Reinfiltration in Fractured Porous Media: Part 1 - One Dimensional Model

The reinfiltration process in fractured porous media could be so significant that the oil phase (in gas-oil two phase flow) would mainly flow through the rock matrix.

Abbas Firoozabadi, Reservoir Engineering Research Institute,
Koichiro Ishimoto, Reservoir Engineering Research Institute

SUMMARY

Conventional fractured reservoir simulation models often neglect some fundamental aspects of multiphase flow in fractured porous media. Reinfiltration of the drained liquid from the upper blocks to the lower blocks is a key process which is often neglected. In this study, we present a theory of reinfiltration in fractured porous media in a one-dimensional (1-D) space. Based on this theory, it is established that the rate of drainage in a matrix block is always less than or equal to the reinfiltration rate. This implies that the drained liquid (for a gas-liquid system) from an upper block, reinfiltrates to the lower block and does not flow through the fracture network.

INTRODUCTION

Fractured petroleum reservoirs are thought of as matrix blocks and fracture networks. The matrix blocks are perceived to provide the storage and the fractures are believed to provide the flow path for the liquid produced from the matrix blocks. In other words, the primary flow in fractured porous media is assumed to be in the fractured network, with local exchange of fluids between the fractures and the matrix blocks. This simple picture of flow is valid for single phase flow. It may also be valid for water injection in fractured petroleum reservoirs under certain conditions. For a gas-oil drainage process, the drained liquid from various blocks does not generally flow in a two-phase manner in the fracture network which surrounds the blocks. The drained liquid from upper blocks may quickly reinfiltrate back to the lower matrix blocks. The process of reinfiltration could, therefore, strongly influence the two-phase gas-oil flow in fractured petroleum reservoirs.

Current tools for the study of fractured petroleum reservoirs include the popular dual-porosity model. In this model, the equations for flow of immiscible fluids in the discretized form are:

Fracture Flow

$$\Delta_t [\tau_{ef} (\Delta P_{ef} - \rho_{ef} g \Delta D_{ef})] - \tau_{oef} \Delta t + q_{ef} = \frac{V_e}{\Delta t} \Delta \frac{\phi A}{B}$$

Matrix Flow

$$\tau_{oef} = \frac{V_e}{\Delta t} \Delta \frac{\phi A}{B}$$

The symbols of Eqs. 1 and 2 are defined in the nomenclature. Eq. 1 implies no reinfiltration. Therefore, if the number of matrix blocks is increased by \( N \), the initial rate of drainage will also increase by a factor of \( N \) (assuming that fracture capillary pressure is zero). Some proposals to account for reinfiltration in a dual-porosity model have recently been made. These proposals are reviewed in Ref. 12 of this study. In the following, the literature on reinfiltration relevant to a one dimensional system will be reviewed.

LITERATURE REVIEW - In an earlier paper, du Prey attempted to answer questions such as: 1) what happens to oil released from a matrix block high in the secondary gas cap as a result of pressure increase, and 2) to what extent the drained liquid tends to reinfiltrate back into partially drained blocks on the way down. The thrust of the present paper could be divided into two parts: 1) experimental observation, and 2) the analytical and the numerical solutions to the 1-D two-phase flow problem. In the scouting experiments, when oil was supplied at the top of a (fit) block via a valve, it was observed that the oil entered into the block at a rate comparable with the block's gravity drainage rate. They observed that when oil was supplied to a fully saturated block with a height of less than the capillary pressure rise, the oil preferred to travel through the block instead of passing along the surface. They also observed full reinfiltration across a stack of four blocks when the horizonal faces were scaled. No quantitative data for these experiments were reported. Saidi et al. used a 1-D steady state model to calculate the oil saturation in a block for various values of reinfiltration rate. The drainage performance of a stack of blocks with a capillary hold-up was approximated by that of a continuous column without capillary effort. A semi-steady state approach and some kind of pseudo-relative permeability to incorporate the capillary hold-up were used in the 1-D model.

Fryer and van Celf-Rachman used fine grid simulation studies to compute the rate of drainage from a stack of two blocks. Their results confirmed du Prey's finding that the oil flowing to the fracture at the bottom of one block reinfiltrates into the block underneath.

The above literature addresses mainly the reinfiltration across the horizontal matrix block face. The literature on reinfiltration from the vertical and subvertical fractures to the matrix is reviewed in Ref. 12.
Ref. 10 provides ample evidence that reinfiltration is an important process for the gas-liquid flow in fractured porous media. What we find lacking in the literature is a theory for the reinfiltration mechanism. In particular, we are interested to know the maximum reinfiltration rate, and its transient behavior. The objective of this work is to: 1) provide a theory for the reinfiltration in fractured porous media, and 2) quantify the transient reinfiltration rate. A 1-D model is used to address reinfiltration across the horizontal face of matrix block of a fractured porous media.

THEORY OF REINFILTRATION

Consider a single block in a 1-D space. The oil velocity at each point in the matrix is given by:

\[ v_{ox} = \frac{k_{ror} d \Phi_o}{\mu_o} \frac{d x}{d z} \]  \hspace{1cm} (3)

where

\[ \Phi_o = p_o + \rho_o g z \]  \hspace{1cm} (4)

Assuming infinite gas mobility

\[ P_g = p_g|_{z=0} - p_g|_{z} \]  \hspace{1cm} (5)

The capillary pressure is defined as:

\[ P_c = p_o - p_g \]  \hspace{1cm} (6)

Combining Eqs. 5 and 6:

\[ p_o = p_o|_{z=0} - P_c - \rho_o g z \]  \hspace{1cm} (7)

Substituting \( p_o \) from Eq. 7 into Eq. 4 gives

\[ \Phi_o = p_o|_{z=0} + \Delta p_g - P_c \]  \hspace{1cm} (8)

where \( \Delta p = p_o - p_g \).

Eqs. 3 and 5 are combined to get

\[ v_{ox} = \frac{k_{ror} d}{\mu_o} \frac{d P_c}{d x} \]  \hspace{1cm} (9)

Let's assume the downward flux of oil (rate per unit area) to be positive and represented by \( q \), then,

\[ q = \frac{k_{ror} {\Delta P_c}}{\mu_o} \]  \hspace{1cm} (10)

Since \( P_c = P_c(S_c) \), Eq. 10 can be written as

\[ q = \frac{k_{ror} {\Delta P_c}}{\mu_o} \]  \hspace{1cm} (11)

Eq. 11 gives the drainage rate at the bottom face (\( z = 0 \)), and the reinfiltration rate at the top face (\( z = l \)), of a matrix block. The term \( \frac{d P_c}{d x} \) could be positive, zero, or negative. The term \( \frac{d P_c}{d x} \) is always less than zero if the threshold pressure is assumed to be zero, and it is never positive. At \( z = 0 \), when the block is fully saturated \( \frac{d P_c}{d z} = 0 \), and if the threshold pressure is zero, the maximum rate of drainage (i.e., the initial rate) is:

\[ q|_{z=0} = \frac{k_{ror} {\Delta P_c}}{\mu_o} \]  \hspace{1cm} (12)

As the block desaturates, \( \frac{d S_g}{d z} |_{z=0} < 0 \) and the rate of drainage decreases. Therefore, the rate of drainage from the bottom face of the matrix block is always

\[ q|_{z=0} \ll \frac{k_{ror} {\Delta P_c}}{\mu_o} \]  \hspace{1cm} (13)

Now let's examine the reinfiltration rate at the top face of the block. If enough liquid is provided, \( S_g|_{z=l} = 1 \), and \( \frac{d S_g}{d z} |_{z=l} = 0 \), which implies a reinfiltration rate of \( \frac{k_{ror} {\Delta P_c}}{\mu_o} \). However, if a block is partially saturated and \( S_g|_{z=l} = 1 \), then \( \frac{d S_g}{d z} |_{z=l} > 0 \) and, therefore,

\[ q|_{z=l} \ll \frac{k_{ror} {\Delta P_c}}{\mu_o} \]  \hspace{1cm} (14)

Based on the above reasoning, we conclude that as \( \frac{d S_g}{d z} |_{z=l} \) increases, the rate of reinfiltration also increases. The implication of relationships 13 and 14 is that initially the rate of drainage and reinfiltration for a 1-D matrix block (fully saturated) are equal, but due to desaturation, the maximum rate of reinfiltration is higher than the rate of drainage. A schematic of saturation profile in Fig. 1 at the drainage and reinfiltration faces further clarifies the above point.

ANALYTICAL SOLUTION TO THE 1-D PROBLEM FOR A SINGLE BLOCK

Gravity and capillarity, as we saw in the preceding section, are two important forces that influence the rate of drainage and reinfiltration from a matrix block. Various forces also influence drainage and reinfiltration rates. Oil viscosity is directly accounted for in the Darcy equation. The assumption of zero gas mobility is viewed as reasonable (in most cases) due to the low viscosity of gas relative to that of oil. Around the wellbores, since gas velocity could be high (especially in the fracture), an infinite gas mobility may not be a good assumption. To analytically solve the equations of flow for a single 1-D matrix certain assumptions have to be made. These are: 1) infinite gas mobility, 2) two-phase gas-oil system to be incompressible, and 3) oil relative permeability and gas-oil capillary pressure can be expressed as:

\[ k_{oro} = C_{oro}(1 - S_g) \]  \hspace{1cm} (15)

\[ P_c = -P_c \ln(1 - S_g) \]  \hspace{1cm} (16)

The above assumptions were made by Pavone, Bruzzi, and Verze13 to solve the 1-D gravity drainage problem analytically. The usefulness of Eqs. 15 and 16 is that they linearize the flow equations. (Pavone, et al. used a more general form of Eqs. 15 and 16.)

The fractional flow equation for a 1-D incompressible gas-oil system is given by:

\[ \frac{\partial S_o}{\partial t} + u \frac{\partial S_g}{\partial x} \frac{\partial S_o}{\partial x} - \frac{\partial}{\partial x} \left[ \Psi(S_g) \frac{\partial S_o}{\partial x} - \chi(S_o) \right] = 0 \]  \hspace{1cm} (17)

where

\[ \psi(S_o) = \frac{1}{1 + \frac{k_{oro} \mu_o}{k_{gor} \mu_o}} \]  \hspace{1cm} (18)

\[ \Psi(S_g) = \frac{k_{gor} \mu_o}{k_{gor} \mu_o} \]  \hspace{1cm} (19)

and

\[ \chi(S_o) = \frac{k_{gor} \mu_o}{k_{gor} \mu_o} \]  \hspace{1cm} (20)

Substituting Eqs. 18, 19, and 20 into Eq. 17 and assuming infinite gas mobility, gives

\[ \frac{\partial S_o}{\partial t} - D \frac{\partial^2 S_o}{\partial x^2} - V \frac{\partial S_o}{\partial x} = 0 \]  \hspace{1cm} (21)

where \( D = \frac{k_{gor} \mu_o}{\rho_o X_o} \), and \( V = \frac{\rho_o \mu_o}{\rho_o X_o} \).
Eq. 21 in dimensionless form is:

\[
\frac{\partial S_e}{\partial \tau} - \frac{\partial^2 S_e}{\partial x^2} - 2\gamma \frac{\partial S_e}{\partial x} = 0
\]  

(22)

Where the dimensionless variables are given by:

\[
x_D = \frac{x}{L}
\]

(23)

\[
t_D = \frac{t}{T}
\]

(24)

And

\[
\gamma = \frac{V L}{2D}
\]

(25)

Let the dimensionless downward flow flux, \(q_D\), be positive, then

\[
q_D = -\frac{\partial S_e}{\partial x} + 2\gamma (1 - S_e)
\]  

(26)

The maximum drainage flux when the matrix is fully saturated is given by \(q_D = 2\gamma\) which is also \(q_D = \frac{2\gamma}{\lambda_D}\).

The dimensionless cumulative flux is given by:

\[
N_{D_D} = \int_0^{t_D} q_D \, dt_D
\]  

(27)

In the following, the solution to Eq. 22 with various boundary and initial conditions will be presented.

**Case 1 - Simple Drainage: \(q_D|_{x=D}=0\)**

The boundary and initial conditions for this case are:

\[\begin{align*}
\text{At} & \quad \text{At} = 1 & \frac{\partial S_e}{\partial x} + 2\gamma (1 - S_e) = 0 \\
\text{At} & \quad \text{At} = 0 & S_e = 0 \quad \text{(i.e.,} \quad P_e = 0) \\
\text{At} & \quad \text{At} = 0 & \text{At} e (0, 1)
\end{align*}\]

(28)

The solution to Eq. 22 and boundary and initial conditions 28 to 30 is given by Pavone, et al.\textsuperscript{15}

\[
S_e = 1 - e^{-2\gamma t_D} + \sum_{k=1}^{\infty} A_k e^{-2\gamma t_D} e^{-\left(\omega_k^2 + \gamma^2\right) t_D} \sin(\omega_k x_D)
\]

(31)

Where

\[
A_k = \frac{4\gamma e^{-\omega_k^2}}{\gamma + \gamma^2 + \omega_k^2}
\]

And \(\omega_k\)'s are the roots of

\[-\omega_k^4 + \gamma \omega_k^2 = 0;
\]

\[-\pi/2 - \pi/2 < \omega_k < -\pi/2 + (k+1)\pi \quad k = 1, 2, \ldots, \infty\]

(32)

Using Eqs. 26 and 31 the rate of drainage flux at \(x_D = 0\) is obtained.

\[
N_{D_D}|_{x_D=0} = -\sum_{k=1}^{\infty} A_k e^{-\omega_k^2 t_D}
\]

(33)

And dimensionless cumulative flux at \(x_D = 0\) is:

\[
N_{D_D}|_{x_D=0} = -\sum_{k=1}^{\infty} \frac{A_k e^{-\omega_k^2 t_D}}{\gamma + \gamma^2 + \omega_k^2} \left[ e^{-\left(\omega_k^2 + \gamma^2\right) t_D} \right]
\]

(34)

Pavone, et al.\textsuperscript{15} used the method of separation of variables to derive Eq. 31. The Laplace transform technique could also be used to solve Eqs. 22, and 28-30. Appendix A gives the solution when the Laplace transform technique is used.

**Case 2 - Reinfiltiration Rate Is Specified: \(q_D|_{x=D}=q(t_D)\)**

The boundary condition at \(x_D = 1\) for this case is:

\[-\frac{\partial S_e}{\partial x} + 2\gamma (1 - S_e) = q(t_D)
\]

(35)

The Laplace transform technique is used to solve the equations for this case. The results are given in Appendix B. For the following subcases, the reinfiltiration rate is assumed to be a constant during various intervals. The method of separation of variables solution for Case 1 is used to obtain an analytical expression for the inversion of the Laplace space solution.

i) \(q_D|_{x_D=0} = q_1 = \text{constant} \quad S_e, \quad t_D, \quad \text{and} \quad N_{D_D}\) are given by:

\[
S_e = \left( \frac{1 - G_s}{2\gamma} \right)
\]

\[
\times \left\{ 1 - e^{-2\gamma t_D} + \sum_{k=1}^{\infty} A_k e^{-2\gamma t_D} \sin(\omega_k x_D) e^{-\left(\omega_k^2 + \gamma^2\right) t_D} \right\}
\]

(36)

\[
q_D|_{x_D=0} = q_1 = \left( \frac{G_s}{2\gamma} \right) \sum_{k=1}^{\infty} A_k e^{-\omega_k^2 t_D} \quad \left[ e^{-\left(\omega_k^2 + \gamma^2\right) t_D} \right]
\]

(37)

\[
N_{D_D}|_{x_D=0} = q_1 t_D - \left( \frac{G_s}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \omega_k \left[ e^{-\omega_k^2 t_D} \right]
\]

(38)

ii) \(q(t_D) = q_1; \quad 0 \leq t_D < t_1\), and \(q(t_D) = q_2; \quad t_D \geq t_1\) - For this two-rate case, both \(q_1\) and \(q_2\) are constant. The gas saturation is given by:

\[
S_g = S_g^i(x_D, t_D) \left( \frac{1 - G_s}{2\gamma} \right) - \left( \frac{2\gamma - 2\gamma}{2\gamma} \right) H(t_D - t_1) S_g^f(x_D, t_D - t_1)
\]

(39)

Where \(S_g^i\) is the solution for subcase i, and

\[
H(t_D - t_1) = \left\{ \begin{array}{ll}
0 & \text{if} \quad t_D < t_1 \\
1 & \text{if} \quad t_D \geq t_1
\end{array} \right.
\]

(40)

The expressions for \(q_D\) and \(N_{D_D}\) are provided in Appendix B. The same appendix also gives the solution for the multi-rate case.

**Case 3 - Saturation (or Capillary Pressure) Is Specified at \(x_D = 1\)**

Case 3 represents the situation where a fracture with constant capillary pressure is imposed on the top face of a 1-D matrix block. For this case we assume that the block will be under free drainage initially. The boundary and initial conditions are:

\[\begin{align*}
\text{At} & \quad \text{At} = 1 & \frac{\partial S_e}{\partial x} + 2\gamma (1 - S_e) = 0 \quad \text{if} \quad t_D \leq t_D \quad \text{(41)}
\end{align*}\]

\[
S_g = S_g^f \quad \text{if} \quad t_D \geq t_D
\]

(42)

\[\begin{align*}
\text{At} & \quad \text{At} = 0 & S_e = 0
\end{align*}\]

(43)

\[\begin{align*}
\text{At} & \quad \text{At} = 0 & \text{At} e (0, 1)
\end{align*}\]

(44)

For \(t_D \leq t_D\), the solution for Case 1 can be applied. For \(t_D > t_D\),

\[
S_g - \frac{S_g^f}{1 - e^{-2\gamma t_D}} \left( 1 - e^{-2\gamma t_D} \right)
\]

\[
+ \sum_{m=1}^{\infty} B_m e^{-\omega_m t_D} \sin(m\pi x_D) e^{-\left(\omega_m^2 + \gamma^2\right) t_D - t_D}
\]

(45)
and

\[ B_m = 2 \left( -1 \right)^m \pi r \left[ 1 - \frac{S_0}{1 - e^{-\gamma}} \left( \frac{e^{-\gamma} - e^{-2\gamma}}{\gamma^2 + m^2 \gamma^2} + \sum_{k=1}^{\infty} A_k e^{-\left(k^2 + m^2\right) \gamma^2 \sin \omega_k} \right) \right] \]

where \( \omega_k \) is given by Eq. 32. The dimensionless flux and cumulative flux, \( \psi_0, N_{D_0} \) are given in Appendix C.

**Case 4 - Drainage Behavior of a Stack of N Equal Blocks**

In this case we are interested to study the drainage behavior of a stack of \( N \) equal blocks which are initially saturated and are surrounded by gas. Due to reinfiltiration, the liquid drained from an upper block will be sucked by the lower block as was shown previously. This problem can be solved using the Laplace transform technique if we assume that fracture capillary pressure is zero.

For the first (i.e., top) block, the boundary conditions are:

\[ \psi_{D_0} = 1 - \frac{\partial S_0}{\partial x_D} + 2\gamma(1 - S_p) = 0 \]  

\[ \psi_{D_0} = 0 \quad S_0 = 0 \]

The boundary conditions for the second block are:

\[ \psi_{D_0} = 1 - \frac{\partial S_0}{\partial x_D} + 2\gamma(1 - S_p) = \psi_{D_1} |_{x_D = 0} \]  

\[ \psi_{D_0} = 0 \quad S_0 = 0 \]

where \( \psi_{D_1} |_{x_D = 0} \) is the dimensionless drainage flux at \( x_D = 0 \) for the first block.

The boundary condition for the \( I \)-th and \( N \)-th blocks are:

\[ \psi_{D_0} = 1 - \frac{\partial S_0}{\partial x_D} + 2\gamma(1 - S_p) = \psi_{D_{I-1}} |_{x_D = 0} \]  

\[ \psi_{D_0} = 0 \quad S_0 = 0 \]

\[ \psi_{D_0} = 0 \quad S_0 = 0 \]

The initial condition for all blocks is:

\[ \psi_{D_0} = 0 \quad S_0 = 0 \quad x_D = 0, 1 \]

The gas saturation in the Laplace space for the \( I \)-th block is:

\[ S_{g_I} = \frac{\partial S_0}{\partial x_D} \left[ \frac{2\gamma}{\alpha} \left( \frac{e^{-\gamma}}{\sqrt{\gamma^2 + \alpha^2}} \right) (1 - e^{-\gamma}) \sinh \left( \sqrt{\gamma^2 + \alpha^2} x_D \right) \right] \]

where

\[ \alpha = \left( \sqrt{\gamma^2 + s} \right) \cosh \left( \sqrt{\gamma^2 + s} x_D \right) \sinh \left( \sqrt{\gamma^2 + s} \right) \]

The inversions of the Laplace space solutions for this case are done numerically by using Stahn's Algorithm.

The dimensionless flux and cumulative flux at \( x_D = 0 \) and \( x_D = 1 \) for the \( I \)-th block are given in Appendix D.

**RESULTS**

In order to have an appreciation of reinfiltiration in fractured porous media, numerical examples using the equations of the preceding section will be presented here. For all cases, a block height of 1.97 ft [60 cm], permeability of 400 md., and a porosity of 26% is used. The constants of Eqs. 15 and 16 are assumed to be \( C_m = 1 \), and \( F_m = .85 \) psi [0.5 kPa]. The oil viscosity is assumed to be 1 cp [1 mPa s], and \( \rho_o = 42.5 \) lbm/ft^3 [724 kg/m^3]. These values yield \( \gamma = 0.85 \). Therefore, the maximum (dimensionless) drainage flux for the block is 0.7. The results for various cases are given next.

**Case 1** - Fig. 2 shows gas saturation profile at various dimensionless times, and Fig. 3 shows the dimensionless drainage flux and the cumulative drainage vs. dimensionless time. Fig. 3 illustrates that for \( t_D > 1 \), the drainage rate decreases negligibly. The saturation profile at \( t_D = 1 \) shown in Fig. 2 is, therefore, the equilibrium saturation profile. These two figures are viewed as a reference for the other cases.

**Case 2** - In this case, by providing liquid at the top face of the matrix block (initially fully saturated), the reinfiltiration is allowed to influence the matrix saturation profile. Fig. 4 shows the equilibrium gas saturation for various values of the dimensionless reinfiltiration rate. A reinfiltiration rate of zero corresponds to the previous case, and a reinfiltiration rate of 0.7 corresponds to the maximum drainage rate. As Fig. 4 shows, if the oil is supplied at a rate of 0.7, the matrix block will be fully saturated at all times. Reinfiltiration rates of less than 0.7 cause desaturation of the matrix block. Saidi, et al. arrived at the same conclusion. Fig. 5 shows the drainage rate vs. time for various values of reinfiltiration rates. The plot of the drainage rate for zero reinfiltiration rate is the same as that of Case 1 presented in Fig. 4. Fig. 6 shows the speed at which the drainage rate increases as the matrix block under equilibrium conditions is subjected to various reinfiltiration rates from the top face. There is an assumption inherent in the plots of Fig. 5 and the next figure. The same capillary pressure and relative permeability have been used in the drainage and imbibition processes. Accounting of direction dependent capillary pressure and relative permeability will slow down the rates. Fig. 7 shows the gas saturation profile at various times. The saturation profile at \( t_D = 1 \) is the equilibrium gas saturation. At this time, liquid at a rate of 0.7 is provided at the top face of the matrix block. After a short time (i.e., \( t_D = 5.05 \)), considerable liquid saturation is built up in the upper part of the matrix block. At \( t_D = 5.40 \), a high liquid saturation is developed along the entire length of the block.

**Case 3** - The results of this case are of significant interest. The maximum rate of reinfiltiration, discussed in the theory of reinfiltiration, will be presented here. Fig. 8 shows the saturation profile when a block has reached equilibrium saturation at \( t_D = 5 \). At this time, sufficient liquid is provided at the top face to keep a liquid saturation of unity. As a result, liquid is sucked into the block at a fast rate. After a short time interval of 0.05 (\( t_D = 5.05 \)), the liquid saturation increases considerably at the upper part of the block. At \( t_D = 5.40 \), the block becomes nearly fully saturated. It should be pointed out that the use of scanning capillary pressure and oil relative permeability curves somewhat decreases the rate of reinfiltiration. Fig. 9 shows the saturation profile at various times when the liquid saturation at the top face of the matrix block is held at 60 percent. This figure, similar to Fig. 8, shows a rapid build up in liquid saturation due to the fast reinfiltiration rate. Fig. 10 shows the rate of drainage during the first period, 0 \( \leq t_D \leq 5 \), when the block is under free drainage process. At \( t_D = 5 \), the top face of the block is held at constant liquid saturations of 100% (\( S_g = 0.0 \)), 80% (\( S_g = 0.2 \)), and 60% (\( S_g = 0.4 \)). As
a result, due to a high capillary pressure gradient, the reinfilt- 
tration rate begins at a very fast rate and consequently the 
drainage from the bottom face of the block becomes activated. 
When the top face of the block is kept at a liquid saturation 
of unity, the initial reinfiltiration rate is about 3, which is more 
than four times the maximum drainage rate of the block. 
The reinfiltiration rate drops quickly and at steady state both 
the drainage and reinfiltiration rates become the same — each 0.7. 
If the liquid saturation at the top face is kept at 80 percent instead 
of 100 percent, the initial reinfiltiration rate is about 1.82 which is more than 
2.5 times the maximum drainage rate. As the liquid saturation at which 
the top block face is kept decreases, the maximum reinfiltiration 
rate also decreases.

Case 4 - Figs. 11, 12, and 13 show the gas saturation profile 
in a stack of 10 matrix blocks. At \( t_p = 0.4 \), block 1 and block 2 
has produced most of its oil, whereas block 3 has just begun 
to drain. Fig. 12 shows that at \( t_p = 2 \), blocks 1, 2, and 3 have 
produced most of their oil, whereas block 10 (i.e., bottom block) is 
still nearly fully saturated. Finally at \( t_p = 3 \), the five top blocks 
are approaching equilibrium saturation and the bottom block has 
produced a small portion of its producible liquid. Fig. 14 shows 
the average block gas saturation as a function of time. This 
figure shows a significant difference in block saturations due to 
the process of reinfiltiration. Fig. 15 shows the cumulative production 
for a stack of 3, 5, and 10 blocks. This figure also shows 
the cumulative production when the production of a single block 
is multiplied by the number of blocks in the stack. This latter 
production behavior for a stack is assigned the legend of dual- 
porosity (DP) since conventional dual-porosity simulation could 
give similar results. Fig. 16 shows that there is a significant 
effect of reinfiltiration on the production performance of a stack 
of blocks. As the number of blocks increases, the difference of early 
production performance between the case with zero reinfiltiration 
(i.e., DP), and full reinfiltiration also increases. The final recovery 
for both cases (as expected) is the same.

DISCUSSION

The analytical models that have been derived in this study are 
based on certain assumptions. These include: 1) linear relationship 
between relative permeability and saturation; 2) infinite gas mobility; 3) 
use of saturation direction independent capillary pressure and relative permeability; and 4) zero fracture 
capillary pressure for the case of the stack blocks. The use of 
direction-dependent capillary pressure will slow down the reinfiltiration 
rate. Infinite gas mobility assumption in the matrix is 
believed to be justified for most cases. While zero fracture capillary 
is not a reasonable assumption in the flow of multiphase flow 
in fractured porous media, this assumption was made to study 
the drainage performance of a stack of blocks when reinfiltiration 
is accounted for. We believe the analytical models of this study, 
which are based on the above assumptions will help to develop 
and test more realistic numerical simulation models for fractured 
petroleum reservoirs. The results presented in Cases 1 through 4 
also show that the reinfiltiration process is highly saturation 
dependent and it is a dominant process in a gas-oil gravity drainage. 
The results also imply that the reinfiltiration is a localized phe-
nomenon. This point has already been mentioned in the scouting 
experiment reported in Ref. 10. Further discussion on the topic 
of the reinfiltiration process as a localized phenomenon is made in 
Ref. 12. The specific conclusions of this study are outlined next.

CONCLUSIONS

1. The theoretical analysis shows that the minimum reinfiltiration 
rate from the top face of a matrix block is \( \phi_{m} \), which is equal 
or higher than the maximum drainage rate.

2. Based on the analytical model, the initial rate of drainage 
from a stack of \( N \) matrix blocks is equal to the rate of 
drainage from a single matrix block. The provision for the 
analytical model is the assumption of zero fracture capillary pressure.

NOMENCLATURE

- \( B \) - formation volume factor
- \( C_0 \) - constant of relative permeability curve
  as defined by Eq. 15
- \( D \) - parameter defined by \( \phi_{op} \)
- \( D_f \) - depth of fracture gridblock
- \( g \) - gravitational acceleration
- \( k \) - matrix permeability
- \( k_r \) - relative permeability
- \( L \) - block height
- \( P_{ci} \) - constant of capillary pressure curve
  as defined by Eq. 16
- \( P_c \) - capillary pressure
- \( q \) - oil flow rate
- \( q_D \) - dimensionless oil flow rate
- \( N_{co} \) - cumulative oil production
- \( N_{cp} \) - dimensionless cumulative production
- \( S \) - saturation
- \( S^f_0 \) - fixed gas saturation at \( z_D = 1 \)
- \( S_t \) - saturation for block \( t \) in a stack of \( N \) blocks
- \( \tau \) - Laplace transform variable
- \( t \) - time
- \( t_D \) - dimensionless time defined by Eq. 24
- \( T \) - fluid transmissibility
- \( u_t \) - total oil and gas velocity
- \( v \) - velocity
- \( V \) - parameter defined by \( \phi_{op} \)
- \( V_b \) - gridblock volume
- \( z \) - distance
- \( z_D \) - dimensionless distance defined in Eq. 23

GREEK

- \( \gamma \) - parameter defined by Eq. 25
- \( \mu \) - phase viscosity
- \( \Delta \) - difference operator
- \( \delta \) - difference operator for time derivative
- \( \rho \) - phase mass density
- \( \phi_1 \) - porosity
- \( \phi_p \) - potential defined by Eq. 4
- \( v_{masc} \) - matrix fracture exchange term for phase \( \sigma \)
- \( \phi_{op} \) - parameter defined by Eq. 16
- \( \psi \) - parameter defined by Eq. 19
- \( \chi \) - parameter defined by Eq. 20

SUBSCRIPTS

- \( \alpha \) - phase index
- \( f \) - fracture
The expressions for \( q_D \), and \( N_{pD} \), for the two-rate case are:

\[
q_D|_{td=0} = q_1 + (q_2 - q_1)H(td - t_{d1}) \\
- \left( 1 - \frac{q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right] \\
+ \left( \frac{q_2 - q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right]
\]

(\[B-4\])

\[
N_{pD}|_{td=0} = q_1(t_d) + (q_2 - q_1)(td - t_{d1}) \\
- \left( 1 - \frac{q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right] \\
+ \left( \frac{q_2 - q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right]
\]

(\[B-5\])

For the multi-rate case where

\[
q(t_d) = \begin{cases} 
q_1 & \text{if } 0 \leq t_d < t_{d1} \\
q_2 & \text{if } t_{d1} \leq t_d < t_{d2} \\
q_3 & \text{if } t_{d2} \leq t_d \leq t_{d3} \\
q_4 & \text{if } t_{d3} \leq t_d \leq t_{d4} \\
q_5 & \text{if } t_{d4} \leq t_d \leq t_{d5} \\
q_6 & \text{if } t_{d5} \leq t_d
\end{cases}
\]

(\[B-6\])

the expressions for \( S_g \), \( q_D \), and \( N_{pD} \) are:

\[
S_g = S_g^* (td, t_d) \left( 1 - \frac{q_1}{2\gamma} \right) \\
- \sum_{n=1}^{\infty} \left( \frac{q_{n+1} - q_n}{2\gamma} \right) H(td - t_{d1}) S_g^* (td, t_d - t_{d1})
\]

(\[B-7\])

\[
q_D|_{td=0} = q_1 + \sum_{n=1}^{\infty} (q_{n+1} - q_n) H(td - t_{d1}) \\
- \left( 1 - \frac{q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right] \\
+ \sum_{n=1}^{\infty} \left( \frac{q_{n+1} - q_n}{2\gamma} \right) H(td - t_{d1}) \\
\times \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right]
\]

(\[B-8\])

\[
N_{pD}|_{td=0} = q_1(t_d) + \sum_{n=1}^{\infty} (q_{n+1} - q_n)(td - t_{d1}) \\
- \left( 1 - \frac{q_1}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right] \\
+ \sum_{n=1}^{\infty} \left( \frac{q_{n+1} - q_n}{2\gamma} \right) \sum_{k=1}^{\infty} A_k \frac{\omega_k}{\omega_k^2 + \gamma^2} \left[ 1 - e^{-(\omega_k^2 + \gamma^2)(td - t_{d1})} \right]
\]

(\[B-9\])

In Eq. B-7, \( S_g^* \) is the solution for subcase i of Case 2 given by Eq. 36.

Appendix C: \( q_D \) and \( N_{pD} \) Expressions for Case 3

For \( t_d \leq t_{d1} \), the expressions for \( q_D \) and \( N_{pD} \) of Case 1 can be used. For \( t_D > t_{d1} \), the dimensionless reflushment rate for oil at \( x_D = 1 \) and the dimensionless drainage rate for oil at \( x_D = 0 \) are:

\[
q_D|_{x_D=1} = 2 \gamma \left( 1 - \frac{S_g}{1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})}} \right) \sum_{m=1}^{\infty} m x(-1)^m B_m e^{-r(t_d - t_{d1})} \]

(\[C-1\])

\[
q_D|_{x_D=0} = 2 \gamma \left( 1 - \frac{S_g}{1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})}} \right) \sum_{m=1}^{\infty} m x^2 B_m e^{-r(t_d - t_{d1})} \]

(\[C-2\])

The dimensionless cumulative rates across \( x_D = 1 \) and \( x_D = 0 \) faces are given by:

\[
N_{pD}|_{x_D=1} = 2 \gamma \left( 1 - \frac{S_g}{1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})}} \right) \sum_{m=1}^{\infty} m x B_m e^{-r(t_d - t_{d1})} \left[ 1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})} \right] \]

(\[C-3\])

\[
N_{pD}|_{x_D=0} = 2 \gamma \left( 1 - \frac{S_g}{1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})}} \right) \sum_{m=1}^{\infty} m x^2 B_m e^{-r(t_d - t_{d1})} \left[ 1 - e^{-(\omega_1^2 + \gamma^2)(td - t_{d1})} \right] \]

(\[C-4\])

where \( \gamma \) and \( B_m \) are given by Eqs. 32 and 46, respectively.

Appendix D: Expressions for \( q_D \) and \( N_{pD} \) for Case 4

The dimensionless flow rate at \( x_D = 0 \) for the I-th block is given by:

\[
q_D|_{x_D=0} = L^{-1} \left\{ \frac{2\gamma}{\sqrt{\frac{\omega_1}{A}}} \left[ 1 - \left( \frac{\gamma \sqrt{\frac{\omega_1}{A}}}{\omega_1} - \frac{\omega_1^2}{A} \right) \right] \right\}
\]

(\[D-1\])

where \( L^{-1} \) is the sign for the inverse Laplace and parameter \( A \) is given by Eq. 57.

Since the fracture is assumed to have a negligible storage capacity,

\[
q_D|_{x_D=0} = q_D|_{x_D=0} |_{x_D=0}
\]

(\[D-2\])

The dimensionless cumulative rate across \( x_D = 0 \) face for the I-th block is given by:

\[
N_{pD}|_{x_D=0} = L^{-1} \left\{ \frac{2\gamma}{\sqrt{\frac{\omega_1}{A}}} \left[ 1 - \left( \frac{\gamma \sqrt{\frac{\omega_1}{A}}}{\omega_1} - \frac{\omega_1^2}{A} \right) \right] \right\}
\]

(\[D-3\])

The dimensionless cumulative rate across $x_D = 1$ face for the $k$-th block is:

$$ N_{pD} |_{x_D=1} = N_{pD_{k-1}} |_{x_D=0} $$

(D-4)

**Authors**

Abbas Firoozabadi is a senior scientist at the Reservoir Engineering Research Institute in Palo Alto, CA. His research interests include miscible and immiscible gas injection in fractured and layered reservoirs, reservoir fluids phase behavior, and gas reservoirs. Firoozabadi previously was a senior researcher and manager of the EOR Research Center at the National Iranian Oil Co., a petroleum engineering faculty member at Stanford University, and Research advisor with Norsk Hydro A/S. He holds a B.S. degree from the Abadan Institute of Technology and M.S. and Ph.D. degrees from the Illinois Institute of Technology, all in gas engineering. Koichiro Ishimoto is a senior petroleum engineer with Nippon Oil Exploration Co. Ltd., in Japan. He was a visiting scientist at the Reservoir Engineering Institute in 1996. Ishimoto has a B.S. degree in engineering from Tokyo University and an M.S. degree in petroleum engineering from the University of Texas-Austin.

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**Figure 2:** Gas saturation profiles at various times (Case 1).

**Figure 1:** Schematic representation of saturation profile in a matrix block for different boundary conditions.

**Figure 3:** Drainage flux and cumulative production versus time (Case 1).
Figure 4: Equilibrium gas saturation profiles for various re-infiltration rates (Case 2).

Figure 5: Drainage rate versus time (Case 2).

Figure 6: Drainage rate versus time (Case 2).

Figure 7: Saturation profiles at various times for $T_{l_{gas}} = 0.70$ (Case 2).

Figure 8: Saturation profiles at various times for $T_{l_{gas}} = 0$ (Case 3).

Figure 9: Saturation profiles at various times for $T_{l_{gas}} = 0.4$ (Case 3).

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Figure 10: Rate versus time for various $S_{p_{ini}}$ values (Case 3).

Figure 11: Gas saturation profiles in the stack of blocks at $t_D = 0.4$ (Case 4).

Figure 12: Gas saturation profiles in the stack of blocks at $t_D = 2$ (Case 4). Figure 13: Gas saturation profiles in the stack of blocks at $t_D = 3$ (Case 4).

Figure 14: Average gas saturation in each block versus time (Case 4).

Figure 15: Cumulative production from the stack of blocks (Case 4).