Creation of new lasing modes with spatially nonuniform gain

Jonathan Andreasen^{1,2,*} and Hui Cao^{1,3}

¹Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA ²Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA ³Department of Physics, Yale University, New Haven, Connecticut 06520, USA *Corresponding author: jonathan.andreasen@yale.edu

Received September 24, 2009; revised October 14, 2009; accepted October 15, 2009; posted October 22, 2009 (Doc. ID 117659); published November 13, 2009

We report on the creation of new lasing modes with spatially nonuniform profiles of optical gain in a onedimensional random structure. It is demonstrated numerically that even without gain saturation and mode competition, the spatial nonuniformity of gain can cause dramatic and complicated changes. New lasing modes appear with frequencies between those of the lasing modes with uniform gain. We examine new modes in detail and find that they exhibit high output directionality. Our results show that random lasing properties may be modified significantly without changing the underlying structure. © 2009 Optical Society of America

OCIS codes: 140.3460, 140.3430, 220.4830.

The available lasing modes of a conventional laser are typically fixed once the cavity is made. Finer control over lasing properties may be obtained, for example, by carefully placing the gain medium in a cavity to reduce the lasing threshold [1] or by using specific pumping profiles to select lasing modes with desirable properties [2-5]. However, once the laser cavity is made, it is difficult to obtain new lasing modes that have no correspondence to the resonant modes of the cold cavity if nonlinearity is negligible. A random laser is made of disordered media, and lasing modes are determined by the random distribution of refractive index. Owing to the randomness, it is difficult to intentionally produce lasing modes with desirable properties. To have more control, the structures themselves may be adjusted by selecting the scatterer size [6-8] and separation [9,10], changing the scattering structure with temperature [11,12] or electric field [13], or creating defects [14]. For random lasers in the localization regime, spatially nonoverlapping modes may be selected for lasing by local pumping [15,16]. In the case of diffusive random lasers far above the lasing threshold, nonlinear interaction between the light field and the gain medium alters the lasing modes [17]. Without gain nonlinearity, local pumping and absorption in the unpumped region can also change the lasing modes significantly [18], because the system size is effectively reduced. Recent experiments [19,20] and numerical studies [21] show that even without absorption in the unpumped region, the spatial characteristics of lasing modes may vary with local pumping. Although the spatial distributions are distorted, lasing modes have a one-to-one correspondence to resonant modes of the passive system. However, spatial inhomogeneity in the refractive index can introduce a linear coupling of resonant modes mediated by the polarization of the gain medium [22].

In this Letter, we demonstrate that new lasing modes can be created by nonuniform profiles of gain in 1D random systems without absorption and nonlinearity. These new lasing modes do not correspond to modes of the passive system nor to any lasing modes in the presence of uniform gain. New lasing modes can lase independently of other lasing modes when gain saturation is taken into account. They appear at various frequencies for different gain profiles and can have highly directional output. These findings may offer an easy and fast way of dramatically changing random laser properties without modifying the underlying structure.

We consider a 1D random system composed of 161 layers. Dielectric material with index of refraction $n_1=1.05$ separated by air gaps $(n_2=1)$ results in a spatially modulated index of refraction n(x). Outside the random media $n_0=1$. The system is randomized by specifying thicknesses for each layer as $d_{1,2} = \langle d_{1,2} \rangle (1 + \eta \zeta)$, where $\langle d_1 \rangle = 100$ nm and $\langle d_2 \rangle = 200$ nm are the average thicknesses of the layers, $\eta=0.9$ represents the degree of randomness, and ζ is a random number in (-1,1). The length of the random structure L is normalized to $\langle L \rangle = 24.1 \ \mu$ m. The above parameters give a localization length of $\xi \approx 240 \ \mu$ m at a vacuum wavelength $\lambda = 600$ nm.

The transfer matrix (TM) method developed in [21] is used to simulate lasing modes at threshold with linear gain. A real wavenumber $k=2\pi/\lambda$ describes the lasing frequency. Propagation of the electric field through the structure is calculated via the 2×2 matrix M. Boundary conditions with only emission out of the system require $M_{22}=0$. Linear gain is simulated by appending an imaginary part to the index of refraction $\tilde{n}(x)=n(x)+in_if_E(x)$, where $n_i<0$. We neglect the change to n(x) in the presence of gain. Spatial nonuniformity of gain is implemented by multiplying n_i by a step function $f_E(x)=H(-x+l_G)$, where x=0 is the left edge of the structure and $x=l_G$ specifies the right edge of the gain region.

Lasing frequencies and thresholds are located by determining which values of k and n_i , respectively, satisfy $M_{22}=0$. Re $[M_{22}]=0$ (Im $[M_{22}]=0$) forms real

(imaginary) "zero lines" in the (k, n_i) plane. The crossing of a real and imaginary zero line pinpoints a solution. Zero lines are visualized in Fig. 1 by plotting $\log_{10}|\text{Re}\,M_{22}|$ and $\log_{10}|\text{Im}\,M_{22}|$ to enhance the regions near $M_{22}=0$. Changes of zero lines are monitored as the right edge of the gain region moves gradually from $l_G = L$ (uniform gain). In the wavelength range 500 nm $< \lambda < 750$ nm, new lasing modes appear between existing lasing modes. Figure 1(a)shows the only two lasing modes (marked 1 and 2) found within a smaller frequency range for l_G =14.961 μ m. Mode 1 (λ_1 =593 nm) has a lower threshold than mode 2 (λ_2 =598 nm). Both modes correspond to resonant modes of the passive system. The spatial intensity distributions are distorted by nonuniform gain as reported previously [21]. However, for $l_G = 14.559 \ \mu \text{m}$ [Fig. 1(c)] the zero lines are joined. An entirely new lasing mode (λ_{nm} =596 nm), encircled in white, appears between modes 1 and 2. The spatial intensity distribution of the new lasing mode differs from those of modes 1 and 2. As l_G decreases further, the zero lines forming mode 2 and the new mode pull apart. The solutions approach each other in the (k, n_i) plane [Fig. 1(f)] until becoming identical. The zero lines then separate; mode 2 and the new mode disappear [Fig. 1(h)]. The lines cross again for $l_G = 14.295 \ \mu \text{m}$; the solutions reappear and move away from each other [Fig. 1(j)].



Fig. 1. (Color online) Left, real (green) and imaginary (red) zero lines of M_{22} in the (k, n_i) plane. Right, phase θ of M_{22} in the (k, n_i) plane. The ranges of axes are 10.48 μ m⁻¹ < k < 10.64 μ m⁻¹ and -0.0326 < n_i < -0.0130. The gain region length l_G is, from top to bottom; (a) and (b) 14.961 μ m, (c) and (d) 14.559 μ m, (f) and (g) 14.523 μ m, (h) and (i) 14.472 μ m, (j) and (k) 14.295 μ m.

Verification of lasing mode solutions is provided by the phase of M_{22} , $\theta = \arctan 2(\text{Im}[M_{22}], \text{Re}[M_{22}])$. Locations of vanishing M_{22} give rise to phase singularities. The phase change around a closed path surrounding a singularity is referred to as topological charge. Two phase singularities are seen in Fig. 1(b) at the same locations as the zero line crossings in Fig. 1(a), verifying the solutions. The singularity at the location of the new lasing mode [Fig. 1(d)] also confirms that it is a genuine lasing mode in the presence of linear gain. The singularity associated with the new mode has opposite topological charge. As l_G is reduced, two oppositely charged singularities move closer [Fig. 1(g)], eventually annihilate each other at $l_G = 14.472 \ \mu \text{m}$ [Fig. 1(i)], and reappear at smaller l_G [Fig. 1(k)].

For a more thorough study of the new lasing modes and confirmation of their existence in the presence of gain saturation, we switch to a more realistic gain model. The Bloch equations of two-level atoms [23] are solved together with Maxwell's equations via the finite-difference time-domain method [24]. In the resulting Maxwell-Bloch (MB) equations, nonuniform gain is simulated with two-level atoms only in the region $0 \le x \le l_G$. The rate of atoms being incoherently pumped from the ground state to the excited state is proportional to the ground-state population. The proportional coefficient P_r is called the pumping rate. The gain spectrum is centered at the atomic transition wavelength λ_a with a spectral width $\Delta \lambda_a$.

To individually investigate the three lasing modes for $l_G=14.295 \ \mu$ m, we set $\Delta \lambda_a = (\lambda_2 - \lambda_{nm})/2$. Figure 2 shows the steady-state output intensity with $\lambda_a = \lambda_1$, λ_2 , λ_{nm} as P_r is varied. The lasing threshold for mode 1 is reached first at $P_r=1.9$, then mode 2 at $P_r=2.1$ and the new mode at $P_r=3.0$, agreeing qualitatively with the TM calculation. When $\lambda_a = \lambda_{nm}$ the first lasing mode is the new mode instead of mode 1 or 2. Figure 3(a) shows the output emission spectrum just above the threshold at $P_r=3.0$. It consists of a single lasing peak with the same wavelength as the new mode calculated with the TM method. The spatial intensity distribution from the MB calculation $|\psi_{\text{MB}}(x)|^2$ is compared with that from the TM calculation $|\psi_{\text{TM}}(x)|^2$ in Fig. 3(b). They are normalized as $\int_0^L |\psi_{\text{MB}}(x)|^2 dx = \int_0^L |\psi_{\text{TM}}(x)|^2 dx$. The two distributions



Fig. 2. (Color online) Steady-state output intensity versus pumping rate P_r from MB simulations with different gain spectra. The atomic transition wavelength $\lambda_a = \lambda_1$ (red crosses), $\lambda_a = \lambda_2$ (blue open diamonds), and $\lambda_a = \lambda_{nm}$ (black circles).



Fig. 3. (Color online) Steady-state emission spectra from two MB simulations with $P_r=3.0$, $\lambda_a=\lambda_{nm}$ (solid curve) and $P_r=2.1$, $\lambda_a=\lambda_2$ (dashed curve). For both, $\Delta\lambda_a=0.71$ nm, l_G = 14.295 μ m. The MB intensity distribution $|\psi_{MB}(x)|^2$ (black dashed curve) is compared with the TM intensity distribution $|\psi_{TM}(x)|^2$ (red solid curve) for the new lasing mode (b) and mode 2 (c). Agreement is so good that the MB curve mostly covers the TM curve. The inset in (b) expands the range 0 μ m < $x < 10 \mu$ m.

are almost identical; the difference between them is 8%. When λ_a shifts from λ_{nm} to λ_1 or λ_2 , the first lasing mode switches to mode 1 or 2. Figure 3(c) plots the MB and TM distributions of mode 2 for $\lambda_a = \lambda_2$ and $P_r = 2.1$, which are also nearly identical. Comparing Figs. 3(b) and 3(c) the intensity distribution of the new lasing mode differs significantly from that of mode 2 within the gain region. Outside the gain region the two distributions are similar, because their wavelengths are very close.

With gain on the left side of the structure, we observe that new lasing modes are concentrated on the right side of the gain region. Thus the emission through the right structure boundary is much larger than that through the left boundary. We calculate the ratio of right-to-left output flux $S \equiv |\psi(x=L)|^2/|\psi(x=0)|^2$. For the new lasing mode S=40, indicating the laser output is highly directional. In comparison, S = 1.1 for mode 1 and S=3.3 for mode 2.

Because of the excellent agreement found between the MB and TM calculations, we conclude that new lasing modes do appear in random structures with spatially nonuniform gain. These new lasing modes are sensitive to the spatial gain profile and disappear if the profile is altered slightly. We have verified their existence in numerous random structures as well as dielectric slabs of uniform refractive index. New lasing modes offer more control over random laser properties, since their frequency and output directionality can be quite different from that of existing lasing modes. Moreover, new lasing modes can be easily manipulated by varying the spatial profile of the pump beam without modifying the random structure. We anticipate that such new lasing modes also exist in higher dimensions, which may yield further advantages for laser control.

The authors thank C. Vanneste, P. Sebbah, L. Ge, A. D. Stone, J. Wiersig, and D. Savin for stimulating discussions and acknowledge support from the Yale Faculty of Arts and Sciences HPC facility and staff. This work was supported partly by the National Science Foundation (NSF) under grants DMR-0814025 and DMR-0808937.

References

- 1. R. J. Horowicz, H. Heitmann, Y. Kadota, and Y. Yamamoto, Appl. Phys. Lett. **61**, 393 (1992).
- C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, J. Faist, D. L. Sivco, and A. Y. Cho, Science 280, 1556 (1998).
- T. Fukushima, T. Harayama, P. Davis, P. O. Vaccaro, T. Nishimura, and T. Aida, Opt. Lett. 27, 1430 (2002).
- G. D. Chern, H. E. Türeci, A. D. Stone, R. K. Chang, M. Kneissl, and N. M. Johnson, Appl. Phys. Lett. 83, 1710 (2003).
- 5. M. Hentschel and T. Y. Kwon, Opt. Lett. 34, 163 (2009).
- X. H. Wu, A. Yamilov, H. Noh, H. Cao, E. W. Seelig, and R. P. H. Chang, J. Opt. Soc. Am. B 21, 159 (2004).
- C. Vanneste and P. Sebbah, Phys. Rev. E 71, 026612 (2005).
- S. Gottardo, R. Sapienza, P. D. García, A. Blanco, D. S. Wiersma, and C. López, Nat. Photonics 2, 429 (2008).
- J. Ripoll, C. M. Soukoulis, and E. N. Economou, J. Opt. Soc. Am. B 21, 141 (2004).
- T. Savels, A. P. Mosk, and A. Lagendijk, Phys. Rev. Lett. 98, 103601 (2007).
- 11. D. S. Wiersma and S. Cavalieri, Nature **414**, 708 (2001).
- K. Lee and N. M. Lawandy, Opt. Commun. 203, 169 (2002).
- S. Gottardo, S. Cavalieri, O. Yaroshchuk, and D. S. Wiersma, Phys. Rev. Lett. 93, 263901 (2004).
- 14. H. Fujiwara, Y. Hamabata, and K. Sasaki, Opt. Express 17, 3970 (2009).
- C. Vanneste and P. Sebbah, Phys. Rev. Lett. 87, 183903 (2001).
- P. Sebbah and C. Vanneste, Phys. Rev. B 66, 144202 (2002).
- H. E. Türeci, L. Ge, S. Rotter, and A. D. Stone, Science 320, 643 (2008).
- A. Yamilov, X. Wu, H. Cao, and A. L. Burin, Opt. Lett. 30, 2430 (2005).
- R. C. Polson and Z. V. Vardeny, Phys. Rev. B 71, 045205 (2005).
- X. Wu, A. Yamilov, A. A. Chabanov, A. A. Asatryan, L. C. Botten, and H. Cao, Phys. Rev. A 74, 053812 (2006).
- X. Wu, J. Andreasen, H. Cao, and A. Yamilov, J. Opt. Soc. Am. B 24, A26 (2007).
- 22. L. Deych, Phys. Rev. Lett. 95, 043902 (2005).
- R. W. Ziolkowski, J. M. Arnold, and D. M. Gogny, Phys. Rev. A 52, 3082 (1995).
- 24. A. Taflove and S. Hagness, *Computational Electrodynamics* (Artech House, 2005).