Co-current and Countercurrent Imbibition in a Water-Wet Matrix Block

Mehran Pooladi-Darvish, SPE, and Abbas Firozabadi, SPE, Reservoir Engineering Research Inst.

Summary
Imbibition in water-wet matrix blocks of fractured porous media is commonly considered to be co-current. The modeling studies of this paper indicate that when a matrix block is partially covered by water, oil recovery is dominated by co-current imbibition, not countercurrent. It is also found that the time for a specified recovery by the former can be much smaller than that by countercurrent imbibition. Consequently, use of the imbibition data by immersing a single block in water and its scale-up may provide pessimistic recovery information. Moreover, it is shown that the application of the diffusion equation for modeling of oil recovery by co-current imbibition leads to a large error. Through a detailed study of the governing equations and boundary conditions, significant insight is provided into the mathematical and physical differences between co- and countercurrent imbibition.

Introduction
Co-current imbibition, in which water and oil flow through the same face in opposite directions, has received considerable attention in the literature. The mathematical formulation of co-current imbibition is of the form of a nonlinear diffusion equation. Analytical and semianalytical solutions of co-current imbibition have been recently emphasized. These solutions assume a semi-infinite domain and are valid before the saturation front reaches the far boundary.

Much experimental work on co-current imbibition has been reported in the literature. In these experiments, the oil-saturated cores are either immersed in water, or sealed such that water in-flow and oil out-flow occur through the same faces. Some imbibition studies have reported oil production from a face not covered with water (i.e., co-current imbibition). The majority of these studies were insensitive regarding the differences between co-current and countercurrent imbibition. In a detailed experimental study, Bourbiaux and Kalaydjian examined the counterc and countercurrent imbibition processes on a laterally coated core. When the two opposing faces of a cylindrical core were open to flow, and water was in contact with one face, oil was mostly produced by countercurrent imbibition from the face in contact with oil, oil production from the water-contacted face was very small—about 3%. The authors expressed uncertainty about the source of oil. It was not clear if this small amount was produced from the rock, or was the oil from the dead volume. The countercurrent experiment had a slower recovery than the co-current experiment; the half-recovery time for co-current imbibition was 7.1 hours and that for countercurrent imbibition was 22.2 hours, for one set of experiments. The measured saturation profiles for the two processes were also different.

In a recent experimental study, a stack of Berea and chalk matrix blocks were used to study water injection in fractured porous media. Visual observation and recovery data from a number of tests revealed that (1) the injected water did not fill the fracture space surrounding the matrix blocks rapidly, and (2) the recovery mechanism was mainly countercurrent imbibition before a block was fully covered by water. Once the rock was covered by water, the oil recovery rate decreased due to low efficiency of countercurrent imbibition.

The above experimental studies show that there are significant recovery differences between co- and countercurrent imbibition. To our knowledge, there is currently no theoretical study comparing the two imbibition processes. The major objective of this paper is to study the similarities and differences of co- and countercurrent imbibition and point out the consequences for practical applications. In the following, we first investigate the one-dimensional diffusion equation and boundary conditions for co-current and countercurrent imbibition. A numerical model is then developed to compare the behavior and recovery efficiency of the two processes. Scaling studies are used to draw general conclusions.

Mathematical Investigation
The one-dimensional (1D) countercurrent imbibition process can be described by a nonlinear diffusion equation of the form

$$\frac{\partial}{\partial s} \left( D(S_w) \frac{\partial S_w}{\partial s} \right) = \frac{\partial S_u}{\partial t},$$

(1)

where

$$D(S_w) = -k \frac{k_{sw}}{\phi \mu_w} f(S_w) \frac{dP_r}{dS_w},$$

(2)

and

$$f(S_w) = \frac{1}{1 + k_{sw} \mu_w \mu_o}.$$

(3)

The initial and boundary conditions are

$$S_w = S_{w,0}, \quad t = 0, \quad 0 < x < L,$$

(4)

$$S_w = 1 - S_{w,0}, \quad t = 0^+, \quad x = L,$$

(5)

$$q_w = 0, \quad t = 0^+, \quad x = L.$$  

(6)

The symbols are defined in the nomenclature. Eq. 1 assumes that the fluids are incompressible, and the effect of gravity is neglected. The diffusion coefficient of Eq. 2 is bell shaped with respect to water saturation, attaining a value of zero at $S_{w,0}$ and $S_{w,0}$ (see Ref. 2). Eq. 5 expresses the continuity of capillary pressure at the inlet face ($P_r = 0$), and Eq. 6 is the no-flow boundary condition at the outlet.

Formulation of countercurrent imbibition includes an additional convective term

$$\frac{\partial}{\partial s} \left( D(S_w) \frac{\partial S_w}{\partial s} - q_w f(S_w) \right) = \frac{\partial S_u}{\partial t},$$

(7)

where the functions $D$ and $f$ are given by Eqs. 2 and 3, respectively. In Eq. 7, $a - q_w \cdot a_w$ is unknown, and an additional equation, i.e., the pressure equation, with initial and boundary conditions is required to complete the formulation.

Some studies of the imbibition process have assumed that the pressure gradient in the displaced oil phase may be neglected. This assumption is based on the common practice in hydrology, where the mathematical formulation of unsaturated water flow ignores the air pressure gradient (see Ref. 18 for an account of this assumption). Under this assumption, the problem is formulated as in Eq. 1 with initial and boundary conditions 4 to 6. However, the corresponding diffusion coefficient is

$$D(S_w) = -k \frac{k_{sw}}{\phi \mu_w} dP_r.$$  

(8)

---

1. Now at the U. of Calgary.

Copyright © 1999 Society of Petroleum Engineers

Original SPE manuscript received for review 27 February 1999. Revised manuscript received 16 March 1999. Paper (SPE 58452) first approved 30 August 1999.

SPE Journal 5 (1), March 2000 1086-55X/200051/03/055.00 + 0.50 3
The diffusion coefficient of Eq. 8 does not exhibit the bell-shape behavior of Eq. 2. In the absence of pressure gradients in the displaced phase, this formulation applies to both co- and countercurrent imbibition. The boundary condition 6 is valid for countercurrent imbibition because imbibition continues so long as the water pressure inside the core is less than that outside, and no water breakthrough will occur. Eqs. 1, 4–6, and 8 may be good approximations for unsaturated flow, where viscosity of the displaced nonwetting phase (i.e., air) is much smaller than the displacing viscosity, water. It will be shown later that for the wateroil system, where oil and water viscosity can be of the same order, the oil pressure gradient may not be neglected.

Numerical Model

As mentioned, the analytical solution for countercurrent imbibition is limited to an infinite-acting solution, that is, before water saturation is reached at the outer boundary, x = L. Therefore, we need a numerical model for the general case; 1D and two-dimensional (2D) finite-difference models were developed to study counter- and countercurrent imbibition in finite-size porous media. Peaceman and co-workers’ approach was used where the continuity equation is coupled with the generalized form of Darcy’s law for two-phase flow to obtain

\[ \nabla \left[ k \frac{k_w}{k_o} \nabla p_w \right] = -\phi \frac{dS_o}{dP_w} \frac{\partial p_w}{\partial t} \frac{\partial p_w}{\partial t}, \]  

(9)

\[ \nabla \left[ k \frac{k_o}{k_w} \nabla p_o \right] = \phi \frac{dS_w}{dP_o} \frac{\partial p_o}{\partial t} \frac{\partial p_o}{\partial t}. \]  

(10)

The initial and boundary conditions are considered for a physical problem in which the viscous forces have no effect and flow is purely capillary-driven. Consider a cylindrical core at irreducible water saturation, closed all around except one (two) end facet(s) that is (are) open to flow. First, consider countercurrent imbibition, in which the only open end is initially in contact with the oil at the ambient pressure, say, zero pressure. The water pressure in the core is given by the capillary pressure relationship, which at \( t = 0 \) leads to \( p_w = P_w(S_w) = P_S(S_w) \). Imbibition begins when the oil outside the core in the open end is replaced by water at the ambient pressure. If we assume \( P_S = 0 \) at this face, both oil and water pressures will be zero at the inlet, for \( t = 0^+ \). (This boundary condition is detailed later.) Water will imbibe into the core due to the low water pressure in the core, and oil flows out of the core from the same face. These conditions are represented by Eqs. 11–16.

\[ p_o = 0, \quad t = 0^+, \quad 0 \leq x \leq L, \]  

(11)

\[ p_w = 0, \quad t = 0^+, \quad 0 \leq x \leq L, \]  

(12)

\[ p_w = 0, \quad t = 0^+, \quad x = 0, \]  

(13)

\[ p_o = 0, \quad t = 0^+, \quad x = L, \]  

(14)

\[ q_o = 0, \quad t = 0^+, \quad x = 0, \]  

(15)

\[ q_w = 0, \quad t = 0^+, \quad x = L. \]  

(16)

For countercurrent imbibition, consider a situation in which the oil pressure at both ends of the core is initially fixed at zero, for example, by exposing it to oil at the ambient pressure. Water at the ambient pressure is then introduced at one end. Water will imbibe into the core and oil might be produced from one or two end faces depending on the inlet boundary condition, for which we consider either of the following two cases: zero-capillary pressure. Eq. 17a or zero-oil flow. Eq. 17b. The corresponding initial and boundary conditions are given by Eqs. 11–14 and 17 and 18:

\[ p_w = 0, \quad t = 0^+, \quad x = 0, \]  

(17a)

\[ q_o = 0, \quad t = 0^+, \quad x = 0, \]  

(17b)

\[ p_w = 0, \quad t = 0^+, \quad x = L. \]  

(18)

Note that the only difference in the mathematical description of co- and countercurrent imbibition, i.e., Eqs. 9–18, is related to the inlet and outlet boundary conditions of the oil phase. It will be pointed out later how this difference leads to the absence of a convective term in Eqs. 1 and 7, respectively.

In order to accurately incorporate the boundary conditions in the numerical model, the space discretization was performed using the control volume approach. For the base case, a 1D matrix block with absolute permeability of 20 md (0.02 μm²) and length of 20 cm (0.2 m) is considered. Oil and water viscosity values are 1 cp (1 mPa-s). The relative permeability and imbibition capillary pressure functions are expressed by

\[ k_r = k_{rw}(1 - S_w), \quad k_i = k_{iw} S_w, \]  

(19)

\[ P_o(S) = -B \ln(S), \]  

(20)

where \( S \) is the normalized saturation (see the nomenclature). The parameters \( k_{rw}, k_{iw}, n_o, n_w, \) and \( B_o \) are constant. Table 1 gives the values considered for the base case example.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>20 cm (0.2 m)</td>
</tr>
<tr>
<td>( k )</td>
<td>20 md (0.02 μm²)</td>
</tr>
<tr>
<td>( \mu_o )</td>
<td>1 cp (1 mPa-s)</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>1 cp (1 mPa-s)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.3</td>
</tr>
<tr>
<td>( B )</td>
<td>1.45 psi (10 kPa)</td>
</tr>
<tr>
<td>( \varepsilon_1 )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 1 – Data for the Base Case Example

Fig. 1 – Oil and water pressure vs. distance for 1D countercurrent imbibition.

4 M. Poochil-Darvish and A. Firoozabadi: Imbibition in Water-Wet Matrix Blocks

duction begins by the appearance of oil drops on the rock surface covered by water. Ref. 24 provides a vivid picture of the process in several figures. Since the diameter of water drops on the rock surface are much larger than the mean-pore radius, capillary pressure is nearly zero. Visual observations show that when the water level in the fractures around a matrix block is rising and oil is predominantly being produced from the faces in contact with the oil (i.e., countercurrent imbibition), some of the oil is expelled through the water-contacted surfaces.\textsuperscript{11,14} Hence, a similar argument to that of countercurrent imbibition would require \( P_i = 0 \) at the water-contacted surface (Eqs. 13 and 17a). Another plausible boundary condition is the no-oil flow boundary condition at the inlet, Eq. 17b. We have tested both of the boundary conditions in our numerical model and observed little difference in the recovery performance. For the sake of brevity, the results presented in the paper are based on \( P_i = 0 \) at the inlet, which is in line with our visual observations.\textsuperscript{15} The effect of the no-oil flow boundary condition is presented in Fig. 6 (to be discussed later).

**Validation.** The numerical model was validated against the analytical solution of countercurrent imbibition.\textsuperscript{2} The saturation and pressure profiles and the recovery (based on total recoverable oil in place, ultimate recovery would always be 1) from the analytical model are shown in Figs. 1, 2, and 5, and are compared with the numerical solution with 300 gridblocks. In all the figures, a close match is observed.

For comparison, the recovery curve with 50 gridblocks is shown in Fig. 5. The recovery at the early time is overestimated somewhat by the coarser grid, and the saturation plots (not shown) indicate a limited numerical dispersion. All the 1D results presented are performed using 300 gridblocks and the 2D calculations with 20\times 30 gridblocks. A small effect of numerical dispersion was observed in the 2D calculations due to the coarse grids.

**1D Results**

In this section, the behavior of the 1D countercurrent and cocurrent imbibition is illustrated using the numerical model described above.
Comparison of Figs 2 and 4 suggests that saturation profiles advance further in cocurrent imbibition compared with that in countercurrent imbibition, which shows the superiority of cocurrent over countercurrent imbibition. Cocurrent imbibition takes advantage of oil pressure gradient downstream of the front in the single-phase region, and results in an increased recovery rate.

Heuristically, there is a contribution of a convective term for cocurrent imbibition in Eq. 7. For cocurrent imbibition, due to the no-flow boundary conditions at the outlet, the convective term is zero everywhere, $q_i = q_w = 0$, and Eq. 7 simplifies to Eq. 1. Physically, in cocurrent imbibition, oil is produced downstream of the water front through the single-phase region, whereas in countercurrent imbibition, oil flows through the two-phase region, reducing oil recovery efficiency.

The recovery performance of co- and countercurrent imbibition is depicted in Fig. 6. If the residual oil saturations for co- and countercurrent imbibition are equal, as assumed here, recovery curves at very late time approach the same value—1. At earlier times, however, especially before the saturation front reaches the far boundary, there is a substantial difference between the two curves. For example, the half-recovery time for countercurrent imbibition is more than five times that of cocurrent imbibition.

We used the same relative permeability curves for co- and countercurrent imbibition. Several studies suggest that due to viscous coupling between the flowing phases, relative permeability curves for the two processes could be different. The recovery curve for countercurrent imbibition when relative permeability curves are reduced by 30%. Half-recovery time for countercurrent imbibition is then 32 days, compared with 4.5 days for cocurrent imbibition.

Fig. 6 also shows the contribution of oil recovery from the face in contact with water (the other face is in contact with oil). The contribution of the backflow production at a recovery of 80% is less than 5%, a large portion of the backflow recovery is obtained at a very early time. Very fine grid studies with small time steps indicate the early high backflow is a characteristic of the process. The small recovery from the inlet face suggests that if a
no-oil flow boundary condition is imposed (Eq. 17b), there will be only a small change in the recovery performance. This is clearly observed in Fig. 6.

The results presented above show that oil recovery by co-current imbibition is much faster than by countercurrent imbibition. The scaling studies of the following section will show that this conclusion is not limited to the data used for the base case example.

**Scaling Studies.** Rapoport\(^{20}\) presented the scaling criteria for two-phase incompressible flow through porous media. Using inspectional analysis of the differential equations of water/oil flow through porous media, he found that saturation distribution is a function of dimensionless time provided certain similarities are present. Rapoport’s dimensionless time is given by

\[
t = \frac{k_t}{\phi \mu_w L^2} \frac{\partial P_o}{\partial S_w} \frac{1}{S_w^*}.
\]

(21)

The parameter \(S_w^*\) is any characteristic saturation. In our calculation we used \(S_w^* = 1 - S_{sw}\), such that \(\partial P_o/\partial S_w|_{S_w^*=1} = \beta\). We varied the value of absolute permeability \(k\), length \(L\), derivative of the capillary pressure curve with respect to water saturation at \(S_w^*\), and water viscosity \(\mu_w\) (at a constant viscosity ratio). Some of the results are depicted in Fig. 7, which shows that the above scaling law applies to both co- and countercurrent imbibition, and all the data fall on a pair of curves. Results of other cases, not shown here, fall on the same pair of curves. The results also indicate that the contribution of the back-flow production is independent of length (and the other parameters), and remains at about 5% (not shown).

To compare the scale sizes of the two imbibition processes, a dimensionless time ratio is defined as the time ratio of countercurrent to co-current imbibition to achieve a specific recovery; the results are plotted in Fig. 8 by the solid line. At the very early time, the recovery performance of co- and countercurrent imbibition is similar, and the time ratio is equal to 1. The time ratio increases rapidly such that half-recovery time for countercurrent imbibition is more than five times that of co-current imbibition.

After the saturation front reaches the far boundary, at about 50% recovery for countercurrent imbibition, the recovery rate decreases drastically (see Fig. 7). For co-current imbibition, however, the saturation front does not reach the far boundary until about 70% recovery. Hence, the time ratio between the two processes increases sharply in this interval. Beyond this time, the recovery rate for countercurrent imbibition drops, and the time ratio decreases. Fig. 8 indicates that for the most part, oil recovery for co-current imbibition is more than four times faster than the countercurrent imbibition. Note that the data labeled as base case in Fig. 8 encompass all the variations shown in Fig. 7.

**Sensitivity Studies.** In the following, the effects of some of the variables that are not included in Rapoport’s scaling law are investigated. These include the oil and water relative permeability exponents, viscosity ratio, and initial water saturation. The effect of these parameters is most obvious on the saturation and pressure profiles (not shown here). When the water and oil exponents were individually reduced to 2, the saturation gradients at the water front and at the inlet were reduced, respectively. When the initial water saturation was increased to \(S_w = 0.2\), a tongue was organized at the leading edge of the saturation front. This behavior in countercurrent imbibition was previously studied by Barenblatt et al.\(^{2}\) The scaling study of the previous section suggests that a pair of recovery curves, similar to those of Fig. 7, can be presented for each of the above cases. These, of course, will be independent of the parameters included in Eq. 21. From such recovery calculations, co- and countercurrent imbibition are compared and the results are shown in Fig. 8. Again, a large difference between the two processes is observed. Among the parameters varied, the exponent of oil relative permeability has the largest effect (see the thin-dashed line in Fig. 8).

It is interesting to note that the time ratio at large recoveries approaches 4. All parameters being the same, a four-time permeability increase is required for co- and countercurrent imbibition to behave similarly at high recoveries. The high recoveries of Fig. 8 are at extremely low rates.
Length Effect. Fig. 4 indicates that the oil pressure gradient downstream of the front depends on the length of the core, seemingly suggesting that the superiority of cocurrent imbibition over countercurrent imbibition reduces as the length of the core increases. Here, we demonstrate that this conclusion is incorrect, and as shown in Fig. 7, the superiority is independent of the length. Fig. 9 shows the total oil produced for co- and countercurrent imbibition for a cross-sectional area of 1 cm² (0.0001 m²) and lengths of 20, 100, and 1000 cm (0.2, 1, and 10 m). Fig. 9 indicates that as the length of the core increases, oil production for cocurrent imbibition decreases at any time, and approaches that of countercurrent imbibition. In fact, for the time considered in Fig. 9, most of the oil is produced due to backflow when the core is 1000 cm (10 m) long. The answer is evident by realizing that for longer cores a longer time scale should be considered to achieve a fixed dimensionless time. The superiority of cocurrent vs. countercurrent imbibition for any specific rock and fluid property is a function of dimensionless time only (see Fig. 7 or 8). For the longer samples in Fig. 9, the large difference between co- and countercurrent imbibition will be observed at later times.

2D Results

In the previous section, co- and countercurrent imbibition were studied for a 1D geometry. In this section, we examine them in a 2D medium. The base case properties are used for a square porous medium [the dimensions are 20\times20 cm (0.2\times0.2 m) and very long]. The left and bottom faces are in contact with water at ambient pressure, and the top and right faces are either closed or in contact with oil at the ambient pressure for counter- and cocurrent imbibition, respectively.

Co- and Countercurrent Imbibition. Fig. 10 displays the recovery curve for the two processes. Similar to the 1D case, cocurrent imbibition is more efficient than countercurrent imbibition. At \( t = 4 \) days, oil recoveries for co- and countercurrent imbibition are 70 and 37%, respectively. Recovery due to backflow production for the cocurrent imbibition is about 5%. Fig. 11 depicts the time ratio for the two imbibition processes. Figs. 10 and 11 for 2D imbibition are similar to Figs. 7 and 8 for 1D imbibition with minor differences.

By varying the parameters in Eq. 21, we tested the scaling law of Rapoport\(^\text{10}\) in the 2D geometry. Similar results to the 1D case are obtained (not shown here). Thus, Figs. 10 and 11 can be used for other 2D systems, if the required similarities are met. Time in
Figs. 10 and 11 should then be replaced by the corresponding dimensionless values.

In order to study further the similarities and the differences between 1D and 2D imbibition, the saturation, pressure, and velocity profiles of the 2D imbibition were examined. Study of saturation profiles indicate that similar to the 1D case, there are high gradients at the inlet and at the front (results not shown). Water and oil pressures have high gradients at the front and at the inlet, respectively. Figs. 12 and 13 show oil velocity distributions for countercurrent and cocurrent imbibition, respectively. (The arrows in Figs. 12 and 13 show the velocity magnitude.) Oil velocity profiles, especially for the cocurrent process shown in Fig. 13, indicate that oil rates are higher where there is small distance between the water front and the open face. As the saturation front moves from the bottom to the top, oil is mostly produced from the right face. This is in contrast with the 1D problem, where oil is either produced from the inlet or has to flow the length of the core to be produced from the opposing end.

**Superposition.** Experimental and mathematical studies of 1D imbibition cannot be used for multidimensional predictions due to lack of geometric similarity, a requirement from the scaling law of Rapoport. Superposition of 1D solutions to 2D and three-dimensional solutions (3D) has been suggested to address this point, although superposition does not hold for nonlinear problems. Figs. 14 and 15 show the comparison between the recovery obtained from the superposition of 1D solutions to 2D with the results of 2D calculations for co- and countercurrent imbibition. The 1D and 2D calculations are performed using 50 gridblocks. These figures indicate that a good approximation is obtained at early times, however, the late-time behavior is significantly overpredicted by the superposition solution.

**Discussion**

In a water-wet naturally fractured reservoir, the matrix blocks experience an advancing fracture-water level (FWL). This study points out that so long as the parts of the rock surface are in
contact with the oil, oil recovery would be dominated by co-current imbibition. For such a matrix, it is easy to envision that the oil displaced by water adjacent to the FWL will be produced above the FWL into the oil zone, not below. The results of this study show that capillary imbibition leads to the production of oil, predominantly from the rock surface in contact with the oil, should such an open surface exist. For gravity-segregated waterdrive, such open surfaces may exist for a significant recovery period. For the zero-capillary pressure inlet-boundary condition, the percentage of the backflow recovery, i.e. the countercurrent component, was shown to be small and independent of length. (For the no-oil flow boundary condition at the inlet, backflow recovery would be zero.) It was also shown that countercurrent imbibition leads to faster oil recovery.

Simple calculations show that for some of the North Sea reservoirs, which exhibit small block size and high imbibition capillary pressure, a large portion of the oil is recovered from the matrix before the block becomes fully surrounded by water. Under these circumstances, the proper formulation of countercurrent imbibition should be considered. The above findings have important implications for the experimental studies of water injection in fractured reservoirs in that the scaling experiment of countercurrent imbibition, which is commonly used to evaluate water injection in fractured reservoirs, may lead to very pessimistic results. In a recent modeling study, two different time scales were needed to match the experimental and fine-grid simulation results of oil recovery from matrix blocks experiencing advancing FWI.

In fractured gas reservoirs it is possibly unimportant to distinguish between co- and countercurrent imbibition. Due to low gas viscosity, the gas-phase pressure gradient would be small and Eq. 1 with the diffusion coefficient of Eq. 8 would describe both co-current and countercurrent imbibition.

**Conclusions**

1. Modeling results presented in this paper reveal that when a water-wet porous media is partially in contact with water, oil recovery is dominated by co-current imbibition, not countercurrent. Moreover, co-current imbibition can be much more efficient than countercurrent imbibition, and the time for a specific recovery by countercurrent imbibition is a fraction of that by countercurrent imbibition. Further studies are warranted for the study of co-current and countercurrent imbibition in a multiblock stack and for field applications.

2. The scaling criterion of Rapoport is valid for both co- and countercurrent imbibition. However, oil recovery calculations based on scaling studies of countercurrent imbibition, when used for a countercurrent process, lead to pessimistic recovery predictions.

3. The diffusion equation may be inappropriate for the description of oil recovery by countercurrent imbibition; the oil pressure gradient must be included.

**Nomenclature**

**Latin Letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>capillary pressure constant, psi (kPa), m/L$^2$</td>
</tr>
<tr>
<td>$D$</td>
<td>diffusion coefficient, cm$^2$/s (m$^2$/s), L$^2$/t</td>
</tr>
<tr>
<td>$L$</td>
<td>length, cm (m), L</td>
</tr>
<tr>
<td>$P_c$</td>
<td>capillary pressure, psi (kPa), m/L$^2$</td>
</tr>
<tr>
<td>$S = \frac{S_w - S_{iw}}{1 - S_{or} - S_{iw}}$</td>
<td>normalized water saturation, dimensionless</td>
</tr>
<tr>
<td>$S_i$</td>
<td>normalized initial water saturation, dimensionless</td>
</tr>
<tr>
<td>$S_w$</td>
<td>water saturation, dimensionless</td>
</tr>
<tr>
<td>$S_{sat}$</td>
<td>initial water saturation, dimensionless</td>
</tr>
<tr>
<td>$S_{iw}$</td>
<td>irreducible water saturation, dimensionless</td>
</tr>
<tr>
<td>$f$</td>
<td>fractional flow, dimensionless</td>
</tr>
<tr>
<td>$k$</td>
<td>permeability, md ($\mu$m$^2$), L$^2$</td>
</tr>
<tr>
<td>$n$</td>
<td>relative permeability exponent, dimensionless</td>
</tr>
<tr>
<td>$q$</td>
<td>flow rate, cm$^3$/s (m$^3$/s), L$^3$/t</td>
</tr>
<tr>
<td>$t$</td>
<td>time, hour, second, t</td>
</tr>
</tbody>
</table>

**Greek Letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>porosity, dimensionless</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity, cp (mPa-s), m/Lt</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>gradient operator</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>divergence operator</td>
</tr>
</tbody>
</table>

**Subscripts**

$D$ = dimensionless
\[ \sigma = \text{oil} \]
\[ \sigma_r = \text{residual oil} \]
\[ r = \text{relative} \]
\[ w = \text{water} \]

**Superscripts**

\[ 0 = \text{end-point value} \]

**Acknowledgments**

This research project was supported by U.S. DOE Grant No. DE-FG22-96BC14850 and the members of the Research Consortium on the Fractured/Layered Reservoirs of the Reservoir Engineering Research Institute (RERI). This support is gratefully appreciated.

**References**


**SI Metric Conversion Factors**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>cp</td>
<td>(10^4)</td>
</tr>
<tr>
<td>darcy</td>
<td>(10^9)</td>
</tr>
<tr>
<td>psi</td>
<td>(10^6)</td>
</tr>
</tbody>
</table>

*Conversion factor is exact.*

**Mehran Poolad-Darvish** is an assistant professor of petroleum engineering at the U. of Calgary, Alberta, Canada. e-mail: poolad@ucalgary.ca. His current research interests include experimental and modeling studies of cold production of heavy oil, water imbibition and miscible displacement in naturally fractured reservoirs, and gas production from hydrate reservoirs. He previously did research at the Reservoir Engineering Research Inst. In Palo Alto, California; was a consultant in Calgary; and was a reservoir engineer in Ahvaz, Iran. Mehran holds a B.S degree in chemical engineering, an M.S degree in chemical and petroleum engineering, and a PhD degree from the U. of Alberta in petroleum engineering. Abbas Firoozabadi is a senior scientist and director at the Reservoir Engineering Research Inst. In Palo Alto, e-mail: afr@resinst.org. His current research interests include multiphase flow in fractured and layered media, equilibrium and nonequilibrium thermodynamics, and hydrocarbon reservoir performance. Firoozabadi holds a BS degree from Abadan Inst. of Technology, Iran, and MS and PhD degrees from Illinois Inst. of Technology, all in gas engineering. He has served on the Editorial Review Committee since 1986 and was a 1988–89 member and 1992–93 chairman of the Forum Series in North America Steering Committee, a 1991–92 member of the Western Regional Meeting Program Committee, a 1987–89 member of the Reservoir Simulation Symposium Program Committee, and a 1992–96 Short Course Instructor.