

# Suppressing transverse mode instability through multimode excitation in a fiber amplifier

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High-power fiber laser amplifiers have enabled an increasing range of applications in industry, science, and defense. The power scaling for fiber amplifiers is currently limited by transverse mode instability. Most techniques for suppressing the instability are based on single- or few-mode fibers in order to output a clean collimated beam. Here, we study theoretically using a highly multimode fiber amplifier with many-mode excitation for efficient suppression of thermo-optical nonlinearity and instability. We find that the mismatch of characteristic length scales between temperature and optical intensity variations across the fiber generically leads to weaker thermo-optical coupling between fiber modes. Consequently, the transverse mode instability (TMI) threshold power increases linearly with the number of equally excited modes. When the frequency bandwidth of a coherent seed laser is narrower than the spectral correlation width of the multimode fiber, the amplified light maintains high spatial coherence and can be transformed to any target pattern or focused to a diffraction-limited spot by a spatial mask at either the input or output end of the amplifier. Our method simultaneously achieves high average power, narrow spectral width, and good beam quality, which are required for fiber amplifiers in various applications.

fiber laser amplifier | transverse mode instability | multimode fiber

High-power lasers play a critical role in modern industry and in fundamental science studies. A very attractive and efficient type of high-power laser is based on multistage fiber amplifiers (1-6), which have been broadly deployed for industrial, scientific, and defense applications such as laser welding, gravitational-wave detection, and directed energy (7-9). Great opportunities in science and technology will be opened up by achieving higher power with such fiber amplifiers (10). A major obstacle for further power increase is the transverse mode instability (TMI), induced by thermo-optical nonlinearity (11). It is one of the most industrially important outstanding scientific challenges in laser physics. The rich multiscale, multicomponent physics involved in TMI has been extensively studied (12-28), as well as its detrimental effect on the output beam quality (29, 30). Various techniques have been developed over the years to suppress TMI while maintaining good beam quality (31-48). Since TMI originates from nonlinear multimode interactions, the conventional view has been that stable operation could only be achieved by maintaining predominantly single-mode operation. Therefore, most current approaches for TMI suppression strive to minimize the number of fiber modes or reduce mode coupling. In addition, single-mode operation guarantees a high-quality output beam. However, it has proven extremely challenging to maintain single-mode operation at high power, as thermal and/or nonlinear optical effects excite and can even create additional fiber modes.

Here, we propose and validate, using simulation and theory, an efficient robust mechanism for TMI suppression in high-power fiber amplifiers. Since TMI originates from thermo-optical coupling between fiber modes, we intentionally introduce a substantial mismatch between thermal and optical length scales to significantly weaken the coupling. This is realized by coherent excitation of many fiber modes, whose interference creates an intensity variation on a much finer scale than the temperature variation that is dictated by thermal diffusion. Our method of controlling nonlinear light–matter interactions by tuning characteristic length scales may be applied to suppress different types of instabilities in laser systems and even other nonlinear dynamical systems.

Fig. 1 is a schematic illustration of TMI and our scheme of suppression. In a standard quasi-single-mode fiber amplifier, Fig. 1*A*, a higher-order mode (HOM) is excited and its interference with the fundamental mode (FM) creates a spatial modulation of optical intensity (blue). The spatially nonuniform heating results in a temperature variation (red). Due to the thermo-optical effect, a refractive index grating is formed and moves

## Significance

There has been an increasing demand for high-power fiber amplifiers for a wide range of applications. The maximum output power achievable is limited by a nonlinear effect-transverse mode instability. Previous research has pursued single-mode operation as the only route to achieving stable amplification with good beam quality. This work breaks the single-mode paradigm and shows that the transverse mode instability is effectively mitigated in a multimode fiber amplifier when many modes are coherently excited. The output can be focused to a diffraction-limited spot by shaping the input wavefront. Beyond its importance for applications, this work illustrates the potential of multimode fiber amplifiers for fundamental studies on nonlinear complex systems and the possibility of controlling dynamic effects through wavefront shaping.

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Α

Seed laser

In order to take advantage of the increased TMI threshold, the output beam quality must remain high. There is a common impression that if the input light excites many modes, the output beam would necessarily be incoherent, leading to poor beam quality. However, this is true only if the linewidth of the input seed is broader than the spectral correlation width of the output field pattern (for a fixed input wavefront), which need not be the case for narrowband seeds and relatively short length of fibers for high-power lasers, as shown by our simulations below. As long as the output fields from different fiber modes are mutually coherent, two spatial phase masks, placed at the fiber distal end, can convert highly speckled output light to a collimated Gaussian beam without loss (49). Alternatively, the masks may be placed at the fiber proximal end to shape the input wavefront of a coherent seed for focusing the output beam to a diffractionlimited spot (50-52), even in the presence of thermo-optical nonlinearity; then, a lens can be placed after the focal spot to

collimate the beam. Therefore, our method will allow stable, ultrahigh power generation in a many-mode fiber amplifier while maintaining high coherence and beam quality. This multimode approach can also suppress other nonlinear optical effects in fiber amplifiers, such as stimulated Brillouin and Raman scattering (53, 54).

Unstable output beam

Our results are obtained from time-domain numerical simulation and from frequency-domain semianalytic theory. The simulation is performed on optical waveguides with one-dimensional (1D) cross-section, and the results validate the semianalytic TMI theory that can be extended to many modes and fibers with 2D cross-section. It predicts a linear increase of the TMI threshold with the number of excited modes. Even for shaping the input wavefront to focus the output, the linear scaling of the TMI threshold remains, but the slope is lower than that for equal mode excitation if there is no linear mode coupling. In real fiber amplifiers, random linear mode coupling, resulting from fiber imperfections or external perturbations, can further enhance the TMI threshold, while still allowing output focusing. Thus, using highly multimode fibers for amplification may lead to orders of magnitude increase in the TMI threshold.

#### **Numerical Simulation**

We conduct a numerical study on the TMI induced by the thermo-optical nonlinearity in a multimode waveguide amplifier. The reason we simulate a one-dimensional (1D) cross-section waveguide instead of a 2D cross-section fiber is that runtime for simulating TMI in the presence of multiple modes grows rapidly with the number of modes, as finer spatial discretization in both transverse and longitudinal dimensions is required to calculate the nonlinear interactions that involve higher-order modes. Simulation of a realistic fiber with a 2D cross-section



Fiber amplifier

Unshaped seed

Temperature

would take extremely long runtime. Our simplified model of 1D cross-section waveguides not only keeps the runtime tractable but also allows us to clearly identify the physical mechanism underlying the TMI suppression by multimode excitation.

The waveguide has a core of width  $W_c = 40 \ \mu\text{m}$ , and refractive index  $n_c = 1.5$ . Its cladding is 400  $\mu\text{m}$  wide, and its length is  $L = 1 \ \text{m}$ . Monochromatic radiation at wavelength  $\lambda_s = 1,064$ nm is launched into the waveguide as the seed. The pump wavelength is  $\lambda_p = 975 \ \text{nm}$ . Ignoring the evanescent wave in the cladding, the *m*-th guided mode in the core has the transverse (along the *x* axis) field profile  $\phi_m(x) = \sin[m\pi (x + W_c/2)/W_c]$ , and the longitudinal (along the *z* axis) propagation constant  $\beta_m = \sqrt{k^2 - (m\pi/W_c)^2}$ , where m = 1, 2, ..., and  $k = 2\pi n_c/\lambda_s$ . The total field in the waveguide is decomposed as  $\psi(x, z, t) = \sum_m A_m(z, t) \phi_m(x)$ , where *t* denotes time. The scalar paraxial optical wave equation gives

$$\frac{\partial}{\partial z} A_m(z,t) = \left(i\beta_m + \frac{g}{2}\right) A_m(z,t) + ik \sum_{j \neq m} \left[A_j e^{i(\beta_j - \beta_m)z} \times \int_{-W_c/2}^{+W_c/2} \phi_j^*(x) \Delta n_c(x,z,t) \phi_m(x) \, dx\right],$$
[1]

where g is the mode-independent linear-gain coefficient, and  $\Delta n_c(x, z, t)$  is the thermally induced refractive-index change in the core. The last term on the right-hand side of Eq. 1 represents nonlinear mode coupling. Linear mode coupling is neglected here, and its effect on TMI will be discussed later.

The refractive-index change induced by the nonuniform heating is given by  $\Delta n_c(x, z, t) = (dn/dT)\Delta T(x, z, t)$ , where dn/dT is the thermo-optical coefficient and  $\Delta T = T(x, z, t) - T(x, z, 0)$  is the local deviation of the temperature from its value at t = 0. The latter is obtained by solving the heat diffusion equation:

$$\rho C \frac{\partial}{\partial t} T(x, z, t) = Q(x, z, t) + \kappa \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) T(x, z, t),$$
[2]

where  $\rho$ , *C*, and  $\kappa$  are the mass density, specific heat capacity, and thermal conductivity of the waveguide, respectively. The quantum-defect heating  $Q(x, z, t) = g(\lambda_s/\lambda_p - 1) I(x, z, t)$ , where  $I(x, z, t) = n_c |\sum_m A_m(z, t) \phi_m(x)|^2$  is the local intensity. The outer boundaries of the cladding are set to be perfectly thermally conducting (the Dirichlet boundary condition).

Based on the input field amplitude and phase in each waveguide mode, the coupled optical and thermal equations are solved numerically in the time domain (*Materials and Methods* for details). In practice, there is always broadband noise, which commonly arises from the power fluctuation of the seed laser; some amount of such noise is required to simulate TMI. After finding the steady-state solution for a time-invariant input signal, we add a perturbation—temporal fluctuation at frequency  $\Omega$ —to the seed to simulate the effect of the noise. We assume that the noise power in each mode is proportional to the time-invariant signal power. We compute the temporal fluctuation of the output field and examine its dependence on the perturbation frequency  $\Omega$ .

We consider linear gain with a constant coefficient for all modes and throughout the fiber. We ignore gain saturation and pump depletion, which would greatly increase the simulation time. Their effects on TMI suppression will be discussed later.

#### **Two-Mode Amplifier**

We first show how dynamic mode coupling can be suppressed by equal modal excitation. For simplicity, we consider only two modes, the FM and a HOM, excited with distinct power ratios at the input: I) FM:HOM = 99:1, II) FM:HOM = 50:50. I) corresponds to the typical situation that a HOM is created in a single-mode fiber at high power due to thermal and/or nonlinear effects and excited slightly either at the fiber input or via coupling with the FM. The effective number of excited modes is  $M_e \approx 1$ in (I) and  $M_e = 2$  in (II). Ignoring linear mode coupling and mode-dependent gain, the signal mode content is retained throughout the waveguide. For the FM-dominant excitation (I), the steady-state light intensity  $I_{ss}(x, z)$  and temperature  $T_{ss}(x, z)$ distributions, in Fig. 2 A and C, both exhibit a small undulation on top of the FM background, as a result of weak modal interference. In the case of equal-mode excitation (II), a 50times increase in the HOM content leads to a much stronger contrast of light intensity grating in Fig. 2B. The temperature grating in Fig. 2D is of the same period and shows a similarly high contrast. However, the resulting refractive-index grating is static in time and does not cause any dynamic power transfer between the modes.

Next, a sinusoidal perturbation at  $\Omega$  = 1 kHz is added to the input field, with strength  $3 \times 10^{-5}$  of the signal power. With the FM-dominant excitation, both intensity and temperature distributions start changing in time [Fig. 2E and SI Appendix, Movie S1], as is known (11); this leads to a moving refractiveindex grating, which enables dynamic power transfer between the modes. For the FM-dominant excitation, even though the initial HOM content is very low, it grows rapidly along the waveguide due to nonlinear coupling with the FM (Fig. 2G). Close to the waveguide end, the FM and HOM contents exhibit strong antiphase oscillations in time, leading to strong fluctuations of the output beam profile in Fig. 21. Such dynamic power transfer between the FM and HOM is clearly indicative of TMI. In sharp contrast, when the two modes are equally excited with the same total power at the waveguide input, the dynamic part of the temperature grating is much weaker, (Fig. 2F vs. E, note change of color scale), suppressing dynamic mode coupling and power transfer as shown in Fig. 2H. Consequently, at the same power level, TMI is suppressed, and the output beam profile remains stable (Fig. 2J). Therefore, the thermo-optical stability of the amplifier has been greatly improved by evenly distributing the seed power between the modes.

To gain a further physical understanding of the time-domain simulation results, we utilize a semianalytic theory of TMI in the frequency domain. For highly multimode excitations (next section), a new and more general theory of TMI for multimodal excitation of arbitrary mode content is needed, as existing theories consider the interaction between only two optical modes (signal in the FM and noise in a HOM) (17, 19). For the two-mode case, one can understand our time-domain simulation results by examining the existing frequency-domain theories of TMI. As in the simulation, we consider a monochromatic (frequency  $\omega_0$ ) signal of fixed total power that is launched into the FM and a HOM in a certain ratio, along with noise at frequency  $\Omega$  proportional to the signal power in each mode. As described above, the interference between the signal and noise creates a moving intensity grating. Due to the quantum defect heating and the thermo-optical nonlinear index change, a moving refractiveindex grating is generated and induces power transfer between the signal and noise. Two sidebands are formed at  $\omega_0 \pm \Omega$ . Only the downshifted component  $(\omega_0 - \Omega)$  gains power from



**Fig. 2.** Two-mode amplification. The input power ratio of the FM and HOM is 99:1 in the *Left* column ( $M_e \approx 1$ ), and 50:50 in the *Right* column ( $M_e = 2$ ). Panels (*A–D*) have no perturbation (noise) added to the input signal; (*E–J*) have a sinusoidal perturbation at 1 kHz. The SD of temporal fluctuations of the input power in each mode is  $3 \times 10^{-5}$  of the time-invariant signal power. (*A* and *B*) Steady-state light intensity distribution  $I_{SS}(x, z)$  with  $M_e \approx 1$  in (*A*) displays much weaker mode interference than that of  $M_e = 2$  in (*B*). The input and output powers are 23 W and 206 W, respectively. (*C* and *D*) Steady-state temperature distribution  $I_{SS}(x, z)$  with  $M_e \approx 1$  in (*C*) has much lower contrast than that of  $M_e = 2$  in (*D*). (*E* and *F*) The temperature change  $\Delta T = T(z, t) - T(z, 0)$  (at  $x = 10 \ \mu$ m, *z* near the output) induced by the input perturbation is much larger for  $M_e \approx 1$  in (*E*) than  $M_e = 2$  in (*F*). Note that the scale for  $\Delta T$  in (*E*) is increased by 10 times compared to (*F*). (*G* and *H*) Power ratio of HOM and FM (averaged over t = 5 to 10 ms) increases rapidly along the waveguide with  $M_e \approx 1$  in (*G*), while it stays nearly invariant throughout the waveguide for  $M_e = 2$  in (*H*). (*I* and *J*) Output beam profile ( $I_{Out} = I(x, L, t)$ ) with  $M_e \approx 1$  begins to oscillate after 1 ms of perturbation in (*I*), indicative of the onset of TMI. By contrast, the output profile with  $M_e = 2$  remains almost unchanged over time in (*J*), and its shape is determined by the relative phase of input fields in the two modes.

the signal and manifests as output-power fluctuation, while the upshifted component  $(\omega_0 + \Omega)$  is suppressed. We thus focus on the fluctuation at  $\omega_0 - \Omega$  in the HOM, which grows exponentially, owing to both linear optical gain and thermo-optical gain, according to ref. 19:

$$\tilde{P}_2(z) = \tilde{P}_2(0) \cdot e^{gz} \cdot e^{\chi_{21}(\Omega)[P_1(z) - P_1(0)]},$$
[3]

where  $\tilde{P}_2(z)$  denotes the noise power in the HOM at  $\omega_0 - \Omega$ ,  $P_1(z)$  is the signal power in the FM, and  $\chi_{21}(\Omega)$  is the nonlinear susceptibility due to thermo-optically induced coupling between the FM and HOM at frequency downshift  $\Omega$  (*Materials and Methods*). The final factor on the right-hand side of Eq. **3** shows that the exponential growth rate of the noise power increases linearly with the signal power in the FM. This expression is valid below the TMI threshold ignoring nonphase-matched terms and the interaction of noise and signal in the same mode; the latter of which occurs at much lower frequencies (17).

A critical insight from Eq. 3 is that the HOM power fluctuation depends linearly on its own input noise power  $\tilde{P}_2(0)$ , which is proportional to its signal power  $P_2(0)$ , but it also depends exponentially on the thermo-optical gain, which is proportional to the FM signal power  $P_1(z)$ . Hence, the noise power at the output, z = L, is much more sensitive to the latter. Therefore, by putting signal power in the HOM and correspondingly more noise in the HOM, we are trading less input noise in the HOM for a much slower growth rate, which is highly favorable for reducing TMI. Specifically, for the parameters relevant to our simulation, changing from the FMdominant to equal-mode excitation increases the HOM input noise power  $\tilde{P}_2(0)$  by 50 times, but the halving of the FM signal power  $P_1(0)$  in the exponent leads to a reduction of the final term in Eq. 3 by  $3.7 \times 10^5$ . Consequently, the HOM noise power at the waveguide end,  $\tilde{P}_2(L)$ , is  $1.35 \times 10^{-4}$  smaller with equal-mode excitation than with FM-dominant excitation.

#### **Multimode Excitation**

The previous section shows that equal power division between two modes effectively suppresses TMI. A natural question arises: Will TMI be further suppressed if even more modes are excited? The answer is not obvious a priori. When the seed power is divided among multiple modes, thermo-optical coupling between any mode pair is weaker, but there are more mode pairs. The increase in the number of coupled mode pairs might counter the decrease of coupling between each mode pair. The noise in Downloaded from https://www.pnas.org by YALE UNIVERSITY CTR FOR SCI & SOCIAL SCI INFO on May 22, 2023 from IP address 130.132.173.15.

one mode may acquire thermo-optical gain equal or greater from signals in all other modes, despite the gain from each individual mode being lowered. However, consistent with our argument about mismatch of spatial scale, we will show below that only a fraction of total mode pairs have strong thermo-optical coupling, leading to significant suppression of the TMI upon many-mode excitation.

Below we consider five-mode amplification in the waveguide with a 1D cross-section. A monochromatic seed coherently excites the five lowest-order modes equally. The multimodal interference produces a fine-grained intensity pattern  $I_{ss}$ , as shown in Fig. 3A. The first and fifth modes have the largest difference in transverse and longitudinal wave numbers, and their beating periods are the shortest in both directions, e.g., their longitudinal beating period is 7 times shorter than that between the first and second modes. However, as we expected, the fine grains in the light intensity pattern  $I_{ss}$  disappear in the steady-state temperature distribution  $T_{ss}$  in Fig. 3B because the heat diffusion responds on a longer spatial scale and cannot follow the short-scale intensity variations that act as local heat sources. This effect is evident when comparing the longitudinal Fourier spectra of  $I_{ss}$  and  $T_{ss}$  in Fig. 3C. The high-frequency peaks in  $I_{ss}$ due to longitudinal beating of nonadjacent modes are strongly suppressed in  $T_{ss}$ , which contains mostly the low-frequency peaks from adjacent-mode beating. The resulting refractive-index grating couples mainly adjacent modes in propagation constant, and the thermo-optical interaction between nonadjacent modes is greatly damped.

Next, we introduce an input perturbation at  $\Omega = 1$  kHz to all five modes. While the ratio of the total input perturbation power to the total signal power is kept the same as that for twomode excitation, the signal output-power is raised to 315 W by increasing the seed power. At this higher power level, as discussed below, TMI does occur under two-mode equal excitation, but the five-mode amplification remains stable, and the output pattern barely changes over time in Fig. 3D, indicating that the TMI has been further suppressed.

To further show the importance of equal excitation of all modes, we simulate different power ratios for the five modes: (I) 96:1:1:1:1; (II) (97/2):(97/2):1:1:1; (III) (98/3):(98/3):(98/3):1:1; (IV) 20:20:20:20:20. The effective numbers of excited modes are  $M_e \approx 1, 2, 3$  and  $M_e = 5$  in the four cases. The total input power is kept constant, and the noise power is proportional to the signal power in each mode. For quantitative comparison of amplifier stability, we compute the relative output-power fluctuation:  $F \equiv \sum_m \sigma_m / \sum_m P_m(L)$ , where  $\sigma_m$  is the SD of temporal fluctuation of output power in the *m*-th mode, and  $\sum_m P_m(L)$  is the total output power (in all modes) averaged over time t. Fig. 3E reveals that F grows exponentially with increasing  $\sum_m P_m(L)$ . However, the growth rate drops rapidly as  $M_e$  increases from 1 to 5. We also vary the input perturbation at a fixed output power and show in *SI Appendix*, Fig. S2 that the amplifier becomes much more robust against input perturbation with more-mode excitation.

To understand and further validate these results, we have developed a semianalytic theory of TMI for multimodal excitation by solving the coupled optical and heat equations in the frequency domain. The crucial difference from existing theories (17, 19)is that our model allows arbitrary input power distribution, along with noise, among various modes of a multimode fiber amplifier. In this case, the noise power in the *m*-th mode grows exponentially as

$$\tilde{P}_m(z) = \tilde{P}_m(0) \cdot e^{gz} \cdot e^{\sum_{j \neq m} \chi_{mj}(\Omega) [P_j(z) - P_j(0)]}.$$
 [4]



Fig. 3. Five-mode amplification. (A-D)  $M_e = 5$ , equal excitation of five lowestorder guided modes. (A) Steady-state light intensity distribution  $I_{SS}(x, z)$ displays multimode interference. The input and output powers are 35 W and 315 W, respectively. (B) Steady-state temperature distribution  $T_{SS}(x,z)$ exhibits much smoother and longer modulation than  $I_{SS}(x, z)$ . (C) Fourier spectrum of longitudinal intensity  $\langle |I_{SS}(x, k_Z)| \rangle_X$  (blue) consists of peaks from mode-beating. The black (#) above each peak denotes the spacing |m-j| of the corresponding mode pair. The high-frequency peaks from nonadjacent mode pairs are strongly suppressed in the longitudinal spectrum of temperature  $\langle |T_{SS}(x, k_Z)| \rangle_X$  (orange). (D) The output beam profile ( $I_{out} = I(x, L, t)$ ) under input perturbation ( $\Omega = 1$  kHz) remains stable in time. The SD of temporal fluctuation of input power in each mode is  $3 \times 10^{-5}$  of the signal power. (E) Relative output-power fluctuations for different numbers of excited modes, Me. The fluctuation F grows exponentially with average output-power  $\sum_{m} P_m(L)$  and with a smaller growth rate for larger  $M_e$ , while input fluctuation is kept the same. Crosses are numerical data, and lines represent exponential fitting. The gray dotted line at F = 0.01 marks the TMI threshold, which increases with Me.

The nonlinear growth rate of noise power in each mode is equal to the sum of signal powers in all the other modes weighted by the corresponding thermo-optical susceptibility  $\chi_{mj}$ . For a mode pair *m* and *j*,

$$\chi_{mj}(\Omega) = \alpha \sum_{l} \frac{\Omega}{\Omega^2 + \Gamma_l^2} \left| \int_{-W_c/2}^{+W_c/2} \psi_m^*(x) \psi_j(x) T_l(x) \, \mathrm{d}x \right|^2,$$
[5]

where  $\alpha$  is a material-dependent coefficient (Materials and *Methods*),  $T_l(x)$  is the transverse temperature profile for the *l*-th eigenmode of the heat diffusion equation, and  $\Gamma_l$  is its decay rate, which is on the order of a few kHz. For the five-mode waveguide, we calculate the nonlinear gain susceptibilities  $\chi_{mj}$ of all mode pairs. For a given number of equally excited modes  $M_e$ , the effective thermo-optical susceptibility for mode m is defined as  $\overline{\chi}_m \equiv \sum_{j \neq m} \chi_{mj}/M_e$ , where  $1/M_e$  is the fraction of signal power in each of the other modes. Finally, in Fig. 4A, we plot the average of  $\overline{\chi}_m$  over all *m*, denoted by  $\overline{\chi}$  (*Materials* and Methods for details of the calculation); the theory (red dots) and the numerical results from the time-domain simulations (blue crosses) show excellent agreement. The value of  $\overline{\chi}$  depends on the perturbation frequency downshift,  $\Omega$ , and reaches its maximum at around 1 kHz. Five-mode excitation ( $M_e = 5$ ) greatly reduces the thermo-optical susceptibility, compared to FM-dominant excitation ( $M_e \approx 1$ ). Fig. 4B depicts in a color scale the maximal value of  $\chi_{mj}(\Omega)$  for each pair of modes. The thermo-optically induced mode coupling is the strongest for the adjacent modes which have the smallest difference in propagation constant  $\Delta \beta_{mj} = |\beta_m - \beta_j|$ .  $\chi_{mj}$  decreases monotonically with increasing mode spacing |m - j|. This short-ranged coupling is characteristic of a diffusion-mediated process; physically, it arises from the mismatch between length scales between thermal diffusion and optical interference. This enters the theory through the overlap integral in Eq. 5, which dictates that only the thermal mode l with its spatial scale matching the beating period of two optical modes m and j can cause significant coupling between mand j. The contribution to  $\chi_{mj}$  from thermal mode l peaks at frequency downshift  $\Omega = \Gamma_l$ , and the peak value is proportional to the mode lifetime  $1/\Gamma_l$ . Thus, for a pair of optical modes with large spacing |m - j|, their beating period  $1/\Delta\beta_{mj}$  is short, so only the thermal mode with a high spatial frequency (large l) contributes to the thermo-optical coupling. However, such a thermal mode has a short lifetime,  $1/\Gamma_l$ , leading to weak coupling strength for optical modes with substantially different propagation constants.

This short-ranged nature of the thermo-optical coupling ensures that the noise growth rate in any waveguide mode is effectively determined by the signal power in its two neighboring modes in propagation constant (for 1D cross-section). Therefore, if the signal power is equally divided into  $M_e \gg 2$  modes, the noise growth rate in Eq. 4 drops by a factor of  $\sim M_e/2$ . Alternatively, for the same noise at the output, the signal power is raised by a factor of  $\sim M_e/2$ . Note that nonadjacent mode couplings are weak but nonzero; thus, the slope of the enhancement factor is slightly less than 1/2.

In Fig. 4*C*, we evaluate the TMI threshold quantitatively for the waveguides which we have simulated. Defining the threshold as the signal output power when the relative outputpower fluctuation reaches F = 0.01, we find that threshold output power does scales linearly with  $M_e$  for  $M_e \ge 5$ , with slope  $\approx 0.36$ . The exact simulation results agree with the theory quantitatively, with no fitting parameters, for  $M_e \le 5$ ; for  $M_e > 5$ , the simulations are impractical, but the theory (which requires only numerical evaluation of the overlap integrals in Eq. 5) is straightforwardly extended to any relevant number of modes. We define a threshold enhancement factor as the ratio of the threshold for  $M_e$  equally excited modes to that under the FMdominant excitation ( $M_e \approx 1$ ). We find that choosing  $M_e = 20$ leads to a seven-fold increase of the TMI threshold.

The excellent agreement of theory and simulation validates our semianalytic frequency-domain theory which requires only the guided mode profiles and propagation constants as numerical inputs. Since no discretization of space and time is required, our theory can predict the TMI threshold for realistic fibers with 2D cross-sections. We calculate the thermo-optical coupling matrix and the TMI threshold of a step-index, circular-crosssection multimode fiber with parameters similar to commercially available Yb-doped silica fibers (Fig. 5). The fiber, with a core diameter of 40  $\mu$ m and numerical aperture (NA) of 0.145, supports 78 modes per polarization at  $\lambda_s = 1.064 \ \mu m$ . Fig. 5A shows that the thermo-optical coupling matrix for this highly multimode fiber is sparse, similar to the 1D-cross-section case, (Fig. 4B), due again to the fact that the mode pairs with a large difference in the propagation constant couple very weakly. Because of such robust sparseness, the TMI threshold still increases linearly with the effective number of equally excited modes  $M_e$ , (Fig. 5B). Note that the slope of the linear increase (0.16) is lower than that (0.36) for a 1D-cross-section waveguide of similar dimensions since there are more adjacent modes in terms of propagation constant in a 2D-cross-section fiber. With all 78 modes equally excited, the TMI threshold power is  $\sim$ 13 times the threshold under the FM-dominant excitation



**Fig. 4.** Thermo-optical mode coupling and TMI threshold. (*A*) Effective thermo-optical susceptibility  $\overline{\chi}$  for  $M_e = 5$  and  $M_e \approx 1$  varies with frequency downshift  $\Omega$  from  $\omega_0$  due to sinusoidal input perturbation and peaks around 1 kHz. The maximum of  $\overline{\chi}$  for equal excitation of five modes ( $M_e = 5$ ) is  $\sim M_e/2 = 2.5$  times weaker than that under FM-dominant excitation ( $M_e \approx 1$ ). Circles are numerical data from time-domain simulation; lines are from semianalytic frequency-domain theory. (*B*) Maximal value of  $\chi_{mj}$  for every pair of modes ( $m \neq j$ ) decreases monotonically with increasing mode spacing |m - j|. (C) TMI threshold increases linearly with the effective number of excited modes  $M_e$  in a waveguide of 1D cross-section. Blue crosses are time-domain simulation results, red dots are predictions of frequency-domain theory, and the black line is a linear fit with a slope of 0.36 for the threshold enhancement factor over the FM-dominant excitation ( $M_e \approx 1$ ) for  $5 \leq M_e \leq 20$ . The threshold enhancement is about 7× at  $M_e = 20$ , and the output power exceeds 1,200 W.



**Fig. 5.** TMI in a step-index fiber amplifier. The fiber has a circular crosssection (core diameter = 40  $\mu$ m, NA = 0.145) and supports 78 modes per polarization. (*A*) Calculated matrix of the maximal values of thermo-optical coupling coefficients ( $\chi$ *mj*) for all pairs of modes ( $m \neq j$ ) is sparse. Modes are ordered according to their propagation constants. (*B*) The TMI threshold increases linearly with effective number of excited modes  $M_e$ . Red dots are predictions of our frequency-domain theory; the black line is a linear fit for the threshold enhancement over the FM-dominant excitation ( $M_e \approx 1$ ) with a slope of 0.16. With all 78 modes equally excited, the TMI threshold is enhanced 13×, reaching an output power of ~3.2 kW.

 $(M_e \approx 1)$ , outputting 3.2 kW power before the onset of instability. The TMI threshold enhancement can be further increased by using a fiber with more modes.

### **Output Beam Quality**

One serious negative consequence of dynamical mode coupling is the resulting temporal fluctuations of the transverse field profile, which corresponds to a loss of spatial coherence. While multimode excitation itself produces a speckled field, the spatial coherence is not necessarily lost. Let us fix the input field profile to a multimode waveguide and scan the wavelength  $\lambda_s$ ; the output field pattern  $E_{out}(x, \lambda_s)$  will change and eventually decorrelate. The spectral correlation function is defined as  $C(\Delta\lambda_s) \equiv \langle |\langle E_{out}^*(x, \lambda_s) E_{out}(x, \lambda_s + \Delta\lambda_s) \rangle_x | \rangle_{\lambda_s}$ , where  $\langle ... \rangle_x$  denotes averaging over x, and  $\langle ... \rangle_{\lambda_s}$  over  $\lambda_s$ .  $C(\Delta\lambda_s)$  decays with  $\Delta\lambda_s$ , and its width at half maximum is defined as the spectral correlation width. If the input seed has a bandwidth narrower than the spectral correlation width, the output field pattern stays nearly constant in time, except for a global amplitude and/or phase change. Thus, the output field is spatially coherent, even if it displays a complex interference pattern. This will be reflected in the degree of spatial coherence:  $C_s \equiv \langle |\langle E_{out}^*(x,t)E_{out}(x,t+\Delta t)\rangle_x|\rangle_{t,\Delta t}$ , where the output field  $E_{out}(x,t)$  is normalized:  $\int_{-W_c/2}^{+W_c/2} |E_{out}(x,t)|^2 dx = 1$  at any t.

Fig. 6 shows the degree of spatial coherence of the output field for different modal excitation in a five-mode amplifier. Below the TMI threshold,  $C_{\rm s} \simeq 1$  reflects nearly perfect spatial coherence in the presence of a small perturbation. Above the TMI threshold,  $C_{\rm s}$  drops quickly, as the output pattern changes with time.

Below the TMI threshold, since the output field pattern  $E_{out}(x)$  is constant in time, a spatial mask placed at the amplifier output end can convert it to any desired pattern  $E_d(x) = T(x) E_{out}(x)$ , where T(x) is the field transmission coefficient of the spatial mask. Since field amplitude modulation introduces power loss, phase-only modulations by two masks (with free-space propagation/diffraction between them) may be employed for lossless conversion (49). To avoid high-power handling, the masks may be placed at the amplifier input end to shape the spatial wavefront of a coherent seed in order to create a desired output profile. Again lossless shaping of output field amplitude and phase requires two separate phase masks for input modulation.

Since our approach to suppressing TMI relies only on nearly equal excitation of all waveguide modes, it does not impose any restriction on the phase of the input field in each individual mode. Therefore, the input phases of the excited modes may be adjusted to shape the output beam profile. As an example, we numerically show that the output beam can be refocused to a diffraction-limited spot by wavefront shaping of the input light, while simultaneously achieving a high TMI threshold. In Fig. 7, we show a 270-W output beam from the five-mode amplifier focused to a diffraction-limited spot, using the input intensity and phase in each mode shown in Fig. 7*A*. A sharper and cleaner focus than Fig. 7*C* should be attainable in a fiber with more modes excited.

Fig. 7*D* reveals that the output intensity pattern is invariant over time, with the spatial coherence  $C_s = 0.9999$ . This performance contrasts with that of the conventional quasi-single-mode amplifier, whose output fluctuates drastically in time at the same power level (*SI Appendix*, Fig. S1). Compared to the 153-W threshold for the FM-dominant excitation, the TMI threshold in



**Fig. 6.** Spatial coherence of a multimode amplifier. Plot of the spatial coherence of the output field  $C_S$  vs. average output-power  $\sum_m P_m(L)$  showing near-perfect spatial coherence below the TMI threshold (marked by gray dotted line) but a rapid deterioration, as expected, above the threshold. The SD of the temporal fluctuations of the input power in each mode is  $3 \times 10^{-5}$  of the signal power. At a fixed average output-power,  $C_S$  is higher for a larger effective number of excited modes  $M_e$ .



**Fig. 7.** Adjusting the seed wavefront to focus through a multimode amplifier. (*A*) Power and phase of input light in each of five modes in the waveguide for focusing the output light to a spot at  $z_f = 174 \,\mu\text{m}$  away from the end facet and  $x_f = 2.2 \,\mu\text{m}$  off the waveguide axis, as shown schematically in (*B*). (*C*) Transverse intensity distribution ( $l_f(x)$ ) at  $z_f$ , showing the focal spot with a diffraction-limited width of 10.4  $\mu$ m, given by the full width at  $e^{-2}$  of the maximum intensity. (*D*) The output beam profile at the focal plane ( $l_f(x, t)$ ) remains stable in time under input perturbation ( $\Omega = 1 \,\text{kHz}$ ). The SD of temporal fluctuations of the input power in each mode is  $3 \times 10^{-5}$  of the signal power. (*E* and *F*) Focusing the output light to one or two spots at different positions ( $z_f$  and  $x_f$  are labeled in the panels) by shaping the input seed wavefront. The side lobes can be greatly reduced by increasing the number of waveguide modes that contribute to the focusing.

the case of output focusing with five modes reaches 270 W. This threshold is below that for equal mode excitation (cf. Fig. 3*E*), due to the nonuniform power distribution among the modes (Fig. 7*A*). Nevertheless, the linear threshold scaling with the number of modes remains valid for output focusing (*SI Appendix*, Note 3 and Fig. S4), but the slope of linear increase is smaller than that of equal mode excitation, reflecting a reduced efficacy of TMI suppression.

Although the mode content for output focusing is not optimal for TMI suppression, it is noteworthy that random mode coupling due to fiber imperfections and external perturbations promotes equipartition of power among modes and thus increases the TMI threshold (*SI Appendix*, Note 2). As shown in *SI Appendix*, Table S1, with strong random mode coupling, the TMI threshold approaches that under equal mode excitation (~330 W). Since random mode coupling is a linear, deterministic process, mutual coherence among the modes does not deteriorate throughout the waveguide, and thus, the output field remains spatially coherent. That is, output focusing can still be achieved by shaping the input wavefront, in the presence of excessive mode coupling, as shown in *SI Appendix*, Fig. S3.

Moreover, the capability of shaping a coherent seed at fiber input, rather than output, could avoid modulating a high-power output light while achieving beam steering, shaping, or other advanced manipulations. As an illustration, in Fig. 7 E and F, we show two additional examples of output focusing to one or two spots at various transverse and longitudinal positions. Their TMI thresholds are both slightly higher than that in the focusing case shown in Fig. 7C (270 W).

#### Discussion

In summary, equal excitation of many guided modes stabilizes a fiber amplifier against the thermo-optical nonlinearity leading to TMI. The maximum growth rate for the noise power is reduced by distributing the power into many modes because any given mode only experiences strong gain from signal power in the adjacent modes (i.e., modes with the closest propagation constants). Nonadjacent modes have weak thermo-optical coupling and contribute little to the dynamic power transfer. The substantially reduced coupling strength of nonadjacent modes is due to the large mismatch between the thermal and optical length scales. The linear scaling of the TMI threshold with the number of equally excited modes provides a powerful path toward robust TMI suppression, as a multimode fiber can easily support hundreds of modes. In principle, the threshold enhancement is only limited by the total number of modes in a fiber.

We have also shown by numerical simulations that the output beam profile of a multimode amplifier with thermo-optical nonlinearity can be tailored by wavefront shaping of a coherent seed with spatial masks. The capability of focusing, collimation, and shaping the output beam is crucial for practical applications. Several techniques (50-52, 55, 56) are readily available for the experimental implementation of wavefront shaping for fiber amplifiers. For high-power amplification, lossless conversion of coherent beam profiles is important, and it can be realized by using two separate phase masks (with some distance between them) (49). Both of them should be dynamic phase masks (spatial light modulators), and their phase patterns will be constantly adjusted to compensate for temporal drift and fluctuation of the fiber amplifier (below the TMI threshold) and the static phase mask. The feasibility and possible configurations for wavefront shaping are discussed in more detail in SI Appendix, Note 4.

In our study, we have cross-validated the numerical simulations and semianalytic theory on a waveguide with a 1D cross-section and further adapted the theory for waveguides with 2D crosssection to confirm that our method works for realistic multimode fiber amplifiers. In our simulations and theory, random mode coupling, mode-dependent gain/loss, and pump depletion are neglected. Random mode coupling tends to spread power evenly among all modes and facilitates our equal-excitation scheme, leading to stronger suppression of TMI (as confirmed by our numerical simulations shown in SI Appendix, Note 2). Moreover, as a linear, deterministic process, random mode coupling does not reduce the spatial coherence of the output field; thus, output focusing can still be achieved by input wavefront shaping. Therefore, our method will work well in the presence of fiber bending and imperfections, which induce random mode coupling (45). Mode-dependent gain/loss generally reduces the effective number of excited modes. We have considered a uniform pump over the waveguide cross-section (mode-independent gain) due to the highly multimode and incoherent nature of the pump light commonly used for fiber amplifiers. We have also neglected the decrease in pump power along the longitudinal direction of the waveguide (pump depletion). This is because the TMI threshold does not depend on the longitudinal variation of the gain coefficient, as the heat load varies proportionally to the outputpower change. Since the thermo-optical gain depends only on the extracted signal power (output power minus input power), the net noise growth by thermo-optical gain is independent of the longitudinal pump depletion for a fixed output power. Another effect that is not accounted for in this study is optical gain saturation, which reduces heating and raises the TMI threshold (35, 38). We expect the threshold increase we have found under multimode excitation to be present even if gain saturation is taken into account. As noted above, the advantage of multimode excitation stems from the mismatch in the length scale for optical mode interference and the much larger scale over which the induced temperature variations occur. Because the characteristic length scale of optical mode interference is not increased by gain saturation, it should not alter the short-range nature of thermooptical coupling, which leads to the linear scaling of the TMI threshold with the effective number of excited modes.

Finally, our method may be combined with other schemes for TMI suppression. Since the mismatch between optical and thermal length scales is almost universal among multimode fibers, our multimode excitation scheme will work for specialty fibers that have been designed to increase the TMI threshold (32, 36, 47). This also opens a vast space for fiber structure design to enable output-beam engineering from tailored mode structures or to offer different functionalities. We envision that, with optimized modal excitation, multimode fiber amplifiers can operate stably with high beam-quality at extreme powers that would be unachievable with their single-mode counterparts.

#### **Materials and Methods**

**Time-Domain Numerical Simulation.** We choose the parameters in our simulation to be comparable with other numerical studies (14, 17-19, 25, 27). The thermo-optical coefficient is  $dn/dT = 1.17 \times 10^{-5} \,^{\circ}\text{C}^{-1}$ , the product of mass density and specific heat capacity is  $\rho C = 1.67 \times 10^{-12} \,\text{J K}^{-1} \mu\text{m}^{-3}$ , and the thermal conductivity is  $\kappa = 1.4 \times 10^{-6} \,\text{W K}^{-1} \mu\text{m}^{-1}$ . We ignore the pump depletion and assume that the optical gain is constant for all modes throughout the waveguide. The power gain coefficient is  $g \approx 2.2 \,\text{m}^{-1}$ .

Since heat diffusion is much slower than light propagation through the amplifier, the temperature distribution throughout the waveguide is treated as static during the time that light travels the entire length of the amplifier (14). At each time step, Eq. **1** is solved with a static  $\Delta n_c$ , which is given by the current temperature distribution. The resulting modal amplitudes along the waveguide are then used to calculate the heat load to update the temperature

distribution. More specifically, we use the Crank–Nicolson scheme to solve Eq. **2** for the temperature distribution at the next time step. The longitudinal (along z) heat diffusion is neglected in our simulations because the temperature gradient in z is much smaller than that in x. Note that the longitudinal heat diffusion will become comparable to the transverse heat diffusion (along x) when a large number of modes are excited.

We carefully adjust the temporal and spatial discretization steps in the simulations and test numerical accuracy and convergence. The time step is  $\Delta t \sim 100$  ns. The longitudinal step is  $\Delta z \sim 1 \,\mu$ m for solving Eq. **1**. The spatial grids are  $\Delta x \sim 0.1 \,\mu$ m and  $\Delta z \sim 100 \,\mu$ m for solving Eq. **2**.

**Frequency-Domain Semianalytic Theory.** Eq. **4** is a direct generalization of previously studied cases of noise growth upon FM-dominant excitation (17). It ignores any nonphase-matched terms and the self-coupling, which occurs on a frequency scale much lower than standard TMI.  $T_l$  is the *l*-th eigenmode of the transverse Laplacian  $\nabla^2_{\perp}$  with Dirichlet boundary conditions at the outer cladding surface.

The coefficient in Eq. **5** is  $\alpha = 2\pi \eta D(\lambda_s - \lambda_p)/n_c \kappa \lambda_s \lambda_p$ , where  $\eta \approx 2n_c(dn/dT)$  is the thermo-optical constant, and  $D = \kappa/\rho C$  is the heat diffusion constant.

To extract the effective thermo-optical susceptibility from the time-domain simulation of five-mode excitation ( $M_e = 5$ ) in Fig. 4A, we perturb only one mode at a time and examine the output-power fluctuation of that mode. From the time-domain simulation data, we first calculate the noise power in each mode at position z,  $\tilde{P}_m(z)$ , by taking the ratio between the temporal variance and mean of the mode power. We then calculate the relative intensity noise (RIN) of mode *m* at *z* as  $RIN_m(z) = \tilde{P}_m(z)/P_m(z)$ . The effective thermo-optical susceptibility  $\overline{\chi}$  is calculated by taking the log of noise amplification factor (given as the ratio of the output and input RINs), dividing by the total extracted power, and averaging over the five modes,  $\overline{\chi} = \langle \overline{\chi}_m \rangle_m = \langle \log[RIN_m(L)/RIN_m(0)] \rangle_m / \sum_m [P_m(L) - P_m(0)].$  This formula is obtained by inverting Eq. 4. For comparison, we calculate the effective thermo-optical susceptibility under the FM-dominant excitation from the fluctuation of the HOM in a two-mode waveguide,  $\overline{\chi}(M_e \approx 1) = \chi_{21} =$  $\log[\text{RIN}_2(L)/\text{RIN}_2(0)]/[P_1(L) - P_1(0)]$ . For the theoretical calculation of  $\overline{\chi}$ , we first calculate pairwise thermo-optical susceptibility  $\chi_{mj}(\Omega)$  using Eq. 5 and then the mean for mode m,  $\overline{\chi}_m = \sum_{j \neq m} \chi_{mj} / M_e$ , and finally average  $\overline{\chi}_m$  over m. Since only the noise power at downshifted frequency ( $\omega_0 - \Omega$ ) experiences growth due to the thermo-optical effect, we plot only the downshifted spectrum of both theoretical and numerical  $\overline{\chi}$  in Fig. 4A, and the perturbation frequency  $\Omega$  corresponds to the frequency downshift.

**Data, Materials, and Software Availability.** All numerical and theoretical findings can be reproduced using the information presented in the paper or *SI Appendix*. All the data are available at https://doi.org/10.5281/zenodo. 7927514 (57).

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