Reinfiltration in Fractured Porous Media: Part 2 - Two Dimensional Model

A basic analytical framework to study reinfiltration in a 2-D matrix block has been established

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SUMMARY

The fractional flow equation in a two-dimensional space with mixed boundary conditions is solved, and the expression for the reinfiltration rate across the faces of a matrix block is derived. The results show that there is a considerable variation in rate of reinfiltration across a vertical face of a matrix block. For a matrix block with an inclined face, the lower portion could drain liquid, but the upper portion could be under reinfiltration. The results also show that reinfiltration could be effective in a small area at the top portion of a matrix block.

INTRODUCTION

Reinfiltration of the produced liquid from the upper region of a fractured petroleum reservoir to lower matrix blocks could be either from top or side faces. In part 1, the reinfiltration through the top horizontal face of a matrix block was modeled. The subject of this paper is the modeling of reinfiltration in a two dimensional (2-D) space. In particular, we are interested in studying the reinfiltration rate for both the vertical and the subvertical faces of a matrix block. A review of the literature on this subject shows very few attempts.

Saad, Tehrani, and Wit observed that when oil was supplied to the side face of a (oil saturated) matrix block, it was sucked into the block. When the block had a much larger lateral than vertical dimension, the rate of oil reinfiltration was observed to be less than the rate of drainage of such an extended block.

Cositas discussed the subject of reinfiltration by using fine grid simulation. The fractures between a stack of six matrix blocks were assumed to be horizontal for one case, and were given an angle of 11° with the horizontal plane in another case. With the fractured fractures between the matrix blocks, Cositas' calculations show almost no reinfiltration. Ref. 3 may imply that for slightly sloped fractures, the recovery of a stack of matrix blocks is similar to dual-porosity simulation results.

Two recent papers have attempted to incorporate the reinfiltration mechanism in the dual-porosity and the dual-permeability models. For, Beurigter, Maas, and de Vries suggested additional connections between the matrix and the fracture nodes. The connections allow the oil to drain from an upper block to flow into the fractures and also from the fractures into the lower block. These authors placed the center of the fracture node at the bottom face of the associated block. The matrix-fracture and fracture-fracture liquid phase relative permeability curves should be provided to account for the reinfiltration mechanism.

Fang uses another approach to incorporate reinfiltration in a dual-porosity model. His mass balance equation includes term(s) to represent the contribution of reinfiltration. Through a parameter "θ", fractional reinfiltration parameter, the reinfiltration rate is supposed to be controlled. This parameter is apparently assumed to be independent of saturation.

da Silva and Meyers have also used fine grid simulation to study the influence of oil reinfiltration across a stack of matrix blocks which were initially saturated with the oil. They found out that for the sloped fractures, the calculated rate of block desaturation will depend on the model discretization. Their numerical results for a certain rock and fluid data show that the oil reinfiltration effect decreases as the fracture dip angle increases. For a stack of small blocks and the assigned rock and fluid data, the influence of reinfiltration becomes negligible for a fracture dip angle of around 45°.

The purpose of this paper is to provide a basic understanding of the reinfiltration in a 2-D fractured porous media. Questions which are important to address include: 1) the variation of the reinfiltration rate across a vertical face of a matrix block, and 2) the variation of the reinfiltration rate across a vertical face of a matrix block as a function of time. The topic of drainage and reinfiltration for a tilted matrix block is also of special importance because part of the block face could be reimbibing the oil and part of the same face could be draining the oil. In this paper, we derive the analytical equations for the reinfiltration and drainage rates across various faces of a matrix block. These equations will then be used to study the variation of reinfiltration and drainage rates both as a function of time and position.

PROBLEM FORMULATION

Consider the following 2-D matrix block of dimensions of $L_x$ and $L_y$ (see Fig. 1), and the dip angle of $θ$. The fractional flow equation for the gas phase in a two-phase gas-oil system assuming incompressible fluids can be written as:

$$
\frac{\partial S_g}{\partial t} + \frac{\partial \psi(S_g)}{\partial y} \left[ \frac{\partial S_g}{\partial y} + u \frac{\partial S_g}{\partial x} \right] - \frac{\partial}{\partial x} \left[ \psi(S_g) \frac{\partial S_g}{\partial x} - \chi(S_g) \sin \theta \right] - \frac{\partial}{\partial y} \left[ \psi(S_g) \frac{\partial S_g}{\partial y} - \psi(S_g) \cos \theta \right] = 0
$$

(1)

where

$$
\psi(S_g) = \frac{1}{1 + \kappa(S_g) \psi_{min}}
$$

(2)
\[ \psi(S_p) = \frac{k \frac{dS_k}{dS_p}}{S_k + \frac{S_k^2}{S_p}} \]  
(3)

\[ X(S_p) = \frac{\mu_c \rho_o - \mu_o \rho_c g}{\mu_c + \mu_o} \]  
(4)

If the gas mobility is assumed to be infinity and Eqs. 15 and 16 of Ref. 1 are used to represent gas-oil capillary pressure and oil relative permeability, then Eq. 1 can be written as:

\[ \frac{\partial S_k}{\partial t} - D \left( \frac{\partial^2 S_k}{\partial x^2} + \frac{\partial^2 S_k}{\partial z^2} \right) - V \left( \sin \theta \frac{\partial S_k}{\partial x} + \cos \theta \frac{\partial S_k}{\partial z} \right) = 0 \]  
(5)

Dimensionless parameters \( D \) and \( V \) are defined in Ref. 1, which are defined the same way as in Ref. 6.

Let’s define dimensionless variables

\[ x_D = \frac{x}{L_x} \]  
(6)

\[ z_D = \frac{z}{L_z} \]  
(7)

\[ \alpha = \frac{L_x}{D_t} \]  
(8)

\[ \eta_D = \frac{V L_z \cos \theta}{D} \]  
(9)

\[ \tau \eta = \frac{V L_z \sin \theta}{2D} \]  
(10)

Note that \( \tau \eta \) is also given by:

\[ \tau \eta = \sigma \gamma_x \tan \theta \]  
(12)

Using the above dimensionless variables, Eq. 5 becomes

\[ \frac{\partial S_k}{\partial t} - \alpha \frac{\partial^2 S_k}{\partial z_D^2} - \frac{1}{\alpha} \frac{\partial S_k}{\partial x_D} - 2 \gamma_x \frac{\partial S_k}{\partial z_D} - 2 \gamma_z \frac{\partial S_k}{\partial z_D} = 0 \]  
(13)

The oil velocity in \( x \) and \( z \) directions is given by:

\[ v_{oe} = -\frac{k \mu_o}{\mu_c} \left( -\frac{\partial P_o}{\partial S_k} \frac{\partial S_k}{\partial x} + \Delta p \sin \theta \right) \]  
(14)

\[ v_{oz} = -\frac{k \mu_o}{\mu_c} \left( -\frac{\partial P_o}{\partial S_k} \frac{\partial S_k}{\partial z} + \Delta p \cos \theta \right) \]  
(15)

Substitution of \( k_o \) and \( P_z \) expressions from Eqs. 15 and 16 of Ref. 1, and the use of dimensionless variables into Eqs. 14 and 15, gives

\[ v_{oe} \mu_o L_z = \frac{\Delta S_k}{\tau \eta} \]  
(16)

\[ v_{oz} \mu_o L_z = \frac{\Delta S_k}{\tau \eta} - 2 \gamma_x \tan \theta (1 - S_p) \]  
(17)

Let the oil phase velocity toward the origin of the coordinate system of Fig. 1 be positive, and assign dimensionless oil velocities in \( x_D \) and \( z_D \) directions \( v_{o_D} \) and \( v_{z_D} \), respectively. Then,

\[ v_{o_D} = -\frac{\partial S_k}{\partial x_D} + 2 \gamma_x \tan \theta (1 - S_p) \]  
(18)

and

\[ v_{z_D} = -\frac{\partial S_k}{\partial z_D} + 2 \gamma_x (1 - S_p) \]  
(19)

Note that if a block is fully saturated, the initial dimensionless velocities would be \( v_{o_D} = 2 \gamma_x \tan \theta \), and \( v_{z_D} = 2 \gamma_x \).

The flow rate per unit width at \( z = L_z \) (i.e., block face at \( z = L_z \), and \( 0 < x < L_x \)) is given by

\[ q_{oz} \mid z = L_z = \int_0^L v_{oz} \mid z = L_z \, dx \]  
(20)

Since \( v_{oz} = -\frac{k \mu_o L_z}{\mu_c} \rho_o \) and \( dx = L_d x_D \), therefore,

\[ q_{oz} \mid z = L_z = -\frac{k \mu_o L_z}{\mu_c} \rho_o \left( \frac{L_x}{L_d} \right) \int_0^1 \theta \mid x_D = 1 \, dx_D \]  
(21)

If the dimensionless oil flow rate across the face at \( x_D = 1 \) is represented by \( q_{oz} \mid x_D = 1 \), then,

\[ q_{oz} \mid x_D = 1 = -\frac{\mu_o}{\kappa \phi \mu_c} \sigma \gamma_x \tan \theta \]  
(22)

Similarly, the oil rate at \( z = 0 \) face (i.e., \( x_D = 0 \) face) per unit width is,

\[ q_{oz} \mid z = 0 = \int_0^{x_D} v_{oz} \mid z = 0 \, dx \]  
(23)

and the dimensionless oil rate across this face is given by:

\[ q_{oz} \mid x_D = 0 = -\frac{\alpha \mu_o}{\kappa \phi \mu_c} \sigma \gamma_x \tan \theta \]  
(24)

**ANALYTICAL SOLUTION**

In order to study the process of refilfiltration across various faces of a matrix block, a solution to Eq. 13 would be needed. Two cases will be considered in this paper. For Case 1 we will assume \( \theta = 0 \). For such a matrix block only vertical and horizontal faces will exist. Case 2 corresponds to a tilted block with \( \theta \) varying from 0 to 90°.

**Case 1: \( \theta = 0 \)**

In this case, we are interested to study the rate of refilfiltration across the vertical face of a matrix block. We will assume that the matrix block has an initial saturation given by:

\[ S_p \mid z = 0 = 1 - \exp(\gamma_x x_D) \]  

\[ + \sum_{k=1}^{\infty} A_k \exp \left( -\frac{(\gamma_x x_D + \omega_k) \Delta t_D}{\alpha} \right) \]  

\[ \times \exp(-\gamma_x x_D) \cos(\omega_k x_D) \]  

\[ x_D < (0,1) \]  

where

\[ A_k = -\frac{\sigma \gamma_x}{(\gamma_x \lambda_0 + (\gamma_x \lambda_1)^2 + \omega_k^2)} \]  

\[ \omega_k + (\gamma_x \lambda_0 \tan \omega_k = 0 \]  

\[ = \frac{\omega_k}{2} + k \pi < \omega_k < -\frac{\pi}{2} + (k + 1) \pi \]  

\[ k = 1, 2, \ldots, \infty \]  

Eq. 25 is the solution to the 1-D problem (Eq. 31 of Ref. 1). Note that \( \Delta t_D \) of Eq. 25 changes the initial saturation distribution in the block and should be viewed as a precautionary parameter.

For the top horizontal face, the oil flow rate is assumed to be zero. One vertical face will be held at an oil saturation of unity. This face will, therefore, allow the oil to reinfiltrate. The other vertical face will have no-oil flow boundary condition. The bottom face will be kept at zero capillary pressure which implies a liquid saturation of unity. Details for these two cases are given in SPE 25516 (supplement to this paper), and are omitted for brevity.

**Case 2: Tilted Block**

In this case, no oil is supplied at \( x_D = 0 \) block face. The block faces at \( x_D = 0 \) and \( x_D = 1 \) are kept at unit oil saturation. If we change \( \theta \) from 0 to 90 degrees the block face at \( x_D = 0 \) may drain oil in lower parts and reinfiltrate in the upper regions. The block face at \( x_D = 0 \) will also be kept at an oil saturation of unity. Initially, we will assume the matrix block is fully saturated with oil. In this case, we seek a solution to Eq. 13 and the above initial and boundary conditions.

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RESULTS

The equations that were derived in the preceding part of this paper will be used to examine the rate of drainage and reinfiltation across various faces of a matrix block. The basic properties of the matrix are assumed to be the same as in Ref. 1. First, the results for Case 1 will be presented.

Case 1. For this case, the side faces of the matrix block are vertical. Also \( L_z = L_u \) (i.e., \( \alpha = 1 \)) and, therefore, Eq. 10 gives \( \tau = 0.35 \).

Figs. 3, 5, and 4 show various results for a matrix block which is initially saturated with oil. Fig. 2 shows both the rate of drainage from the bottom face of the block (i.e., \( \delta_{\text{in}} \)) and the rate of reinfiltiration from the vertical face at \( z_p = 1 \) (i.e., \( \delta_{\text{out}} \)). This figure shows that the initial rate of drainage is 0.7, which is the same for the 1-D model of Ref. 1. The initial rate of reinfiltiration from the vertical face is zero, but there is a rapid increase in rate.

At steady state conditions, the drainage and reinfiltiration rates become the same — both around 0.44. Fig. 3 shows the reinfiltiration velocity profile of the vertical face at various dimensionless times. A short time, \( t_D = 0.01 \), the reinfiltiration velocity at the top of the vertical face reaches 2.35 which is more than three and a half times the initial drainage velocity at face \( z_p = 0 \). The reinfiltiration velocity profile at \( t_D = 0.4 \) also shows a sharp change along the vertical face. At \( t_D > 0.5 \), the reinfiltiration velocity at the top of the vertical face is around 2.8 which is four times higher than the maximum drainage velocity. At about \( t_D = 3.0 \), steady state profile is established, and all the vertical face is engaged in reinfiltiration. The velocity profile of Fig. 3 implies that the reinfiltiration of the drained oil coming from the upper blocks takes place mainly in the top portion of the lower matrix blocks. Fig. 4 shows the drainage velocity profile (at \( z_p = 0 \)) at various times. Unlike the reinfiltiration velocity profile of Fig. 3, the velocity variations seem to be gradual for the drainage face.

Fig. 5 shows the rate of drainage and reinfiltiration along the horizontal bottom face and the vertical side face, respectively. The plots on the left-hand side of this figure correspond to a block which is initially saturated. The drainage is from the bottom face, and the side faces have no-flow boundary conditions up to \( t_D > 0.4 \). Therefore, from \( t_D = 0 \) up to \( t_D = 0.4 \), the drainage performance is similar to a 1-D system. At \( t_D > 0.4 \), the side vertical face is kept at an oil saturation of unity. As a consequence, the reinfiltiration becomes active and the drainage rate increases. The initial reinfiltiration rate is very high — around 36 and decreases rapidly to the equilibrium value of 0.44. The plots on the right-hand side of Fig. 5 show the drainage and reinfiltiration rates for a block which has drained to equilibrium and then at \( t_D = 5 \) the oil saturation at the vertical side face is brought to unity. The initial reinfiltiration rate for the side face is about 50 which is 70 times higher than the maximum drainage rate. Similar to the left-hand plot, the reinfiltiration rate decreases rapidly and equilibrium is quickly established. Figs. 6 and 7 show the reinfiltiration and drainage velocity profiles for the situation where \( t_{DF} = 5 \). Fig. 6 shows that the initial reinfiltiration velocity for a partially saturated block is very high. The high reinfiltiration velocity extends to the lower part of the block. Due to a high reinfiltiration rate, the oil saturation decreases and consequently the reinfiltiration rate and the reinfiltiration velocity decreases. Fig. 7 shows the drainage velocity profile at various times. Note that the drainage profile velocity has initially sharp changes. The equilibrium profile shows a gradual decrease as one moves away from the reinfiltiration face.

Case 2. The results of this case are basically intended for a tilted block where at the same block face both drainage and reinfiltiration processes might be occurring. For the case of tilted block, we will assume \( L_x = 0.5 L_y \) and, therefore, \( \alpha = 0.5 \). Note that \( \tau \) of Case 2 is twice \( \tau \) of Case 1 when \( \theta = 0 \). The dimensionless time for Case 1 would also be twice the dimensionless time for Case 2 when \( \theta = 0 \). The actual times will, however, be equal for both cases.

For Case 2, the matrix block is initially assumed to be fully saturated. The angle \( \theta \) (see Fig. 1) is varied to examine its influence on drainage and reinfiltiration rates. Figs. 8, 9, and 10 show the velocity profile for various faces of the matrix block which has an angle of 30 degrees with the horizontal. The initial drainage velocity for the block face at \( z_p = 0 \) is around 0.6 everywhere along the \( z_p \)-axis. As the matrix block deaerates, the drainage velocity decreases in the middle part of this face. Note that the velocity profile is slightly asymmetric. Fig. 9 shows the reinfiltiration velocity profile at face \( z_p = 1 \) for various times. The reinfiltiration velocity at the bottom of this face is 0.7 and increases along the \( z_p \)-axis. Towards the top of this face, there is a sharp increase in the reinfiltiration velocity. Fig. 10 shows the velocity profile at block face \( z_p = 0 \). Note that both drainage and reinfiltiration processes are present in this case. The reinfiltiration takes place in the upper part, and the drainage takes place in the lower part of this face. The surface of reinfiltiration grows with time. The velocity profile at \( t_D = 5.0 \) is the equilibrium profile.

Figs. 11, 12, and 13 show velocity profiles for an angle of 30°. The initial drainage velocity at face \( z_p = 0 \) is uniform at a value of 0.35 and decreases in the middle part of the block face due to desaturation (see Fig. 11). The velocity profile is also slightly asymmetric. Fig. 12 shows the reinfiltiration velocity at \( z_p = 1 \) face. Note that the reinfiltiration rate at the bottom of this face is a constant — about 1.2. The sharp increase in the reinfiltiration velocity in the upper part of this face is similar to the previous cases. Fig. 13 shows the velocity profile at \( z_p = 0 \) face. The reinfiltiration surface area is considerably smaller than the previous case for which \( \theta = 0° \) (see Fig. 10). Comparison of Figs. 10 and 13 indicates that at this block face the drainage process is predominant at all times when \( \theta = 30° \), whereas for \( \theta = 0° \), reinfiltiration initially is small but at equilibrium reinfiltiration contribution to total flow rate is higher than drainage.

DISCUSSION AND CONCLUSIONS

A basic analytical framework to study reinfiltiration in a 2-D matrix block has been established. The solution of the fractional flow equation in a 2-D space with the boundary and initial conditions assigned for Case 1 and 2 can be used to study the sensitivity of reinfiltiration rate to capillarity, matrix block dimensions, rock permeability, and other parameters.

The practical implication of this study is that in a two-phase gas-liquid system, the fracture network is basically a conduct for the gas and the liquid path is through the matrix block. Other specific conclusions of this study are:

1. A two-dimensional analytical solution to the problem of gas-oil gravity drainage and reinfiltiration in a single matrix block was developed for testing reservoir simulation models and to provide insight into this process for tilted blocks.

2. Reinfiltiration in fractured porous media is a localized phenomenon.

3. There is considerable variation in the rate of reinfiltiration across a vertical face of a block. The rate is high at the upper portion of the face and decreases rapidly toward the lower part.

4. On the lower face of a tilted matrix block, reinfiltiration may take place in one portion and drainage may take place in another portion of the same face.
NOMENCLATURE

\( C_p \) - constant of relative permeability curve
\( D \) - parameter defined by \( \frac{A_p a_{m}}{\phi \phi_{m}} \)
\( g \) - gravitational acceleration
\( h \) - matrix permeability
\( k_r \) - relative permeability
\( L_z \) - block width
\( L_s \) - block height
\( P_{c0} \) - constant of capillary pressure curve
\( P_0 \) - capillary pressure
\( q_0 \) - oil flow rate
\( \eta \) - dimensionless oil flow rate
\( S \) - saturation
\( t \) - time
\( t_D \) - dimensionless time defined by Eq. 9
\( u \) - total oil and gas velocity
\( v \) - velocity
\( \psi \) - dimensionless velocity
\( V \) - parameter defined by \( \frac{A_p a_{m}}{\phi \phi_{m}} \)
\( z \) - distance
\( z_D \) - dimensionless distance defined by Eq. 7
\( s \) - distance
\( s_D \) - dimensionless distance defined by Eq. 6

Greek
\( \alpha \) - ratio of block height to block width
\( \beta \) - parameter defined by Eq. 10
\( \tau \) - parameter defined by Eq. 11
\( \Delta \rho \) - \( \rho_0 - \rho_1 \)
\( \rho \) - phase mass density
\( \phi \) - porosity
\( \theta \) - the angle between block face and the horizontal surface
\( \varphi \) - parameter defined by Eq. 2
\( \psi \) - parameter defined by Eq. 3
\( \chi \) - parameter defined by Eq. 4

Subscripts
\( g \) - gas phase
\( o \) - oil phase
\( w \) - water phase index
\( x \) - x-direction index

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Figure 1: Two-dimensional matrix block.

Figure 2: Drainage ($x_D = 0$ face) and reinfilttration ($x_D = 1$ face) rates versus time (Case 1).

Figure 3: Reinfilttration velocity profile along $x_D = 1$ face (Case 1).

Figure 4: Drainage velocity profile along $x_D = 0$ face (Case 1).

Figure 5: Drainage ($x_D = 0$ face) and reinfilttration ($x_D = 1$ face) rates versus time (Case 1).

Figure 6: Reinfilttration velocity profile along $x_D = 1$ face (Case 1).
Figure 7: Drainage velocity profile along $z_D = 0$ face (Case 1).

Figure 8: Drainage velocity profile along $z_D = 0$ face for $\theta = 30^\circ$ (Case 2).

Figure 9: Drainage velocity profile along $z_D = 1$ face for $\theta = 30^\circ$ (Case 2).

Figure 10: Velocity profile along $z_D = 0$ face for $\theta = 30^\circ$ (Case 2).

Figure 11: Drainage velocity profile along $z_D = 0$ face for $\theta = 60^\circ$ (Case 2).

Figure 12: Reinfiltaration velocity profile along $z_D = 1$ face for $\theta = 60^\circ$ (Case 2).
Figure 13: Velocity profile along $z_D = 0$ face for $\theta = 60^\circ$ (Case 2).