SUPERVISORY CONTROL OF INTEGRAL-INPUT-TO-STATE STABILIZING CONTROLLERS

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Abstract

A high-level supervisor, employing switching and logic, is proposed to orchestrate the switching between a family of candidate controllers into feedback with an imprecisely modeled process so as to stabilize it. Each of the candidate controllers is required to integral-input-to-state stabilize one particular admissible process model, with respect to a suitably defined disturbance input. The controller selection is made by (i) continuously comparing in real time suitably defined "normed" output estimation errors or "performance signals" and (ii) placing in the feedback-loop, from time to time, that candidate controller whose corresponding performance signal is the smallest. The use of integral-input-to-state stability in the context of supervisory control of nonlinear systems, allowed us to weaken the requirements on the candidate controllers being used. It also seems quite natural when the performance signals are defined as "integral norms" of the output estimation errors.

1 Introduction

This paper deals with the control of poorly modeled nonlinear systems. Our paradigm of choice to undertake this problem consists of an architecture in which a high-level, logic-based supervisor orchestrates the switching between a family of candidate controllers so as to achieve some desired behavior for the closed-loop system. The need for switching arises from the fact that no single candidate controller would be capable, by itself, of guaranteeing good performance when connected with the poorly modeled process.

In [1] it was shown that any stabilizing, certainty equivalence control used within an adaptive control system, causes the familiar interconnection of a controlled process and associated output estimator to be *detectable* through the estimator's output error e_p, for every frozen value of the index or parameter vector p upon which both the estimator and controller dynamics depend. This was shown to be so whenever each of the candidate controllers input-to-state stabilizes the corresponding admissible process model, with respect to a suitably defined disturbance input.

Here it is shown that just requiring that each candidate controller integral-input-to-state stabilize the corresponding admissible process model suffices for a suitably defined "integral" detectability of the interconnection of the candidate controller with the corresponding output estimator {the so-called injected system} through the estimator's output error. In turn, this is used to design a supervisor, employing switching and logic, to orchestrate the switching between the set of candidate controllers into feedback with the imprecisely modeled process so as to stabilize it. The integral version of detectability, introduced in this paper, is dual to the concept of integral-input-to-state stability in [2].

By replacing the requirement of input-to-state stability with that of integral-input-to-state stability, the conditions under which the supervisory control algorithm is proved to achieve stability are significantly weakened. The fact that input-to-state stability was, at times, too strong a requirement was stressed by recent results in [3, 4]. For a discussion on integral-input-to-state stability v.s. input-to-state stability see [2, 5]. The latter reference addresses the question of designing integral-input-to-state stabilizing controllers. In light of the results presented here, this topic becomes quite relevant for the supervisory control of nonlinear systems.

The use of integral-input-to-state stability and integral detectability also seems quite natural when the performance signals are defined as "integral norms" of output estimation errors. In fact, with integral detectability we were able to avoid many of the technical difficulties that arose in [1, 6]. Working with "time-domain" definitions of integral-input-to-state stability and integral detectability—instead of definitions based on dissipation-like inequalities, as in [1, 6]—also helped simplifying the analysis.

This paper is organized as follows. In Section 2 the notion of integral-input-to-state stability is reviewed and a dual definition of integral detectability is introduced. Section 3 describes the overall control problem addressed in this paper—namely the stabilization of poorly modeled processes—and also the basic structure of an estimator-base supervisor. In Section 4 it is shown that the interconnection of the candidate controller with the corresponding output estimator is integral detectable through the estimator's output error. Section 5 outlines the analysis of a supervisory control system in a fairly general setting. Section 6 contains some concluding remarks.

2 Integral-Input-to-State Stability and Detectability

Let

$$\dot{x} = A(x, u), \qquad y = C(x, u) \tag{1}$$

be a finite dimensional dynamical system whose state, input, and output take values in real, finite dimensional spaces $\mathcal{X}$, $\mathcal{U}$, and $\mathcal{Y}$, respectively. Suppose that A and C are at least locally Lipschitz continuous on $\mathcal{X} \oplus \mathcal{U}$. In the sequel we denote by $\mathcal{K}$ the set of all continuous functions $\alpha : [0, \infty) \to [0, \infty)$ which are zero at zero, strictly increasing, and continuous, and by $\mathcal{K}_\infty$ the subset of $\mathcal{K}$ consisting of those functions that are unbounded. We also denote by $\mathcal{KL}$ the set of continuous functions $\beta : [0, \infty) \times [0, \infty) \to [0, \infty)$ which, for each fixed valued of the second argument, are of class $\mathcal{K}$ when regarded as functions of the first argument, and that have $\lim_{\tau \to \infty} \beta(s, \tau) = 0$ for each fixed $s \geq 0$. The following definition extends to nonzero equilibrium states, the concept of "integral-input-to-state stability" in [2].

γ-stability: Given a function $\gamma \in \mathcal{K}_\infty$, the system defined by (1) is said to be *γ-stable*, if $A(\tilde{x}, 0) = 0$ for some state $\tilde{x} \in \mathcal{X}$ and there exists a function $\beta \in \mathcal{KL}$ such that for each initial state $x(t_0) \in \mathcal{X}$ and each piecewise continuous input u,

$$\|x(t) - \tilde{x}\| \leq \beta(\|x(t_0) - \tilde{x}\|, t - t_0) + \int_{t_0}^{t} \gamma(\|u(\tau)\|)d\tau, \quad t \geq t_0 \geq 0, \tag{2}$$

along the corresponding solution to (1). If (1) is γ-stable there can be only one state $\tilde{x} \in \mathcal{X}$ at which $A(\tilde{x}, 0) = 0$. We call $\tilde{x}$ the *stable equilibrium state* of (1). A system that is γ-stable for some $\gamma \in \mathcal{K}_\infty$ is simply called *integral-input-to-state stable.* Integral-input-to-state stability is a weaker notion than the more common input-to-state stability [7] in that any input-to-state stable system is integral-input-to-state stable, but the converse is not true.

It is possible to define detectability in a number of different ways {see [8] and references therein}. An especially useful characterization in terms of an inequality like (2) is as follows.

$\{\alpha, \gamma\}$-detectability: Given two functions $\alpha, \gamma \in \mathcal{K}_\infty$, the system defined by (1) is said to be *$\{\alpha, \gamma\}$-detectable* if $A(\tilde{x}, 0) = 0$ and $C(\tilde{x}, 0) = 0$ for some state $\tilde{x} \in \mathcal{X}$, and there exists a function $\beta \in \mathcal{KL}$ such that for each initial state $x(t_0) \in \mathcal{X}$ and each piecewise continuous input u,

$$\|x(t) - \tilde{x}\| \leq \beta(\|x(t_0) - \tilde{x}\|, t - t_0) + \int_{t_0}^{t} \alpha(\|u(\tau)\|)d\tau + \int_{t_0}^{t} \gamma(\|y(\tau)\|)d\tau, \quad t \geq t_0 \geq 0, \tag{3}$$

along the corresponding solution to (1). If (1) is $\{\alpha, \gamma\}$-detectable there is exactly one state $\tilde{x} \in \mathcal{X}$ at which $A(\tilde{x}, 0) = 0$ and $C(\tilde{x}, 0) = 0$. We call $\tilde{x}$ the *detectable equilibrium state* of (1). A system that is $\{\alpha, \gamma\}$-detectable for some $\alpha, \gamma \in \mathcal{K}_\infty$ is simply called *integral detectable.* In case (3) holds without the term $\int_{t_0}^{t} \alpha(\|u\|)$, (1) is said to be *strongly γ-detectable.* Clearly strong γ-detectability implies $\{\alpha, \gamma\}$-detectability for any $\alpha \in \mathcal{K}_\infty$. It is straightforward to show that if the solution to (1) exists globally, strong γ-detectability implies that $x \to \tilde{x}$ as $t \to \infty$, whenever $\int_0^\infty \gamma(\|y\|)$ is bounded. The preceding definition of $\{\alpha, \gamma\}$-detectability reduces to the familiar one in the event that (1) is a linear system.

3 Overall Problem

The problem formulation is similar to that in [1, 6]. For ease of reference the basic setup is briefly reproduced here. Let $\mathbb{P}$ denote the model of a process of the form

$$\dot{x}_{\mathbb{P}} = A_{\mathbb{P}}(x_{\mathbb{P}}, w, u), \qquad y = C_{\mathbb{P}}(x_{\mathbb{P}}, w), \tag{4}$$

with state $x_{\mathbb{P}}$, control input u, measured output y, and piecewise-continuous disturbance/noise input w that cannot be measured. The signals $x_{\mathbb{P}}$, u, y, and w take values in real, finite-dimensional vector spaces $\mathcal{X}_{\mathbb{P}}$, $\mathcal{U}$, $\mathcal{Y}$, and $\mathcal{W}$, respectively. The functions $A_{\mathbb{P}}$ and $C_{\mathbb{P}}$ are at least locally Lipschitz continuous on $\mathcal{X}_{\mathbb{P}} \oplus \mathcal{W} \oplus \mathcal{U}$ and $\mathcal{X}_{\mathbb{P}} \oplus \mathcal{W}$, respectively[1]. Assume that $\mathbb{P}$ {or equivalently, the pair $(C_{\mathbb{P}}, A_{\mathbb{P}})$} is an unknown member of some suitably defined family of dynamical systems $\mathcal{F}$ that can be written as $\mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p$, where $\mathcal{P}$ is a set of indices and each

[1] Here the exterior direct sum of two real linear spaces $\mathcal{A}$ and $\mathcal{B}$, is denoted by $\mathcal{A} \oplus \mathcal{B}$.

$\mathcal{F}_p$ denotes a subfamily consisting of a given *nominal process model* $\mathbb{M}_p$ together with a collection of "perturbed versions" of $\mathbb{M}_p$.

The overall problem of interest is to devise a feedback control that regulates y about the value 0. To this effect, assume that one has chosen a family of off-the-shelf, candidate loop-controllers $\mathcal{C} \stackrel{\Delta}{=} \{\mathbb{C}_p : p \in \mathcal{P}\}$, in such a way that for each $p \in \mathcal{P}$, $\mathbb{C}_p$ would "solve" the regulation problem, were $\mathbb{P}$ to be any element of $\mathcal{F}_p$. The idea then is to generate a *switching signal* σ taking values in $\mathcal{P}$, which causes the output y of the process model $\mathbb{P}$ in closed-loop with $\mathbb{C}_\sigma$—as shown in Figure 1—to be regulated about zero. We call $\mathbb{C}_\sigma$ a *multi-controller* and we require it to

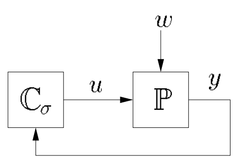

Figure 1: Process and Multi-Controller Feedback Loop

be a dynamical system with a real, finite dimensional state space $\mathcal{X}_{\mathbb{C}}$ and defining equations of the form

$$\dot{x}_{\mathbb{C}} = F_\sigma(x_{\mathbb{C}}, y), \qquad u = G_\sigma(x_{\mathbb{C}}, y), \tag{5}$$

where, for each fixed $p \in \mathcal{P}$, the equations $\dot{\bar{x}}_{\mathbb{C}} = F_p(\bar{x}_{\mathbb{C}}, y)$ and $u_p = G_p(\bar{x}_{\mathbb{C}}, y)$ model $\mathbb{C}_p$, with F_p and G_p locally Lipschitz continuous on $\mathcal{X}_{\mathbb{C}} \oplus \mathcal{Y}$.

Estimator-Based Supervisor

The algorithm used to generate σ is going to be an "estimator-based supervisor". An *estimator-based supervisor* consists of three subsystems: a multi-estimator $\mathbb{E}$, a performance signal generator $\mathbb{PS}$, and a switching logic $\mathbb{S}$ {cf. Figure 2}.

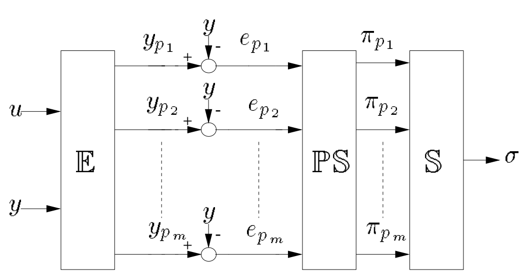

Figure 2: Estimator-Based Supervisor

By a *multi-estimator* $\mathbb{E}$ for a given family of nominal process models $\mathcal{M} = \{\mathbb{M}_p : p \in \mathcal{P}\}$ is meant an integral-input-to-state stable system with finite dimensional state-space $\mathcal{X}_{\mathbb{E}}$, of the form

$$\dot{x}_{\mathbb{E}} = A_{\mathbb{E}}(x_{\mathbb{E}}, u, y), \quad y_p = C_p(x_{\mathbb{E}}), \quad p \in \mathcal{P}, \tag{6}$$

where, for each fixed $p \in \mathcal{P}$, the equations $\dot{x}_{\mathbb{E}} = A_{\mathbb{E}}(x_{\mathbb{E}}, u, y)$ and $y_p = C_p(x_{\mathbb{E}})$ model an "estimator" $\mathbb{E}_p$ for $\mathbb{M}_p$, with $A_{\mathbb{E}}$ and each C_p locally Lipschitz continuous on $\mathcal{X}_{\mathbb{E}} \oplus \mathcal{U} \oplus \mathcal{Y}$ and $\mathcal{X}_{\mathbb{E}}$, respectively. By an *estimator* for a given nominal process model $\mathbb{M}_p$, is meant any finite-dimensional, integral-input-to-state stable dynamical system whose input is the pair $\{u, \; y\}$ and whose output is a signal y_p which would be an asymptotically correct estimate of y, if $\mathbb{M}_p$ were the actual process model and there were no measurement noise or disturbances. For $\mathbb{E}_p$ to have this property, it would have to exhibit {under the feedback interconnection $y \stackrel{\Delta}{=} y_p$ and an appropriate initialization} the same input-output behavior between u and y_p as $\mathbb{M}_p$ does between its input and output. For linear systems such estimators would typically be observers or identifiers [9]. Estimators can also be defined quite easily for certain types of nonlinear systems including those which are linearizable by output injection; in this category is any system whose state and measured output is one and the same [1, 6].

A *performance signal generator* $\mathbb{PS}$ is a dynamical system whose inputs are *output estimation errors*

$$e_p \stackrel{\Delta}{=} y_p - y, \qquad p \in \mathcal{P}, \tag{7}$$

and whose outputs are *performance signals* π_p, $p \in \mathcal{P}$. For each $p \in \mathcal{P}$, π_p is intended to be a suitably defined measure of the size of the e_p.

The third subsystem of an estimator-based supervisor is a *switching logic* $\mathbb{S}$. The role of $\mathbb{S}$ is to generate σ. Although there are many different ways to define $\mathbb{S}$, in each case the underlying strategy for generating σ is more or less that same: From time to time set σ equal to that value of $p \in \mathcal{P}$ for which π_p is the smallest. The motivation for this idea is obvious: the nominal process model whose associated performance signal is the smallest, "best" approximates what the process is and thus the candidate controller designed on the basis of that model ought to be able to do the best job of controlling the process. The origin of this idea is the concept of certainty equivalence. Precise definitions for the performance signal generator and switching logic are deferred to Section 5.

4 Certainty Equivalence

To understand what certainty equivalence actually implies, let us assume that there is a family of functions $\{\gamma_p : p \in \mathcal{P}\} \subset \mathcal{K}_\infty$ such that for each $p \in \mathcal{P}$, $\mathbb{C}_p$ was chosen so that the system shown in Figure 3 is γ_p-stable with respect to the input v. Suppose in addition, that $\bar{y}_p = 0$ at the stable equilibrium state of this system. By this we mean that for each $p \in \mathcal{P}$, the interconnected system

$$\left.\begin{array}{ll} \dot{\bar{x}}_{\mathbb{E}} = A_{\mathbb{E}}(\bar{x}_{\mathbb{E}}, \bar{u}_p, \bar{y}_p - v) & \bar{y}_p = C_p(\bar{x}_{\mathbb{E}}) \\ \dot{\bar{x}}_{\mathbb{C}} = F_p(\bar{x}_{\mathbb{C}}, \bar{y}_p - v) & \bar{u}_p = G_p(\bar{x}_{\mathbb{C}}, \bar{y}_p - v) \end{array}\right\} \tag{8}$$

with input v, is γ_p-stable and that $\bar{y}_p = 0$ at its stable equilibrium state.

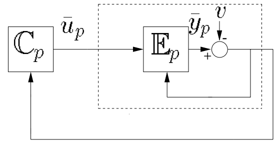

Figure 3: Feedback Interconnection

The intuition behind placing these requirements on $\mathbb{C}_p$ stems from the fact that, when v is equal to zero, the subsystem enclosed within the dashed box in Figure 3 is input-output equivalent to the nominal process model $\mathbb{M}_p$. If one then regards the signal v as a disturbance entering $\mathbb{M}_p$, the requirements above can be restated as demanding $\mathbb{C}_p$ to γ_p-stabilize the nominal process model $\mathbb{M}_p$, with respect to the disturbance[2] v. In view of the control objective—which is to regulate y about zero—the output of $\mathbb{M}_p$ is also required to be zero at the stable equilibrium state of the closed-loop system.

Take any two elements p, p^* in the parameter set $\mathcal{P}$. In analyzing adaptive and supervisory control systems, it is convenient to focus our attention on subsystems of the form

$$\left.\begin{array}{ll} \dot{x}_{\mathbb{E}} = A_{\mathbb{E}}(x_{\mathbb{E}}, u, y) & e_l = C_l(x_{\mathbb{E}}) - y,\ l \in \{p, p^*\} \\ \dot{x}_{\mathbb{C}} = F_p(x_{\mathbb{C}}, y) & u = G_p(x_{\mathbb{C}}, y) \end{array}\right\} \quad (9)$$

These equations describe the dynamics of the multi-controller/multi-estimator subsystem, while σ is held constant and equal to p. Typically, p^* is chosen to be the index of the subfamily $\mathcal{F}_{p^*}$ within which $\mathbb{P}$ resides and therefore the estimation error e_{p^*} is expected to be small in a suitably defined sense [10]. The results which follow do not depend on this fact.

By the $\{p, p^*\}$-*injected system* is meant the system which results when the equation $y = C_{p^*}(x_{\mathbb{E}}) - e_{p^*}$ from (9) is used to eliminate y from $A_{\mathbb{E}}(\cdot)$, $F_p(\cdot)$ and $G_p(\cdot)$ in (9). Once this is done, the $\{p, p^*\}$-injected system can be written as

$$\dot{x} = A_{pp^*}(x, e_{p^*}), \qquad e_p = C_{pp^*}(x) + e_{p^*}, \qquad (10)$$

where $x \triangleq [x'_{\mathbb{E}} \quad x'_{\mathbb{C}}]'$ and

$$A_{\bar{p}\bar{q}}(\bar{x}, e) \triangleq \begin{bmatrix} A_{\mathbb{E}}\Big(\bar{x}_{\mathbb{E}}, G_{\bar{p}}(\bar{x}_{\mathbb{C}}, C_{\bar{q}}(\bar{x}_{\mathbb{E}}) - e), C_{\bar{q}}(\bar{x}_{\mathbb{E}}) - e\Big) \\ F_{\bar{p}}(\bar{x}_{\mathbb{C}}, C_{\bar{q}}(\bar{x}_{\mathbb{E}}) - e) \end{bmatrix},$$
$$C_{\bar{p}\bar{q}}(\bar{x}) \triangleq C_{\bar{p}}(\bar{x}_{\mathbb{E}}) - C_{\bar{q}}(\bar{x}_{\mathbb{E}}),$$

for each $\bar{p}, \bar{q} \in \mathcal{P}$, $\bar{x} \triangleq [\bar{x}'_{\mathbb{E}} \quad \bar{x}'_{\mathbb{C}}]' \in \mathcal{X}_{\mathbb{E}} \oplus \mathcal{X}_{\mathbb{C}}$, and $e \in \mathcal{Y}$. Now, for any fixed $\bar{p}, \bar{q} \in \mathcal{P}$.

$$A_{\bar{p}\bar{q}}(\bar{x}, e) = A_{\bar{p}\bar{p}}(\bar{x}, C_{\bar{p}\bar{q}}(\bar{x}) + e), \quad \forall \bar{x} \in \mathcal{X}_{\mathbb{E}} \oplus \mathcal{X}_{\mathbb{C}},\ e \in \mathcal{Y},$$

thus (10) can also be written as

$$\dot{x} = A_{pp}(x, e_p), \qquad (11)$$

[2] The particular manner in which this disturbance is chosen to enter the nominal model will become clear shortly.

with

$$e_p = C_{pp^*}(x) + e_{p^*}. \qquad (12)$$

But the system defined by (11) is the same as the system defined by (8) when when v, $\bar{x}_{\mathbb{E}}$, and $\bar{x}_{\mathbb{C}}$ are identified with e_p, $x_{\mathbb{E}}$, and $x_{\mathbb{C}}$, respectively. By assumption, the latter is γ_p-stable and $\bar{y}_p = 0$ at its stable equilibrium state $\tilde{x}_p$. From this and (12), one concludes that there exists a function $\beta_p \in \mathcal{KL}$ such that for each initial state $x(t_0)$ and each piecewise continuous signal e_{p^*},

$$\|x(t) - \tilde{x}_p\| \leq \beta_p(\|x(t_0) - \tilde{x}_p\|, t - t_0) + \int_{t_0}^{t} \gamma_p(\|C_{pp^*}(x) + e_{p^*}\|)d\tau, \quad t \geq t_0 \geq 0, \quad (13)$$

along the corresponding solution to (11)-(12). Moreover, $y_p = 0$ at $x = \tilde{x}_p$. The following Lemma is a direct consequence of (13) and the fact that (10) is equivalent to (11)-(12).

Lemma 1. *For each $p, p^* \in \mathcal{P}$, the $\{p, p^*\}$-injected system (10), with input e_{p^*} and output e_p, is strongly γ_p-detectable and $y_p = 0$ at its detectable equilibrium state.*

The implication of Lemma 1 is clear. For each $p \in \mathcal{P}$, the γ_p-stabilization of the system in Figure 3 by $\mathbb{C}_p$ causes the $\{p, p^*\}$-injected system to be strongly γ_p-detectable. In [1] this is summarized by the phrase *certainty equivalence implies detectability*. With the preceding in mind, recall the underlying decision making strategy of an estimator-based supervisor: From time to time select for σ, that value $q \in \mathcal{P}$ such that the performance signal π_q is the smallest among the π_p, $p \in \mathcal{P}$. Justification for this strategy is now clear: By choosing σ to maintain smallness of π_σ and consequently e_σ, the supervisor is also maintaining smallness of the composite state of the interconnection of $\mathbb{C}_\sigma$ and $\mathbb{E}$, because of detectability through e_σ for each fixed value of σ. Moreover, since the input and output of the process can be written in terms of the state of the $\{\sigma, p^*\}$-injected system and e_{p^*} as

$$y = C_{p^*}(x_{\mathbb{E}}) - e_{p^*}, \qquad u = G_\sigma(x_{\mathbb{C}}, C_{p^*}(x_{\mathbb{E}}) - e_{p^*}),$$

these variable should also be small. The above equations were taken from (9).

Because of the equivalence between (10) and (11)-(12), the strong γ_p-detectability of the $\{p, p^*\}$-injected system can be traced directly to the γ_p-stability of (8), with v, $\bar{x}_{\mathbb{E}}$, and $\bar{x}_{\mathbb{C}}$ identified with e_p, $x_{\mathbb{E}}$, and $x_{\mathbb{C}}$, respectively. Now, since both e_p and $x_{\mathbb{E}}$ are available for measurement, the corresponding signals v and $\bar{x}_{\mathbb{E}}$ could be used for control in (8). In practice, this means that the multi-controller $\mathbb{C}_\sigma$ can be of the form

$$\dot{x}_{\mathbb{C}} = \tilde{F}_\sigma(x_{\mathbb{C}}, x_{\mathbb{E}}, e_\sigma), \qquad u = \tilde{G}_\sigma(x_{\mathbb{C}}, x_{\mathbb{E}}, e_\sigma), \qquad (14)$$

where, for each fixed $p \in \mathcal{P}$, the $\tilde{F}_p$ and $\tilde{G}_p$ are locally Lipschitz continuous functions on $\mathcal{X}_{\mathbb{C}} \oplus \mathcal{X}_{\mathbb{E}} \oplus \mathcal{Y}$. In this case, Lemma 1 holds for the appropriate definition of the $\{p, p^*\}$-injected system, when the interconnected system

$$\dot{\bar{x}}_{\mathbb{E}} = A_{\mathbb{E}}(\bar{x}_{\mathbb{E}}, \bar{u}_p, \bar{y}_p - v) \qquad \bar{y}_p = C_p(\bar{x}_{\mathbb{E}})$$
$$\dot{\bar{x}}_{\mathbb{C}} = \tilde{F}_p(\bar{x}_{\mathbb{C}}, \bar{x}_{\mathbb{E}}, v) \qquad \bar{u}_p = \tilde{G}_p(\bar{x}_{\mathbb{C}}, \bar{x}_{\mathbb{E}}, v)$$

with input v, is γ_p-stable and $\bar{y}_p = 0$ at its stable equilibrium state. Since both the disturbance v and the state $\bar{x}_{\mathbb{E}}$ of the system to be γ_p-stabilizable are available for control, the design of the candidate controllers is considerably simpler. Further note that the original multi-controller (5) is a special case of (14) when

$$\tilde{F}_p(\bar{x}_{\mathbb{C}}, \bar{x}_{\mathbb{E}}, e) \stackrel{\Delta}{=} F_p\big(\bar{x}_{\mathbb{C}}, C_p(\bar{x}_{\mathbb{E}}) + e\big),$$
$$\tilde{G}_p(\bar{x}_{\mathbb{C}}, \bar{x}_{\mathbb{E}}, e) \stackrel{\Delta}{=} G_p\big(\bar{x}_{\mathbb{C}}, C_p(\bar{x}_{\mathbb{E}}) + e\big),$$

for each $p \in \mathcal{P}$, $\bar{x}_{\mathbb{C}} \in \mathcal{X}_{\mathbb{C}}$, $\bar{x}_{\mathbb{E}} \in \mathcal{X}_{\mathbb{E}}$, and $e \in \mathcal{Y}$. Although all the results in this paper hold for the general multi-controller (14), we will continue to use the somewhat simpler multi-controller given by (5).

5 Analysis of Supervised System

The intent of this section is to demonstrate how the preceding results can be used to deduce global boundedness and asymptotic convergence in a supervisory control system when $\mathcal{P}$ is a finite set—say $\mathcal{P} \stackrel{\Delta}{=} \{1, 2, \ldots, m\}$. This is done for the special case when the disturbance/noise input w is identically zero and one of the y_p is an asymptotically correct estimate of y.

For the performance signal generator $\mathbb{PS}$ we consider the dynamical system

$$\dot{\pi}_p = -\lambda \pi_p + \gamma_p(\|e_p\|), \qquad p \in \mathcal{P}, \qquad (15)$$

whose state and outputs are the performance signals $\{\pi_1,\ \pi_2, \ldots, \pi_m\}$, λ is a prespecified positive number, and the γ_p are as in Section 4. It is assumed that (15) is initialized so that $\pi_p(0) > 0$, $p \in \mathcal{P}$.

For $\mathbb{S}$ we consider the "scale-independent hysteresis switching logic" [11, 6]. By a *scale-independent hysteresis switching logic* is meant a hybrid dynamical system $\mathbb{S}_{\mathbb{H}}$ whose inputs are the π_p and whose state and output are both σ. To specify $\mathbb{S}_{\mathbb{H}}$ it is necessary to first pick a positive number $h > 0$ called a *hysteresis constant*. $\mathbb{S}_{\mathbb{H}}$'s internal logic is then defined by the computer diagram shown in Figure 4 where, at each time t, $q \stackrel{\Delta}{=} \arg\min_{p\in\mathcal{P}} \Pi(p, x, t)$. The functioning of $\mathbb{S}_{\mathbb{H}}$ is roughly as follows. Suppose that at some time t_0, $\mathbb{S}_{\mathbb{H}}$ has just changed the value of σ to q. σ is then held fixed at this value unless and until there is a time $t_1 > t_0$ at which $(1+h)\pi_p < \pi_q$ for some $p \in \mathcal{P}$. If this occurs, σ is set equal to p and so on.

Three assumptions are made.

Assumption 2. *Each process model in $\mathcal{F}$ has the property that if its inputs and outputs are bounded then so is its state {i.e., each process model in $\mathcal{F}$ is detectable [8]}.*

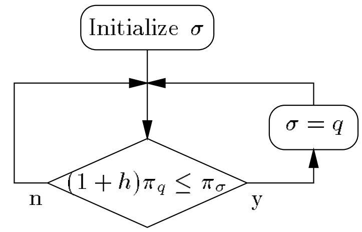

Figure 4: Computer Diagram of $\mathbb{S}_{\mathbb{H}}$.

Assumption 3. *Each γ_p, $p \in \mathcal{P}$, is locally Lipschitz.*

Assumption 4. *There exists an index $p^* \in \mathcal{P}$ such that, for each piecewise-continuous, open-loop control signal u, and each initial state $\{x_{\mathbb{P}}(0),\ x_{\mathbb{E}}(0)\} \in \mathcal{X}_{\mathbb{P}} \oplus \mathcal{X}_{\mathbb{E}}$, $\|e_{p^*}\|$ and $\int_0^t e^{\lambda\tau}\|e_{p^*}(\tau)\|d\tau$ are bounded on the interval of maximal length on which a solution to (4), (6) exists.*

$\mathbb{E}$ can typically be constructed so that Assumption 4 is satisfied in the noise/disturbance free case, provided $\mathbb{P}$ is input-output equivalent to a nominal model {say $\mathbb{M}_{p^*}$} which is linearizable by output injection [6].

Assumptions 3 and 4 enable us to exploit the Hysteresis Switching Lemma [11, 6] and consequently to draw the following conclusion [12].

Lemma 5. *For fixed initial states $x_{\mathbb{P}}(0) \in \mathcal{X}_{\mathbb{P}}$, $x_{\mathbb{E}}(0) \in \mathcal{X}_{\mathbb{E}}$, $x_{\mathbb{C}}(0) \in \mathcal{X}_{\mathbb{C}}$, $\pi_p(0) > 0$, $p \in \mathcal{P}$, $\sigma(0) \in \mathcal{P}$, the system defined by (4), (5), (6), (7), and (15), with σ the output of $\mathbb{S}_{\mathbb{H}}$, has a unique solution $\{x_{\mathbb{P}}, x_{\mathbb{E}}, x_{\mathbb{C}}, \pi_1, \pi_2, \ldots, \pi_m, \sigma\}$ on a nonempty time interval starting at zero. Denoting by $[0, T)$ the largest interval on which this solution is defined, there is a time $T^* < T$ beyond which σ is constant and no more switching occurs. In addition, the scaled performance signal $\bar{\pi}_{\sigma(T^*)} \stackrel{\Delta}{=} e^{\lambda t}\pi_{\sigma(T^*)}$ is bounded on $[0, T)$.*

Let $x_{\mathbb{P}}$, $x_{\mathbb{E}}$, $x_{\mathbb{C}}$, π_1, $\pi_2, \ldots, \pi_m$, σ, T and T^* be as in Lemma 5 and set $q^* \stackrel{\Delta}{=} \sigma(T^*)$. In view of (15) and the observation that $e^{\lambda t}\pi_{q^*}$ must be bounded on $[0, T)$,

$$\int_0^T \gamma_{q^*}(\|e_{q^*}(\tau)\|)d\tau < \infty. \qquad (16)$$

Since σ is frozen at q^* for $t \in [T^*, T)$ and the $\{q^*, p^*\}$-injected system $\dot{x} = A_{q^*p^*}(x, e_{p^*})$, $e_{q^*} = C_{q^*p^*}(x) + e_{p^*}$, determined by (6), (7), and (5), with σ frozen q^* and

$$x \stackrel{\Delta}{=} \begin{bmatrix} x'_{\mathbb{E}} & x'_{\mathbb{C}} \end{bmatrix}', \qquad (17)$$

is strongly γ_{q^*}-detectable, (16) allows one to conclude that x and consequently $x_{\mathbb{E}}$ and $x_{\mathbb{C}}$ must be bounded on $[0, T)$.

In view of (6) and (7), $y = e_{p^*} + C_{p^*}(x_{\mathbb{E}})$. By Assumption 4, e_{p^*} is bounded on $[0, T)$, so y must also be. Boundedness of u on $[0, T)$ then follows from the formula $u = G_\sigma(x_{\mathbb{C}}, y)$. Therefore $x_{\mathbb{P}}$ is bounded on $[0, T)$ because of Assumption 2. So is each e_p, $p \in \mathcal{P}$, because of the defining formula $e_p = C_p(x_{\mathbb{E}}) - y$. Therefore each π_p will be bounded on $[0, T)$ since the differential equations (15) defining the π_p can be viewed as asymptotically linear systems with bounded inputs $\gamma_p(\|e_p\|)$. In other words $x_{\mathbb{P}}$, $x_{\mathbb{E}}$, $x_{\mathbb{C}}$, and the π_p are all bounded on $[0, T)$.

Now if T were finite, the solution to (4), (5), (6), (7), and (15) could be continued onto at least an open half interval of the form $[T, T_1)$ thereby contradicting the hypothesis that $[0, T)$ is the system's interval of maximal existence. By contradiction one can therefore conclude that $T = \infty$ and that $x_{\mathbb{P}}$, $x_{\mathbb{E}}$, $x_{\mathbb{C}}$, and all the π_p are bounded on $[0, \infty)$. With global existence of solution established, (16) and the strong γ_{q^*}-detectability of the $\{q^*, p^*\}$-injected system, allow one to conclude that x converges to the detectable equilibrium state $\tilde{x}$ of the $\{q^*, p^*\}$-injected system. With $\tilde{x}$ partitioned as $\begin{bmatrix}\tilde{x}'_{\mathbb{E}} & \tilde{x}'_{\mathbb{C}}\end{bmatrix}'$ it can therefore be concluded that $x_{\mathbb{E}}$ converges to $\tilde{x}_{\mathbb{E}}$ and $x_{\mathbb{C}}$ converges to $\tilde{x}_{\mathbb{C}}$ as $t \to \infty$ because of (17).

Lemma 1 guarantees that $y_{q^*} = 0$ at $\tilde{x}$ or equivalently that $C_{q^*}(\tilde{x}_{\mathbb{E}}) = 0$. In view of (6) and (7), $y = e_{q^*} + C_{q^*}(x_{\mathbb{E}})$. Therefore $\lim_{t\to\infty} y = \lim_{t\to\infty} e_{q^*}$. Now $\lim_{t\to\infty} \gamma_{q^*}(\|e_{q^*}\|) = 0$ because e_{q^*}, $\dot{e}_{q^*}$, and $\int_0^\infty \gamma_{q^*}(\|e_{q^*}\|)$ are bounded {cf. [13, Lemma 1, p. 58]}. From this it follows that $\lim_{t\to\infty} y = \lim_{t\to\infty} e_{q^*} = 0$ since γ_{q^*} is positive definite and radially unbounded. The following was proved.

Theorem 6. *Let Assumptions 2 to 4 hold. For each initial state $x_{\mathbb{P}}(0) \in \mathcal{X}_{\mathbb{P}}$, $x_{\mathbb{E}}(0) \in \mathcal{X}_{\mathbb{E}}$, $x_{\mathbb{C}}(0) \in \mathcal{X}_{\mathbb{C}}$, $\pi_p(0) > 0$, $p \in \mathcal{P}$, $\sigma(0) \in \mathcal{P}$, the solution $\{x_{\mathbb{P}}, x_{\mathbb{E}}, x_{\mathbb{C}}, \pi_1, \pi_2, \dots, \pi_m\}$ to (4), (5), (6), (7), and (15) {with σ the output of $\mathbb{S}_{\mathbb{H}}$} exists and is bounded on $[0, \infty)$. Moreover, y converges to zero as $t \to \infty$.*

6 Concluding Remarks

In this paper we weaken the conditions under which the supervisory control algorithm proposed in [1] stabilizes a poorly modeled process. This was done by making use of the notion of integral-input-to-state stability [2] and a dual notion of integral detectability introduced here. The use of integral-input-to-state stability and integral detectability seems quite natural when the performance signals are defined as "integral norms" of output estimation errors. In fact, with integral detectability we were able to avoid many of the technical difficulties that arose in [1, 6]. The results in this paper stress the importance of searching for systematic methods to design integral-input-to-stabilizing controllers for large classes of nonlinear processes. On this topic see [5]. The main shortcoming of the present results is that the stability analysis in Section 5 is only valid in the absence of unmodeled dynamics and disturbance/noise. However, simulation results indicate that the overall closed loop system might, in fact, be robust with respect to unmodeled dynamics, disturbance, and noise.

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