{\mathrm{D}}(\cdot)$ can always be assumed to be bounded away from zero. As a matter of fact, by (11), over the interval $[\bar r, \infty)$, $\tau_{\mathrm{D}}(\cdot)$ depends on the continuous, positive function $\tau_\Delta(\cdot)$. If the latter is bounded away from zero, then one can take $\tau_{\mathrm{D}}(\cdot) = \tau_\Delta(\cdot)$ over the interval $[\bar r, \infty)$. If not, then it is always possible to find a function $\tau_{\mathrm{D}}(\cdot)$ which is bounded away from zero and satisfies (11).

Before proceeding, we recall some technical results from De Persis et al. (2003), to which we refer the reader interested in the proofs.

Theorem 1 (Lemma 3, Corollary 1 and Theorem 2 in De Persis et al., 2003). *Let Assumption* 1 *hold and consider system*

$$\dot x = A_\sigma(x, e_\sigma) \qquad (12)$$

with $\sigma(\cdot)$ produced by the switching logic $\mathbb{S}_{\mathrm{SD}}$. The following properties hold:

(i) *There exist a switching time t_i and a class-$\mathscr{K}$ function $\delta(\cdot)$ such that the solution $x(t)$ of* (12) *satisfies*

$$a_1|x(t)|^2 \leqslant a_2|x(t_i)|^2 \mathrm{e}^{-k(t-t_i)} + \frac{a_2}{a_1}\int_{t_i}^{t} \delta(|e_\sigma(\tau)|)\,\mathrm{d}\tau \qquad (13)$$

for all $t \geqslant t_i$, where k is the constant in (11).

(ii) *In the case in which there exists a continuous nonnegative function $\phi: \mathbb{R}^n \to [0, \infty)$, such that $|A_p(x, d) - A_p(x, 0)| \leqslant \phi(x)|d|$ for each $p \in \mathscr{P}$, for all x and d*:

- *if $k = a_3/(2a_2)$, then* (13) *holds with $\delta(r) = \bar{\bar\delta} r$, for some $\bar{\bar\delta} > 0$;*
- *if $k = a_3/(3a_2)$, and for each $p \in \mathscr{P}$, $A_p(x, 0)$ is continuously differentiable for all $|x| \in [0, \bar s]$, then* (13) *holds with $\delta(r) = \tilde{\bar\delta} r^2$, for some $\tilde{\bar\delta} > 0$.*

(iii) *Let $\gamma(\cdot)$ be the function for which* (7) *holds. If there exists a finite $c > 0$ such that $\delta(r) \leqslant c\gamma(r)$ for all $r \geqslant 0$, then, for each $x_0 \in \mathbb{R}^n$, for each input $e_\sigma(\cdot)$ fulfilling*

$$\int_0^\infty \gamma(|e_\sigma(\tau)|)\,\mathrm{d}\tau < \infty, \qquad (14)$$

the solution $x(t)$ of (12) *exists for all $t \geqslant 0$ and is such that* $\lim_{t\to\infty} |x(t)| = 0$.

The interconnected system described by Eqs. (1), (3), (6) and (4) defines a hybrid dynamical system of the form

$$\dot z = \varphi_\sigma(z) \qquad (15)$$

with $z^{\mathrm{T}} = (x_{\mathbb{P}}^{\mathrm{T}}, x^{\mathrm{T}}, \mu_{p_1}, \ldots, \mu_{p_m})$. It is not hard to see (Hespanha & Morse, 1999b) that, for each initial condition $(z(0), \sigma(0))$, there exists a unique pair $(z(\cdot), \sigma(\cdot))$

defined on the maximal interval of existence[5] $[0,T)$ which satisfies (15) with $\sigma(\cdot)$ generated by the state-dependent dwell-time switching logic $\mathbb{S}_{SD}$.

Following Morse (1996), we require the pair $(z(\cdot),\sigma(\cdot))$ to be such that

Assumption 2. There exists at least one $p^* \in \mathscr{P}$ such that e_{p^*} is bounded on $[0,T)$. Furthermore, there exists a finite subset $\mathscr{P}^* \subset \mathscr{P}$ containing p^* with the following properties:

(i) there exists a finite switching time $t^* < T$ such that $\sigma(t) \in \mathscr{P}^*$ for all $t \in [t^*, T)$;
(ii) for all $p \in \mathscr{P}^*$,

$$\int_0^T \gamma(|e_p(\tau)|)\,\mathrm{d}\tau \leqslant C^* < \infty \tag{16}$$

with $\gamma(\cdot)$ the class-$\mathscr{K}$ function appearing in (7).

Remark. The fulfillment of the assumption is closely related to the existence of suitable asymptotic observers. We shall see in Section 4 a considerable example of nonlinear systems—linear systems with input saturation—for which the supervisory control scheme outlined in this section can be designed so as to actually guarantee the fulfillment of Assumptions 1 and 2.

Before stating the main result of this section, we recall a notion of detectability (Sontag & Wang, 1997; Hespanha et al., 2002) for plant (1).

Definition. Plant (1) is input/output-to-state stable (IOSS), if there exist a class-$\mathscr{KL}$ function $\beta(\cdot,\cdot)$ and class-$\mathscr{K}$ functions $\gamma_1(\cdot),\gamma_2(\cdot)$ such that, for every initial condition $x_{\mathbb{P}}(0)$ and every input $u(\cdot)$, the solution of (1) satisfies

$$|x_{\mathbb{P}}(t)| \leqslant \beta(x_{\mathbb{P}}(0),t) + \gamma_1(\|u|_{[0,t]}\|) + \gamma_2(\|y|_{[0,t]}\|)$$

for all $t \geqslant 0$.

Remark. It is a consequence of the definition that the state response of an IOSS system is bounded if inputs and outputs are bounded, and converges to zero if inputs and outputs do the same.

Theorem 2. *Consider the unknown plant* (1) *and assume it is IOSS. Consider multi-estimator* (2) *and multi-controller* (5). *Assume that for each* $p \in \mathscr{P}$ *the multi-estimator/multi-controller closed-loop system* (6) *satisfies Assumption* 1. *In particular, assume that* (7) *holds with a class-*$\mathscr{K}$ *function* $\gamma(\cdot)$ *such that* $\delta(r) \leqslant c\gamma(r)$, *for* $\delta(\cdot)$ *as in Theorem* 1 *and a finite* $c > 0$. *Consider also the ISS monitoring signal generator* (4), *and the switching logic* $\mathbb{S}_{SD}$. *Assume that for each initial condition, the response of the overall system and the switching signal* $\sigma(\cdot)$ *are such that Assumption* 2 *is satisfied. Then, the response of the system exists for all* $t \geqslant 0$ *and all the continuous states converge to zero as t goes to infinity.*

Proof. Due to lack of space the proof is omitted. It can be found in De Persis, De Santis, and Morse (2002b). □

4. Supervisory control of linear systems subject to input saturation

In this section, we specialize the previous results to a relevant class of supervisory adaptive control problems. Plant (1) we consider is a linear system with input nonlinearities. Namely, we are given a process $\mathbb{P}$

$$\dot{x}_{\mathbb{P}} = A_{\mathbb{P}}x_{\mathbb{P}} + B_{\mathbb{P}}\mathrm{sat}(u), \qquad y = C_{\mathbb{P}}x_{\mathbb{P}} \tag{17}$$

with $x_{\mathbb{P}} \in \mathbb{R}^n$, $u \in \mathbb{R}^s$, $y \in \mathbb{R}^p$, and $\mathrm{sat}(\cdot)$—which models constraints on the control magnitude—is an $\mathbb{R}^s$-valued saturation function (see e.g. Isidori, 1999, Definition 14.1.1).

As before (cf. the first part of Section 2), we consider the exact matching case, that is the case in which the actual state-space representation (17) of $\mathbb{P}$ is unknown but it is assumed to belong to a known family of nominal linear model plants $\mathbb{N}_p = (\bar{C}_p, \bar{A}_p, \bar{B}_p)$, with $p \in \mathscr{P} = \{p_1,\ldots,p_m\}$. These systems are assumed to satisfy the following assumption.

Assumption 3. For each $p \in \mathscr{P}$ the pairs $(\bar{A}_p, \bar{B}_p)$ are stabilizable, $(\bar{C}_p, \bar{A}_p)$ are detectable and all the eigenvalues of $\bar{A}_p$ are in the closed left-half plane.

We note that, as a consequence of the exact matching case assumption, there exists a value $p^* \in \mathscr{P}$ such that $(C_{\mathbb{P}}, A_{\mathbb{P}}, B_{\mathbb{P}}) = (\bar{C}_{p^*}, \bar{A}_{p^*}, \bar{B}_{p^*})$.

We next show how to design the family of output-feedback controllers $\mathscr{C} = \{\mathbb{C}_p : p \in \mathscr{P}\}$ and a supervisor (multi-estimator $\mathbb{E}$, monitoring signal generators $\mathbb{M}_p$, switching logic $\mathbb{S}$) which is capable of generating a switching signal $\sigma(\cdot)$ which satisfy Assumptions 1 and 2, i.e. how to design a supervisory control system able to achieve asymptotic regulation to zero of the state of the process (17) and boundedness of all the system signals.

4.1. Identifier-based multi-estimator and monitoring signal generator

The most convenient and simple way to design a multi-estimator is that of designing single estimators for the nominal model plants $\mathbb{N}_p$ and then stack them all together. This results in a multi-estimator of the form

$$\begin{aligned} \dot{x}_{\mathbb{E}} &= A_{\mathbb{E}}x_{\mathbb{E}} + B_{\mathbb{E}}\,\mathrm{sat}(u) + K_{\mathbb{E}}\,y, \\ y_p &= C_{\mathbb{E}p}x, \qquad p \in \mathscr{P}. \end{aligned} \tag{18}$$

[5] System (15) is the interconnection of different subsystems, among which the plant (1) and the multi-estimator/multi-controller (6). Even though the multi-estimator/multi-controller is an iISS system for each $p \in \mathscr{P}$, we know that it is driven by signals coming from the plant, whose existence for all t cannot be a priori guaranteed.

The outputs y_p, $p \in \mathscr{P}$, generated by multi-estimator (18) are used to obtain the *output estimation errors* $e_p = y_p - y$ which feed the monitoring signal generators $\mathbb{M}_p$

$$\dot{\mu}_p = -\lambda\mu_p + |e_p|^\ell, \quad \mu_p(0) > 0, \quad p \in \mathscr{P}, \tag{19}$$

in which ℓ is a fixed integer in the set $\{1,2\}$. The monitoring signal generators are ISS, provided that $\lambda > 0$. Also note that, in the exact matching case, the equations of the output estimation errors show that e_{p^*} decays exponentially to zero, i.e. $|e_{p^*}(t)| \leqslant \bar{c}\exp(-\bar{\lambda}t)$, for some positive numbers $\bar{c}, \bar{\lambda}$.

4.2. Multi-controller

Following Hespanha and Morse (1999b), Hespanha et al. (2002), and Section 2, the controller is designed for a system obtained from the multi-estimator in such a way that it is input–output equivalent to the pth model $\mathbb{N}_p$ (cf. Hespanha & Morse, 1999b, Section 6). Namely, consider multi-estimator (18) under the feedback interconnection $y = y_p - e_p$:

$$\dot{x}_{\mathbb{E}} = (A_{\mathbb{E}} + K_{\mathbb{E}}C_{\mathbb{E}p})x_{\mathbb{E}} + B_{\mathbb{E}}\text{sat}(u) - K_{\mathbb{E}}e_p,$$

$$y_p = C_{\mathbb{E}p}x_{\mathbb{E}}. \tag{20}$$

For each $p \in \mathscr{P}$, the pair of matrices $((A_{\mathbb{E}} + K_{\mathbb{E}}C_{\mathbb{E}p}), B_{\mathbb{E}})$ is stabilizable and all the eigenvalues of $(A_{\mathbb{E}}+K_{\mathbb{E}}C_{\mathbb{E}p})$ are in the closed left-half plane. Then, as a consequence of the results of Teel (1996) and Angeli, Sontag, and Wang (2000), it can be proved that

Lemma 1 (Lemma 4 in De Persis et al., 2003). *For each $p \in \mathscr{P}$, there exist a positive integer v_p, matrices L_i^p, $i = 1, \ldots, v_p$, constants c_j^p, $j = 2, \ldots, v_p$ and a feedback of the form*

$$u = \chi_p(x_{\mathbb{E}})$$

$$:= L_1^p x_{\mathbb{E}} + c_2^p \,\text{sat}(L_2^p x_{\mathbb{E}} + c_3^p \,\text{sat}(+\cdots+ c_{v_p}^p \,\text{sat}(L_{v_p}^p x_{\mathbb{E}}))), \tag{21}$$

such that the closed-loop system (20), (21) *is iISS with respect to the input e_p with gain function $\gamma(r) = r^2$, and is locally exponentially stable when $e_p = 0$.*

That is to say, in this case a static controller $\mathbb{C}_p$ in the form given by Eq. (21) guarantees Assumption 1 to be satisfied.

Remark. The number v_p depends on the eigenstructure of $(A_{\mathbb{E}} + K_{\mathbb{E}}C_{\mathbb{E}p}) := A_{\mathbb{E}p}$. If $A_{\mathbb{E}p}$ is critically stable, i.e. there exists a $P > 0$ such that $A_{\mathbb{E}p}^{\mathrm{T}}P + PA_{\mathbb{E}p} \leqslant 0$, then $v_p = 1$ and feedback (21) is linear.

4.3. Analysis

To complete the supervisory control architecture we need to define the switching logic $\mathbb{S}$. We adopt as in Section 3, the state-dependent dwell-time switching logic, i.e. we set $\mathbb{S} = \mathbb{S}_{\text{SD}}$. Along the same lines of the proof of Lemma 1 in Morse (1996), it is possible to prove the following property for the switching signal $\sigma(\cdot)$ generated by $\mathbb{S}_{\text{SD}}$ driven by the μ_p's.

Lemma 2. *Let $\mathscr{T} := \{0 =: t_0, t_1, \ldots, t_j, \ldots\}$ be the sequence of switching times of $\sigma(\cdot)$ and ℓ as in* (19). *There exists a finite subset $\mathscr{P}^* \subset \mathscr{P}$ containing p^* with the following properties*:

(i) *there exists a finite switching time $t^* \in \mathscr{T}$ such that $\sigma(t) \in \mathscr{P}^*$ for all $t \geqslant t^*$*;
(ii) *there exists a finite real number C^* such that, for each $p \in \mathscr{P}^*$, $\|e_p(\cdot)\|_\ell \leqslant (C^*)^{1/\ell}$.*

Proof. See Morse (1996, Lemma 1). □

Now that all its components are defined, we are ready to prove that the supervisory control achieves regulation to zero of the state of (17).

Theorem 3. *Let $\mathbb{P}$ be process* (17), *unknown member of the family of nominal plant models $\{\mathbb{N}_p : \mathbb{N}_p = (\bar{C}_p, \bar{A}_p, \bar{B}_p),\ p \in \mathscr{P}\}$. Suppose that Assumption* 3 *holds and that the function* sat$(\cdot)$ *is continuously differentiable in a neighborhood of the origin. Consider the supervisory control system described by Eqs.* (18), *and* (21), *with $p = \sigma$, and* (19), *with $\ell = 2$, along with the state-dependent dwell-time switching logic $\mathbb{S}_{\text{SD}}$, with $\tau_{\text{D}}(\cdot)$ satisfying* (11) *and $k = a_3/(3a_2)$. Then, for each set of initial conditions $x_{\mathbb{P}}(0)$, $x_{\mathbb{E}}(0)$, $\mu_p(0) > 0$, $p \in \mathscr{P}$, $\sigma(0)$, the response of the supervisory control system exists for all $t \geqslant 0$ and all the continuous states converge to zero as t goes to infinity.*

Proof. The result descends from Theorem 2. Indeed, the plant is IOSS because, by the exact matching condition and Assumption 3, the pair $(C_{\mathbb{P}}, A_{\mathbb{P}})$ is detectable. The closed-loop multi-estimator/multi-controller (20), (21) is affine in e_p for each $p \in \mathscr{P}$. Because of Lemma 1, it also satisfies Assumption 1. In particular (7) holds with $\gamma(r) = r^2$. Moreover, cf. Theorem 1, (13) holds for $\delta(r) = \bar{\delta}r^2$, for some $\bar{\delta} > 0$ and $k = a_3/(3a_2)$. The inequality $\delta(r) \leqslant c\gamma(r)$ is trivially satisfied with $c = 1/\bar{\delta}$. The first part of Assumption 2 is a consequence of the exact matching condition, as observed in Section 4.1. Lemma 2 and comments thereafter show that, for any initial condition, the switching signal $\sigma(\cdot)$ generated by $\mathbb{S}_{\text{SD}}$ driven by the monitoring signal generators (19) are such that also the second part of Assumption 2 is fulfilled. Finally, note that system (19) is ISS with respect to e_p. Hence, all the assumptions of Theorem 2 are fulfilled and this yields the thesis. □

Remark. We have already noticed in the remark after Lemma 1 that, if systems $\mathbb{N}_p = (\bar{C}_p, \bar{A}_p, \bar{B}_p)$, with $p \in \mathscr{P}$, are *critically stable*, then closed-loop multi-estimator/multi-controller (20), (21) satisfies Assumption 1 with $\gamma(r) = r$

in (7). As a consequence, for this class of systems the conclusion of the previous theorem still holds setting $\ell = 1$ in (19) and $k = a_3/(2a_2)$ in (11). Furthermore, it is not hard to explicitly compute the function $\tau_\Delta(\cdot)$—which allows to define the dwell-time function $\tau_D(\cdot)$ given in (11). We have seen that the calculation of $\tau_\Delta(\cdot)$ amounts to determine the functions $\alpha(\cdot)$, $\tilde{\theta}_1(\cdot)$, $\tilde{\theta}_2(\cdot)$ which appear in Assumption 1, Eq. (7). Details are worked out in De Persis et al. (2002b). As a result, it is shown that function $\tau_\Delta(s)$ grows *polynomially* with s.

5. Conclusion

In this paper, the design of a supervisory control architecture with state-dependent dwell-time switching logic for controlling nonlinear uncertain system has been presented. It was shown that it achieves regulation to zero of the state of the plant in the case each member of the family of candidate controllers renders the corresponding nominal plant iISS and locally exponentially stable. An important application of the design methodology presented in the paper is the control of linear uncertain plants with input saturation in the standard hypothesis of stabilizability and detectability. The precise characterization of how the proposed scheme applies to other classes of nonlinear systems represents an interesting line of possible future investigation. The extension of the results in Section 4 to the case where $\mathscr{P}$ is a continuum of points has been studied in De Persis, De Santis, and Morse (2002a).

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