10 December 2001

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 291 (2001) 208–214

www.elsevier.com/locate/pla

Quantum stochastic trajectory theory of microsuperradiant laser

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Received 31 August 2001; received in revised form 25 October 2001; accepted 25 October 2001
Communicated by P.R. Holland

Abstract

The microsuperradiant laser is studied by the quantum stochastic trajectory approach. Neither discontinuities nor the instabilities are found as in the case of large N limit. The atomic population on the middle level is not always high either. Our © 2001 Published by Elsevier Science B.V.

PACS: 42.50.Fx; 42.50.Ar; 24.10.Lx

1. Introduction

The cooperative spontaneous emission of N identical two-level atoms, which is so-called superfluorescence, had been studied extensively [1] in 1970's. A few years ago, a different type of superradiance, a kind of N identical three-level atom superradiant laser, was proposed and investigated by Haake et al. [2,3]. In contrast to the superfluorescence, these N atoms in this model only interact with two cavity modes. Its line width was found extremely small and proportional to $1/N^2$ while the superfluorescence pulse has a large spectral width $\sim N\gamma$. This superradiant laser is different from normal laser [4] in that the pump and relaxation are also cooperative. The fields are generated collectively by the N atoms so that its intensity is still $\sim N^2$, if p, the effective pumping strength defined by Haake et al., is kept in a fixed value independent of N. Besides, the output light field can be squeezed almost perfectly. However, there were some unusual results [3]: at zero pump some atomic populations show an abruption like first-order phase transition, and as pump rises to a certain value there appears a second-order phase transition to make laser action ceased above this value. These unusual results are obtained by the condition $N \to \infty$, which is implicitly assumed in the linearization process applied in Ref. [3] to solve the superradiant laser's Langevin equations. In the following we shall call such results as semiclassical results.

Recently, there is much interest in microlasers with small number of atoms or excitons. In this Letter we consider the microsuperradiant laser in which only dozens of atoms are involved. It is quite natural to surmise that the abruptions in the semiclassical results will be smoothed out when the number of atoms becomes finite as usually the case in statistical mechanics. The question is, apart from this point, whether

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PII: S0375-9601(01)00719-8

there is any new behavior of microsuperradiant laser different from the semiclassical theoretical results of Ref. [3]. We find that in some region of parameters the situation is indeed so, which will be described in Section 3. In addition, only the steady state is studied in Haake et al. papers [2,3], while in our Letter the whole evolution process of the microsuperradiant laser is also studied, which, among others, will give us an idea as how fast the steady state is established. The method we take for these purposes is the quantum stochastic trajectory approach (QSTA) [5] which was developed about ten years ago. The basic idea of QSTA is to use a non-unitary evolution generated by an effective non-Hermitian Hamiltonian intermingled with successive quantum collapses to describe the dynamics of the system. The collapse superoperator which describes the quantum jumps and the non-Hermitian Hamiltonian are obtained by a decomposition of the corresponding master equation [5]. This approach has some advantages over usual method dealing with master equations: the single trajectory may simulate a concrete sample of the process with such quantum collapses rather than the average expression corresponding to an ensemble of such process. But in this Letter we mainly use QSTA as a calculation means to derive the average values for finite number of atoms. When the collapse superoperator allows the density operator $\rho(t)$ to be factorized as $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, the numerical analysis may be much simplified. The vector $|\Psi(t)\rangle$ is sometimes called Monte Carlo wave function (MCWF).

In Section 2, we will decompose the master equation of a superradiant laser system to get the non-Hermitian operator and the collapse operator. In Section 3, some stationary results are presented. We show explicitly that the discontinuities in semiclassical results is indeed smoothed out. The stationary values of the atomic populations as functions of the pump as well as the stationary photon number distribution are given accordingly. We also show that when the parameters ξ defined in Section 2 becomes large, the atomic populations among the three levels are evidently dissimilar with the semiclassical results of Ref. [3]. In Section 4, we will give a general picture of how the microsuperradiant laser system evolves, as compared with the usual non-superradiant laser system. Besides, we show that, for finite atom number, there do not exist the instabilities found in Ref. [3]. Brief discussion and conclusions are drawn in Section 5.

2. Monte Carlo wave function approach to microsuperradiant laser

The Hamiltonian for the model [3] of superradiant laser in the interaction picture is

$$H = i\hbar g_{12}\big(aS_{21} - a^\dagger S_{12}\big) + i\hbar\Omega(S_{20} - S_{02}) + i\hbar g_{01}\big(bS_{10} - b^\dagger S_{01}\big), \tag{1}$$

in which a and b are the annihilation operators of the light for mode a (lasing mode or active mode) and mode b (passive mode), respectively. The energies for the three atomic levels are E_0, E_1, and E_2. $E_0 < E_1 < E_2$. The collective atomic operators S_{ij} $(i, j = 0, 1, 2)$ are defined as

$$S_{ij} = \sum_{\mu=1}^{N} S_{ij}^{(\mu)} = \sum_{\mu}\big(|i\rangle\langle j|\big)^{(\mu)},$$

which obey the relations $S_{ij}^\dagger = S_{ji}$ and $[S_{ij}, S_{kl}] = \delta_{jk}S_{il} - \delta_{il}S_{kj}$. The parameters g_{01} and g_{12} are the coupling constants both taken as positive real number, and Ω is the pump parameter. We note that the interactions of atoms with mode a and mode b as well as the pump action are all collective. In addition, the photons of lasing mode and passive mode are coupled to their respective reservoirs, leading to the damping of these two modes. In Ref. [3], the problem is studied by Heisenberg–Langevin equations, while we shall study it by master equation. As in Ref. [3], we assume the damping constant κ_b of the passive mode b is the dominant relaxation constant of the system and is large enough so that we may adiabatically eliminate the variable b (and $b^\dagger$). By this way, we arrive at the following master equation [4]:

$$\frac{d}{dt}\rho(t) = L\rho(t) + \Lambda_a\rho(t) + \Lambda_{01}\rho(t), \tag{2}$$

where L, Λ_a and Λ_{01} are superoperators defined by

$$L\rho(t) = \big[\Omega(S_{20} - S_{02}) + g_{12}\big(aS_{21} - a^\dagger S_{12}\big), \rho(t)\big], \tag{3}$$

$$\Lambda_a\rho(t) = \kappa_a(1 + \bar{n}_T)\big(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\big) + \kappa_a\bar{n}_T\big(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger\big), \tag{4}$$

$$\Lambda_{01}\rho(t) = \gamma(2S_{01}\rho S_{10} - S_{10}S_{01}\rho - \rho S_{10}S_{01}),$$
$$\gamma = \frac{g_{01}^2}{\kappa_b}, \qquad \bar{n}_T = \frac{1}{e^{\hbar\omega_a/kT} - 1}. \tag{5}$$

κ_a is the damping constant of mode a and γ is the rate constant related to the collective atom relaxation $1 \to 0$. One can easily convince himself that the equations for $\langle S_{ij}\rangle$ and $\langle a\rangle$ derived by Eqs. (2)–(5) are identical to those derived in Ref. [3].

Now we will treat the master equation by Monte Carlo wave function approach [5]. The non-Hermitian Hamiltonian and collapse operator may be obtained from Eqs. (2)–(5). Assuming $\bar{n}_T \approx 0$, we have

$$H_{\text{nh}} = i\hbar\Omega(S_{20} - S_{02}) + i\hbar g_{12}\big(aS_{21} - a^\dagger S_{12}\big) - i\hbar\kappa_a a^\dagger a - i\hbar\gamma S_{10}S_{01}, \tag{6}$$

and the collapse operators are taken as

$$C_1 = \sqrt{2\kappa_a}\, a, \qquad C_2 = \sqrt{2\gamma}\, S_{01}. \tag{7}$$

C_1 corresponds to annihilation of one mode a photon, and C_2 corresponds to atom transition from level 1 to level 0. Since our numerical simulation takes place over discrete time with a time step Δt, the wave function of the system $|\Psi(t)\rangle$ is represented by $|\Psi(t_n)\rangle$, where $t_n = n\Delta t$. Given the wave function $|\Psi(t_n)\rangle$, the wave function $|\Psi(t_{n+1})\rangle$ may be determined by the following algorithm:

(i) Evaluate the two collapse probabilities during the interval (t_n, t_{n+1}),

$$p_1(t_n) = \big\langle\Psi(t_n)\big|C_1^\dagger C_1\big|\Psi(t_n)\big\rangle\Delta t = 2\kappa_a\big\langle\Psi(t_n)\big|a^\dagger a\big|\Psi(t_n)\big\rangle\Delta t, \tag{8}$$
$$p_2(t_n) = \big\langle\Psi(t_n)\big|C_2^\dagger C_2\big|\Psi(t_n)\big\rangle\Delta t = 2\gamma\big\langle\Psi(t_n)\big|S_{10}S_{01}\big|\Psi(t_n)\big\rangle\Delta t, \tag{9}$$

in which Δt should be small enough to make $p_1(t_n)$and $p_2(t_n)$ much smaller than 1.

(ii) Generate two random number r_1 and r_2 which have uniform probability distribution over the interval $[0, 1]$.

(iii) Compare $p_1(t_n), p_2(t_n)$ with r_1, r_2 and derive $|\Psi(t_{n+1})\rangle$ according to the rule

$$\big|\Psi(t_{n+1})\big\rangle = \frac{C_1|\Psi(t_n)\rangle}{\sqrt{\langle\Psi(t_n)|C_1^\dagger C_1|\Psi(t_n)\rangle}}, \quad p_1(t_n) > r_1 \quad \text{and} \quad p_2(t_n) \leqslant r_2, \tag{10}$$

$$\big|\Psi(t_{n+1})\big\rangle = \frac{C_2|\Psi(t_n)\rangle}{\sqrt{\langle\Psi(t_n)|C_2^\dagger C_2|\Psi(t_n)\rangle}}, \quad p_1(t_n) \leqslant r_1 \quad \text{and} \quad p_2(t_n) > r_2, \tag{11}$$

$$\big|\Psi(t_{n+1})\big\rangle = \frac{C_2C_1|\Psi(t_n)\rangle}{\sqrt{\langle\Psi(t_n)|C_1^\dagger C_2^\dagger C_2C_1|\Psi(t_n)\rangle}}, \quad p_1(t_n) > r_1 \quad \text{and} \quad p_2(t_n) > r_2, \tag{12}$$

$$\big|\Psi(t_{n+1})\big\rangle = \frac{\mathrm{e}^{-(i/\hbar)H_{nh}\Delta t}|\Psi(t_n)\rangle}{\sqrt{\langle\Psi(t_n)|\mathrm{e}^{(i/\hbar)(H_{nh}^\dagger - H_{nh})\Delta t}|\Psi(t_n)\rangle}}, \quad p_1(t_n) \leqslant r_1 \quad \text{and} \quad p_2(t_n) \leqslant r_2. \tag{13}$$

By carrying out the steps above over and over, we will get a quantum stochastic trajectory of the Monte Carlo wave function $|\Psi(t)\rangle$ of the superradiant laser system. And the expectation value of a given operator O at each t_n in respect of this trajectory can be obtained by

$$\big\langle O(t_n)\big\rangle = \mathrm{tr}\big[O\rho(t_n)\big] = \big\langle\Psi(t_n)\big|O\big|\Psi(t_n)\big\rangle. \tag{14}$$

Finally, an ensemble average over a sufficiently large number of trajectory is carried out.

Let $|n_2, n_1, n_0\rangle$ denote the totally symmetric atomic state where n_k $(k = 0, 1, 2)$ is the atomic population on the energy level k, and let $|m\rangle$ denote the state of photon of lasing mode a with photon number m. The state of the system $|\Psi(t)\rangle$ may be expressed as a superposition of all quantum states $|n_2, n_1, n_0\rangle|m\rangle$ in the case that the initial atomic state is totally symmetric as we assume it to be

$$\big|\Psi(t)\big\rangle = \sum_{n_2n_1n_0}\sum_m A_{n_2n_1n_0m}(t)|n_2, n_1, n_0\rangle|m\rangle,$$
$$n_2 + n_1 + n_0 = N, \tag{15}$$

N is the total atom number. The initial state is taken as $|N, 0, 0\rangle|0\rangle$. The operators S_{ij} act on these states as follows [6]:

$$S_{12}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_2(n_1+1)}|n_2-1, n_1+1, n_0\rangle|m\rangle, \tag{16}$$
$$S_{21}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_1(n_2+1)}|n_2+1, n_1-1, n_0\rangle|m\rangle, \tag{17}$$
$$S_{01}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_1(n_0+1)}|n_2, n_1-1, n_0+1\rangle|m\rangle, \tag{18}$$

$$S_{10}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_0(n_1+1)}|n_2, n_1+1, n_0-1\rangle|m\rangle, \tag{19}$$

$$S_{02}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_2(n_0+1)}|n_2-1, n_1, n_0+1\rangle|m\rangle, \tag{20}$$

$$S_{20}|n_2, n_1, n_0\rangle|m\rangle = \sqrt{n_0(n_2+1)}|n_2+1, n_1, n_0-1\rangle|m\rangle. \tag{21}$$

Apart from N, these are four parameters of frequency dimension in the master equation, namely Ω, g_{12}, κ_a and γ, from which three dimensionless parameters can be defined. To compare our results with those of Haake et al., we introduce the same three dimensionless parameters as in Ref. [3]:

$$c = \frac{g_{12}^2\kappa_b}{g_{01}^2\kappa_a} = \frac{g_{12}^2}{\gamma\kappa_a}, \qquad p = \frac{\Omega}{N\gamma\sqrt{c}}, \qquad \xi = \frac{\gamma N}{\kappa_a}. \tag{22}$$

They are called as dimensionless coupling strength, effective pump strength and ratio of decay rates, respectively, ξ is also regarded as the quality factor of the cavity [3].

3. Stationary results for finite atom number N

First we carried out the numerical simulation of an ensemble average of 400 stochastic trajectories for $N = 10, 15, 30$. For fixed value of N, the stationary atomic populations $\langle S_{jj}\rangle$ are determined by the three dimensionless parameters defined by Eqs. (22), hence the curves $\langle S_{jj}\rangle/N$ versus pump parameter p will in general depend on c and ξ, taking $c = 2$ and $\xi = 0.15$. The calculated results are shown in Figs. 1–3, where the dotted lines correspond to the semiclassical result in Ref. [3]. The solid lines are the atomic populations for $N = 10, 15, 30$, respectively.

We see all atoms stay on level 0 at $p = 0$ as surmised above, there is no jump of $\langle S_{22}\rangle$ to $N/(1+c)$ at $p = 0$ as claimed by the semiclassical approach in Ref. [3]. When $p = 1$, both $\langle S_{00}\rangle/N$ and $\langle S_{22}\rangle/N$ are not down exactly to zero and hence no discontinuity of derivatives appears at $p = 1$. In addition, for these values of parameters c and ξ, the bigger N grows, the closer the curve is to the dotted line, namely to the semiclassical result. It is remarkable that the trend towards the infinite N limit can be recognized with atomic numbers as low as 30.

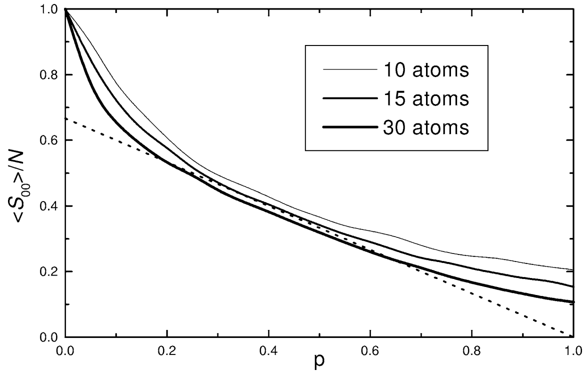

Fig. 1. Ensemble average of the stationary atomic population on level 0 versus effective pump strength at $c = 2$, $\xi = 0.15$ with the step length $\gamma\Delta t = 0.0001$. The dotted line corresponds to the semiclassical result in Ref. [3].

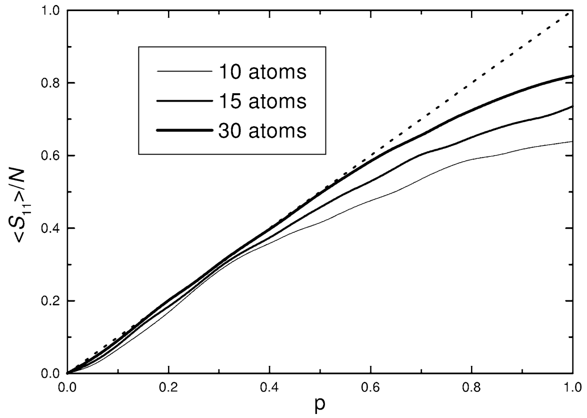

Fig. 2. Ensemble average of the stationary atomic population on level 1 versus effective pump strength at $c = 2$, $\xi = 0.15$ with the step length $\gamma\Delta t = 0.0001$. The dotted line corresponds to the semiclassical result in Ref. [3].

However, the above situation changes when the parameters go over to other range. Since ξ, the so-called quality factor of the cavity [3], is an important parameter of the laser and, somewhat surprisingly, the semiclassical results of $\langle S_{jj}\rangle^{\mathrm{sc}}/N$ given in Ref. [3] is independent of ξ, in this Letter we will mainly study the effect due to variation of ξ. Now when ξ becomes larger, such as taking 10 instead of 0.15, the calculated results presented in Figs. 4–6 indeed show different behavior. We see there is no indication of the trend towards the semiclassical results of Ref. [3] as N changes from 10 to 30, especially for p close

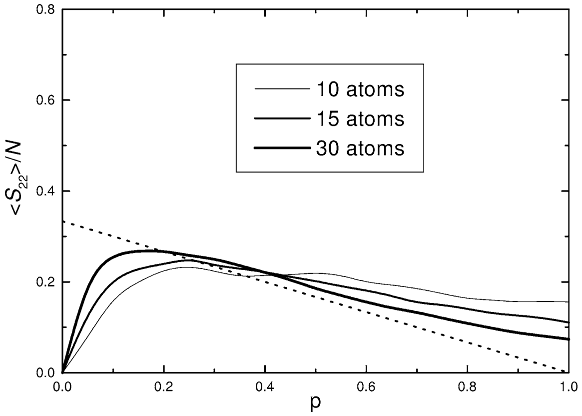

Fig. 3. Ensemble average of the stationary atomic population on level 2 versus effective pump strength at $c = 2$, $\xi = 0.15$ with the step length $\gamma\Delta t = 0.0001$. The dotted line corresponds to the semiclassical result in Ref. [3].

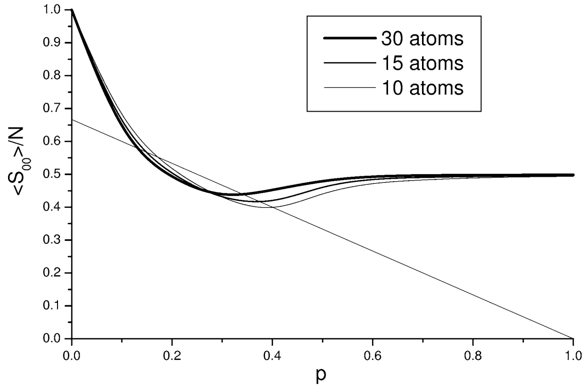

Fig. 4. Ensemble average of the stationary atomic population on level 0 versus effective pump strength at $c = 2$, $\xi = 10$ with the step length $\gamma\Delta t = 0.0001$.

to one. To further highlight this point, we take $p = 1$, $c = 2$ and calculate the atomic population on the middle level 1 as function of $1/\xi$. The result is given by Fig. 7. We see that instead of all the atoms staying on the middle level in the semiclassical results, the atomic population on the middle level in our microsuperradiant case decreases continuously and seems to approach zero for $\xi \to \infty$. Our above results seem physically plausible, since in the case of good cavity, the better lasing corresponds to larger population inversion.

It merits to note that the semiclassical stationary value$\langle S_{jj}\rangle^{\mathrm{sc}}$ of Ref. [3] are derived under two pre-

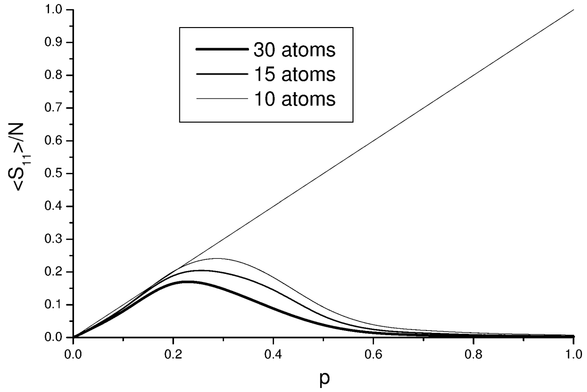

Fig. 5. Ensemble average of the stationary atomic population on level 1 versus effective pump strength at $c = 2$, $\xi = 10$ with the step length $\gamma\Delta t = 0.0001$.

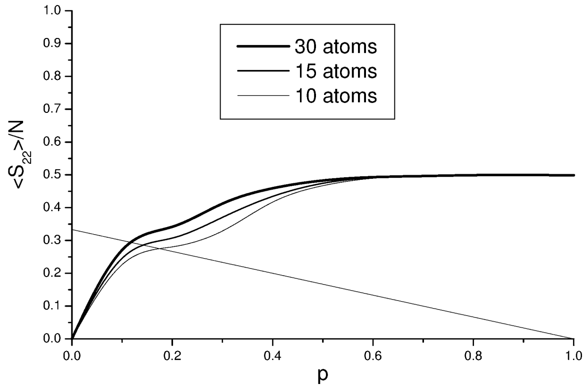

Fig. 6. Ensemble average of the stationary atomic population on level 2 versus effective pump strength at $c = 2$, $\xi = 10$ with the step length $\gamma\Delta t = 0.0001$.

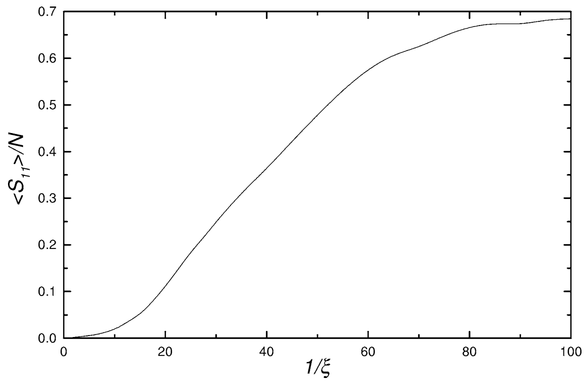

Fig. 7. Ensemble average of the stationary atomic population on level 1 versus $1/\xi$ at $c = 2$, $p = 1$ with the step length $\gamma\Delta t = 0.002$.

suppositions: one is operators z_i satisfying the Bose commutation rules $[z_i, z_j^\dagger] = \delta_{ij}$, which is not an exact relation since, for example, $[z_0, z_0^\dagger]\,|0,0,N\rangle$ equals $-N\,|0,0,N\rangle$ not $|0,0,N\rangle$, the second presupposition claims the average of the products of operators equal to the products of the average of operators, which also needs N very large. When the value of N is about a few dozens, these prerequisites seem more questionable. Furthermore, the derivation of the $\langle S_{jj}\rangle^{\mathrm{sc}}$ in Ref. [3] involves an inner conflict, since at $p = 0$ and $p = 1$ some of their values equal zero and hence violate the initial assumption that the fluctuation can be neglected as compared with the average value. This may provide an explanation why the curves of Figs. 4–6 do not show the tendency to the $\langle S_{jj}\rangle/N$ as N increases.

We have also calculated the stationary photon number distributions of the system starting from following different initial conditions:

$$(1)\quad \big|\Psi(0)\big\rangle = |0,0,50\rangle|0\rangle, \tag{23}$$

$$(2)\quad \big|\Psi(0)\big\rangle = |\beta = 0.2, n_0 = 45\rangle|0\rangle, \tag{24}$$

$$(3)\quad \big|\Psi(0)\big\rangle = |0,0,50\rangle|\alpha = 0.2\rangle\,. \tag{25}$$

Eq. (23) means all the 50 atoms initially stay on level 0 and the light field is vacuum. In Eq. (24), $|\beta, n_0\rangle$ denotes that 45 out of 50 atoms occupy level 0 and the rest 5 atoms are so distributed on level 2 and level 1 as to form a two-level atomic coherent state [7]. In Eq. (25), $|\alpha\rangle$ denotes a photon coherent state.

We find that, as long as the system parameters p, c and ξ remain the same, the stationary photon distributions which we obtain will be precisely identical with no reference to the initial state. These results indicate that in our calculation the steady states are indeed reached. For the case $c = 2$, $p = 0.2$ and $\xi = 5$, the steady photon distribution is characterized by $\langle \Delta n^2\rangle = 8.9$ and $\langle n\rangle = 11.2$ which means the cavity photon has a sub-Poissonian distribution as expected.

4. The time evolution of the superradiant laser system

The quantum stochastic trajectory approach still allows us to see the time evolution of the whole process. The time development of atomic populations

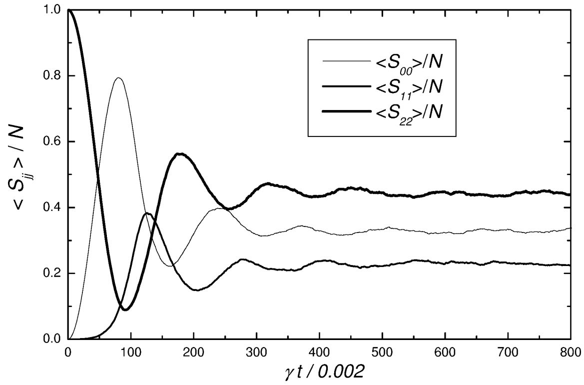

Fig. 8. The time evolution of the atomic populations over an average of 500 trajectories for the initial state $|\Psi(0)\rangle = |0,0,20\rangle|0\rangle$. The simulation parameters are $c = 1$, $p = 0.5$, $\xi = 2$ and step length $\gamma\Delta t = 0.0005$. This figure is also an example to show that the system remains stable for system parameters far out of the stable region in Ref. [3].

$\langle S_{jj}\rangle/N$ $(j = 0,1,2)$ for the initial state $|\Psi(0)\rangle = |0,0,20\rangle|0\rangle$ are shown in Fig. 8. The system parameters are $c = 1$, $p = 0.5$, $\xi = 2$. We can see that the atomic populations display some feature like damped Rabi oscillation and undergo roughly three vibration periods to arrive at their stationary values. The total relaxation time is about $1/\gamma$.

In order to get more physical understanding of the superradiant laser system, we calculate the time evolution of the coherent part of the light field $\langle a(t)\rangle$ corresponding to initial states (23)–(25). When the initial state is given by Eq. (23), the state expectation value $\langle a(t)\rangle$ of the superradiant laser remains zero at any time for any given single stochastic trajectory. This is understandable, since none of the initial states, including atomic state and photonic state, is coherent. The result will be different for initial states (24), (25), in which either the relevant atomic initial state or the photon initial state is coherent. The time evolutions of the coherent part $\langle a(t)\rangle$ in both cases first experience a development then go down gradually to zero due to the phase diffusion [8].

Another question of importance is the stability of the stationary solution. It is shown in Ref. [3] that in certain domain of the three parameters p, c and ξ the stationary solutions become unstable and therefore will not represent true stationary states. However, we have not found any instability for the microsuperradiant laser in a region much larger than those stationary region indicated in Ref. [3]. Fig. 8 also serves as an example. The system parameters are far out of the stable region in the Fig. 2 of Ref. [3], while the atomic populations still evolve to stationary values.

5. Brief discussion and summary

In this Letter, we get the ensemble average value by average over about four hundred trajectories. The question may arise whether such number of trajectories is many enough to attain the true ensemble average value. To check this we have made a comparison with a recent paper by Wiele et al. [9] in which both photon modes are eliminated adiabatically from the master equation and the resultant simplified master equation is treated by a totally different methods from ours to get the stationary values. For comparison, we also treat this simplified master equation by quantum stochastic trajectory approach with parameter $c = 2$ and calculate the average value over four hundred trajectories. Our results are the same as those in Ref. [9] and also similar to our Figs. 1–3, since the two-mode adiabatic approximation corresponds to the limiting case $\xi \to 0$ and in Figs. 1–3 the value of ξ is quite small. The agreement show that taking about four hundred trajectories for ensemble average is likely enough.

Brief summary: we have shown that the discontinuities in the curve of stationary atomic populations versus pump parameter will not appear in microsuperradiant laser with finite atom number N. The authors of Ref. [9] got the same conclusion by adiabatically eliminating both lasing mode and passive mode. Here we not only keep the photons of lasing mode as in Ref. [3], but also carry out further study on the temporal evolutions of the atomic populations. The cavity photon distribution for quite good cavity ($\xi = 5$) is shown sub-Poissonian. Besides, the unstable region described in Ref. [3] seems not to exist, and in the stationary state for $p \sim 1$, not nearly all the atoms populate in the middle level. For large value of ξ, the atomic population in the middle level even becomes very small, hence the corresponding curves do not show the tendency to the semiclassical results of Ref. [3] as N increases from 10 to 30. This may hint that the limit of $\langle S_{jj} \rangle / N$ as $N \to \infty$ may also depend on ξ.

Acknowledgements

This work is supported by National Natural Science Foundation of China (project No. 19774004), and is also partially supported by the International Program of National Science Foundation of USA (ECS-9800068).

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