Crossflow in Fractured/Layered Media Incorporating Gravity, Viscous, and Phase Behavior Effects

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Crossflow across the interfaces of layered reservoirs, and between matrix blocks and fractures in fractured reservoirs may be very pronounced. The literature emphasizes viscous crossflow, however, crossflow due to gravity may be more significant.

In this paper, the crossflow due to various effects in layered and fractured media are presented. The examples selected for the layered media show that most of the oil is first transferred from the low permeability to the high permeability layer and then produced. The examples for fractured media reveal a significant contribution of crossflow to recovery performance. The fractured-media examples imply that a real challenge exists to perform compositional simulation of fractured reservoirs.

Introduction

Crossflow of fluids between a fracture network and rock matrix in fractured reservoirs, and between different layers in layered reservoirs can be very pronounced in displacement processes. The crossflow could be mainly due to: 1) gravity, 2) capillarity, 3) viscous, 4) phase behavior and compressibility, and 5) diffusion effects. As an example, for certain water injection processes in fractured porous media, due to capillary crossflow, the injected water at the water-oil interface in the fracture imbibes into the matrix and the oil is produced just above the interface. On the other hand, in certain high pressure gas injection processes, phase behavior effects reduce the surface tension significantly and, therefore, the capillary crossflow can be neglected.

In a recent work [1], we have demonstrated that the crossflow of the injected fluid from the fracture to the matrix and the crossflow of the matrix oil to the fracture strongly affect the recovery efficiency. Surprisingly, the crossflow due to gravity [2] is the dominant mechanism for high displacement efficiency for first contact miscible fluid injection in fractured porous media. The investigation of crossflow at conditions other than first contact miscibility is one major goal of this work.

Crossflow in both layered and fractured porous media can be a localized phenomena. With a substantial phase behavior effect between the injected gas and the in-place oil, sharp fronts may give amplified numerical dispersion. The numerical dispersion may mask qualitative and quantitative features of the solution and may make the interpretation difficult. The small fracture pore volume (PV) and the high contrast between the matrix and fracture permeability adds to the complexity of the use of a finite difference simulator. An analytical model would be an ideal tool for the study of flow and composition paths in layered and fractured media. In this work, we use the method of characteristics to solve for fluid flow problems by assuming that capillarity and diffusion are negligible. The assumption of negligible capillary pressure is justified for flow at high pressures and for near-miscible conditions where phase behavior effects are important and the surface tension is very low. The method of characteristics is mainly restricted to one-D and a small number of components, but provides a mechanistic understanding of the problem.

In an early paper, Zapata and Lake [3] studied viscous crossflow in layered media for immiscible fluid systems. Pandit and Orr [4] also studied viscous crossflow in a two-layer system incorporating transfer of components between gas and liquid phases but neglected gravity and volume change due to mixing. In a recent study, Tan and Firoozabadi [2] included the effect of gravity in the study of crossflow between a matrix medium and a fracture medium. In all these studies, the effect of volume change on mixing was neglected. When an injected gas dissolves in the oil phase, the volume of the total system may reduce significantly. The subject of viscous crossflow has received considerable atten-
CROSSFLOW IN FRACUTRED/LAYERED MEDIA INCORPORATING GRAVITY, VISCIOUS, AND PHASE BEHAVIOR EFFECTS

Crossflow due to viscous forces is generally not very significant. Gravity contribution to crossflow, on the other hand may be pronounced for fractured reservoirs, and for layered reservoirs with tilt angles in the range of 5-15°. Phase behavior effects which include component transfer and volume change due to mixing and incorporation of these effects in the crossflow term may also be important.

This study addresses a comprehensive examination of crossflow in fractured and layered media incorporating gravity, compressibility, and phase behavior, and viscous effects. In the following, we will first present the problem formulation, and then outline the solution method. The examples of gas injection in two-layer and fractured media will be discussed, and at the end, conclusions will be drawn from the work.

Problem Formulation

The introduction of volume change on mixing complicates considerably the evaluation of the crossflow term, since the sum of flow rates in the layers may change with position. In order to better understand this term, the problem formulation will be presented in two steps. First, flow formulation will be discussed for a homogeneous medium and then in a system with two distinct media consisting of a fracture and a matrix or two layers.

Homogeneous-Medium System. A homogeneous medium can be represented with one set of properties (permeability, porosity, relative permeability, etc.). The governing component molar balance equations for one-dimensional multiphase multicomponent systems are given by,

$$A_i \frac{\partial G_i}{\partial t} + \frac{\partial (\rho_i F_i)}{\partial x} = 0 \quad i = 1, \ldots, n_c \quad (1)$$

where,

$$G_i = G_i(Z) = \sum_{j=1}^{n_p} \chi_{ij} \rho_j S_j \quad i = 1, \ldots, n_c \quad (2)$$

and

$$F_i = F_i(Z, q_T) = \sum_{j=1}^{n_p} \chi_{ij} \rho_j q_T \quad i = 1, \ldots, n_c \quad (3)$$

$G_i$ and $q_T F_i$ represent the number of moles of component $i$ per unit pore volume and molar flow rate of component $i$, respectively. Other symbols are defined in the Nomenclature. The flow rate $q_T$ (in Eq. 1) may not be constant due to volume change on mixing even when the total injection rate at the inlet is held constant. Note that the assumptions of, 1) chemical equilibrium, 2) isothermal flow, 3) negligible capillary and dispersion effects are made in the derivation of Eq. 1. Also note that the thermodynamic properties of the fluid are evaluated at a constant temperature and pressure; this is a good assumption as long as the pressure drop across the system is small.

The boundary and initial overall composition data are given by,

$$\tilde{Z}(x, 0) = \begin{cases} \tilde{Z}_{i,m} & x \leq 0 \\ \tilde{Z}_{i,1} & x \geq 0 \end{cases} \quad (4)$$

The two constant compositions are injection composition, $\tilde{Z}_{i,m}$, and initial composition, $\tilde{Z}_{i,1}$.

The solution of the above Reimann problem with constant injection rate and constant injection and initial compositions is a function of a similarity variable, $z/t$ [5,6]. The solution consists of continuous variations, shocks and constant states. The shock speed, $\Lambda$, is given by [5,6],

$$\Lambda = \frac{(\gamma_T F_i)^{1/2} - (\gamma_T F_i)^{1/2}}{A_i \phi_i (G_i^T - G_i)} \quad i = 1, \ldots, n_c \quad (5)$$

where the superscript $I$ and $II$ refer to opposite sides of the shock, downstream and upstream, respectively. Eq. 5 can be obtained by a material balance across the shock [5,6]. Dupont, Hagee, and Rissereau [7] give a detailed account of the one-D analytical model for a three-component system for a homogeneous medium.

Two-Media System. Consider a two-media flow - one medium, low permeability and the other medium, high permeability. Each medium could represent a layer in layered-media or a fracture or a matrix in fractured media. Two cases are considered; one case in which the two media communicate (crossflow case), the other case in which there is no crossflow between the two media (no-crossflow case). In the following, both cases are discussed.

Crossflow Case. For this case, the problem becomes significantly simplified if we invoke the powerful crossflow equilibrium (CE) assumption between the two media [3,9,10]. Crossflow equilibrium means that the pressures in both media are equal along the displacement length. This is an appropriate assumption for geometries with $R_w = L/H\sqrt{k_w/k_h} > 10$, where $k_w$ is thickness-weighted harmonic vertical permeability and $k_h$ is thickness-weighted average horizontal permeability [3]; L and H are the total system length and the total system thickness, respectively. Crossflow equilibrium is a key assumption and allows the elimination of the pressure dependency of the problem and solving the system of partial differential equations analytically. The validity of this assumption has been demonstrated in Ref. 10 for fractured media. In that work, the results from fine grid simulation and the analytical model for fractured media comprised of a vertical matrix block and a vertical fracture were compared at two different rates.

The material balance equations for the two-media system are obtained by including the crossflow (source/sink) term to the equations for the homogeneous system. The crossflow term represents the transverse communication of the two layers.

Crossflow Term. The molar flow equations for the two layers are given by

$$\Lambda_1 \phi_1 \frac{\partial G_1}{\partial t} + \frac{\partial (\rho_1 F_1)}{\partial x} + CR_{1m} = 0 \quad i = 1, \ldots, n_c \quad (6)$$

$$\Lambda_2 \phi_2 \frac{\partial G_2}{\partial t} + \frac{\partial (\rho_2 F_2)}{\partial x} - CR_{2m} = 0 \quad i = 1, \ldots, n_c \quad (7)$$

where $G_i^T = \sum_{j=1}^{n_p} \chi_{ij} \rho_j^T S_j$, $F_i^T = \sum_{j=1}^{n_p} \chi_{ij} \rho_j^T q_T$, $CR_{1m}$ is the molar crossflow flux of component $i$ from layer 1 to layer 2 (per length), and $k = 1, 2$ is the layer superscript; $k = 1$ is the more permeable medium and $k = 2$ is the less permeable...
medium. For crossflow from layer 2 to 1, the sign of the last term in Eqs. 6 and 7 is reversed. The relationship between molar crossflow flux of component i and volumetric crossflow flux, CRTV, is given by

$$CR_{TV} = F_i^1 CR_{TV} , \quad i = 1, \ldots, n_c$$  \hspace{1cm} (8)

when crossflow is from medium 1 to medium 2. Combining Eqs. 6 and 7 and summing over all the components yields,

$$CR_{TV} = -\frac{1}{n_c} \sum_{i=1}^{n_c} \left[ A^i \frac{\partial G_i^1}{\partial t} + \frac{\partial (Q_i F_i^1)}{\partial x} \right]$$  \hspace{1cm} (9)

Note that the crossflow term given by Eq. 9 depends not only on $\partial Q_i^1/\partial x$ but also varies with $\partial G_i^1/\partial t$ and $\partial F_i^1/\partial x$. These last two terms are due to the volume change on mixing.

Rewriting the molar flow Eqs. 6 and 7 in dimensionless form with the expression of crossflow from Eq. 9,

$$R^1 \frac{\partial G_i^1}{\partial t} + \frac{\partial (Q_i F_i^1)}{\partial x} = \sum_{i=1}^{n_c} \left[ R^1 \frac{\partial G_i^1}{\partial t} + \frac{\partial (Q_i F_i^1)}{\partial x} \right] = 0, \quad i = 1, \ldots, n_c - 1$$  \hspace{1cm} (10)

$$R^2 \frac{\partial G_i^2}{\partial t} + \frac{\partial (Q_i F_i^2)}{\partial x} = \sum_{i=1}^{n_c} \left[ R^2 \frac{\partial G_i^2}{\partial t} + \frac{\partial (Q_i F_i^2)}{\partial x} \right] = 0, \quad i = 1, \ldots, n_c$$  \hspace{1cm} (11)

where

$$G_i^1 = G_i^1(Z^1) = \sum_{k=1}^{n_p} \lambda_i^k \rho_j^k D_j^k, \quad i = 1, \ldots, n_c, \quad k = 1, 2$$  \hspace{1cm} (12)

$$F_i^k = F_i^k(Z^1, Z^2, q_{TD}^k) = \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j^k f_j^k, \quad i = 1, \ldots, n_c, \quad k = 1, 2$$  \hspace{1cm} (13)

The dimensionless variables and parameters are defined by

$$x_D = x / L, \quad t_D = \frac{\varphi_{TD} t_D}{A^k}, \quad q_{TD}^k = \frac{q_{TD}^k}{\varphi_{TD} t_D}, \quad \rho_j = \rho_j / \rho_{mi}$$

$$R^k = \frac{\varphi A^k}{\varphi A^T}, \quad \varphi = \sum_{k=1}^{n_k} A^k, \quad A^T = \sum_{k=1}^{n_k} A^k$$  \hspace{1cm} (14)

where $L$ is the system length.

The total mass balance for component i is the sum of Eqs. 10 and 11,

$$\frac{\partial C_i^T}{\partial t_D} + \frac{\partial F_i^T}{\partial x_D} = 0, \quad i = 1, \ldots, n_c - 1$$  \hspace{1cm} (15)

where $G_i^T = R^1 G_i^1 + R^2 G_i^2$, and $F_i^T = \frac{\partial F_i^1}{\partial x} + \frac{\partial F_i^2}{\partial x}$. The total molar balance of all the components can be obtained by summing Eq. 15 from i = 1 to $n_c$,

$$\frac{\partial C_i^T}{\partial t_D} + \frac{\partial F_i^T}{\partial x_D} = 0$$  \hspace{1cm} (16)

where $F^T = \sum_{i=1}^{n_c} F_i^T$, and $G^T = \sum_{i=1}^{n_c} G_i^T$. We then solve Eqs. 10, 15, and 16 instead of Eqs. 10 and 11. Similar to the homogeneous system, the initial and boundary conditions given by Eq. 4 apply to both media of the two-media system. The total flow rate in layer k, $q_{TD}^k$, can be related to the total flow rate in layer i, $q_{TD}^i$, $Z^k$ and $Z^i$ using the crossflow equilibrium assumption, $dP^i/dx_D = dP^k/dx_D$. From the Darcy law, the pressure gradient of each medium is given by

$$\frac{dP^k}{dx_D} = -\frac{q_{TD}^k}{A^k \lambda_k^2} - g L \sin \theta \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j^k, \quad k = 1, 2$$  \hspace{1cm} (17)

Using the crossflow equilibrium,

$$q_{TD}^k = C_R \frac{L}{\lambda_k^2} \left[ q_{TD}^i + \frac{A^k g \sin \theta}{\lambda_k^2} \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j^k \right]$$

$$-\frac{A^k g \sin \theta}{\lambda_k^2} \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j^k, \quad k, l = 1, 2, \quad k \neq l$$  \hspace{1cm} (18)

where the capacitance ratio, $C_R$, is defined as $C_R = A^k / A^k$. Eq. 18 can be used to calculate $q_{TD}^k$ once $q_{TD}^i$ (l = k), $Z^k$, and $Z^i$ are known.

No-Crossflow Case. The flow equations for the no-crossflow (NC) case are similar to the equations for the CE case (Eqs. 10 and 11) without the crossflow term. In NC, the rates of the two media are related through the pressure drop boundary condition across the displacement length, $0 \leq x_D \leq L$.

$$\Delta P^1 = \Delta P^2$$  \hspace{1cm} (19)

The pressure differences, $\Delta P^1$ and $\Delta P^2$ can be obtained by integrating the Darcy equation using Eq. 17.

$$\int_{x_D=0}^{1} dP^k = -\frac{q_{TD}}{A^k \lambda_k^2} \int_{x_D=0}^{1} q_{TD} \frac{Z_{TD,i}}{\lambda_k^2} dG_D +$$

$$-g L \sin \theta \int_{x_D=0}^{1} \frac{\lambda_{ij}^k \rho_j}{\lambda_k^2} dG_D, \quad k = 1, 2$$  \hspace{1cm} (20)

Combining Eqs. 19 and 20 with the constant total injection rate condition at the inlet, $q_{TD,ini} = q_{TD,ini}^1 + q_{TD,ini}^2$, yields

$$\frac{q_{TD,i}^1}{q_{TD,ini}} = \left\{ \frac{A^k g \sin \theta}{\lambda_k^2} \left[ \int_{x_D=0}^{1} \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j dG_D - \right.ight.$$  

$$\left. \int_{x_D=0}^{1} \sum_{j=1}^{n_p} \lambda_{ij}^k \rho_j dG_D \right] + \frac{A^k}{A^k \lambda_k^2} \int_{x_D=0}^{1} q_{TD} dG_D \} /$$  

$$\left. \frac{A^k}{A^k \lambda_k^2} \int_{x_D=0}^{1} q_{TD} dG_D \right.$$

$$= \left\{ \int_{x_D=0}^{1} \frac{q_{TD}}{\lambda_k^2} dG_D + \frac{A^k}{A^k \lambda_k^2} \int_{x_D=0}^{1} q_{TD} dG_D \right. \}$$  \hspace{1cm} (21)

Eq. 21 indicates that the individual medium injection rate can vary as displacement process continues. Dependence of rate to other parameters (e.g. layer compositions) is implicit.
However, Eq. 21 can be solved explicitly for \( q_{1D,inp} \) over a time step. In the explicit scheme, the parameters at the right side of Eq. 21 are calculated from the old time step. Composition profiles of each layer will still be similar to the profiles of a homogeneous system. Each layer is treated separately at a given time step, and solution is obtained using the same procedure as for the homogeneous systems. The details of the solution procedure for a homogeneous system will not be presented here, and can be found elsewhere [8].

**Solution Technique**

The method of characteristics is used to solve the one-dimensional flow equations with Riemann initial data [11]. We selected the system given by Eqs. 10, 15, and 16 with the initial data and boundary conditions of Eq. 4 and the crossflow equilibrium condition given by Eq. 15. Using the chain rule of differentiation for \( G_i(Z) \) and \( F_1(Z, Z', q_{1D}) \), Eqs. 10, 15, and 16 can be written as

\[
([F] - \lambda_0[G]) \dot{X} = 0.
\]

Eq. 22 is an eigenvalue problem where \( \lambda_0 \) is the eigenvalue and \( \dot{X} \) is the eigenvector of the system. \([G]\) and \([F]\) contain the derivatives of the various terms. Explicit expressions for the entries of Eq. 22 for a binary mixture are presented later.

For a given point in the rate-composition space, continuous variations can be obtained by integrating along the eigenvectors of Eq. 22. The integration along an eigenvector requires a starting rate-composition vector, \( \dot{U}_0 \). The solution procedure is as follows: 1) calculate all the eigenvalues, \( \lambda \), and the associated eigenvectors, \( \dot{X} \), 2) take sufficient small steps, \( s \), along a selected eigenvector to update the starting rate-composition, \( \dot{U}_n = \dot{U}_0 + s \dot{X} \). The full composition path (if permitted) can be obtained by using the same relationship recursively as \( \dot{U}_n = \dot{U}_{n-1} + s \dot{X} \). Details for the case of no-volume change on mixing are given in Ref. 2.

Discontinuities. Eqs. 10, 15, and 16 or the system of equations given by Eq. 22 may yield multivalued solutions. When the transport equations fail to describe physically correct single-valued flow, shocks (i.e., discontinuities) are introduced in the solution. Shocks do not satisfy Eq. 22, but they satisfy mass conservation across them. The shock balance equation with crossflow from medium 1 to medium 2 is derived in Appendix A. Eq. A-5 of that appendix corresponds to Eq. 10. The shock balances without the crossflow term corresponding to Eqs. 15 and 16 are [11,12],

\[
\Lambda_n = \frac{(F^n_1)^I - (F^n_1)^I}{(G^n_1)^I - (G^n_1)^I}, \quad i = 1, \ldots, nc - 1
\]

and,

\[
\Lambda_n = \frac{(F^n_1)^I - (F^n_1)^I}{(G^n_1)^I - (G^n_1)^I},
\]

which are similar to the shock balances of the homogeneous systems, i.e., Eq. 5. Different symbols for the shock speeds are used to distinguish the right hand sides of the shock balances. The shock speeds, \( \Lambda_{n}^D \), (see Eq. A-4), \( \Lambda_{n}^D \), and \( \Lambda_{n}^T \) are all equal to each other. In general, Eqs. A-4, 23, and 24 are coupled and their solution can only yield a maximum of \( 2nc - 1 \) unknowns. Depending on the information at the upstream and downstream of the shock, the shock equations could be coupled with other equations.

The above formulation along with method of characteristics solution procedure is for systems with any number of components. However, as the number of components increases, the degrees of freedom in the composition space also increase, and the solution construction becomes very complicated. In this paper, we restrict our study to binary systems. Even for a binary mixture, the solution usually includes more than two shocks and can become complicated as we will witness next. The shocks with intermediate speeds (intermediate shocks) are generally difficult to resolve, and require iterative schemes. As the qualitative features of the solution change, the solution construction algorithm needs to be modified. Solution of flow systems with volume change on mixing needs an extra level of trial and error procedure, because the total flow velocity in front of the fastest shock is also unknown.

**Binary Flow and Examples**

The material balance equations given by Eqs. 11, 15, and 16 can be written for two components, \( nc = 2 \). The associated eigenvalue problem and the solution procedure are presented in Appendix B.

The fractional flow equation for the gas phase without capillary pressure is given by

\[
f_\theta = \frac{1 - N_{g,\theta} k_{p,\theta}}{1 + N_{g,\theta} k_{p,\theta}}
\]

where \( N_{g,\theta} \) is the gravity number defined by

\[
N_{g,\theta} = \frac{k_{\theta} A_{\theta} (M_{\theta}^2 p_{\theta}^2 - M_{\theta}^2 p_{\theta}^2) \sin \theta}{\mu_{\theta} \rho_{\theta}}
\]

Note that the fractional flow function of Eq. 25 depends on gravity and the total flow rate of the medium. Consequently, the crossflow (see Eq. 9) depends on gravity and the total flow rate of the medium. Due to this dependence, the fractional flow function becomes an implicit function of the composition of both media as can be seen from Eq. 13 and the definition of \( F_s \). This results in the complication of the numerical computations. Gravity number defined by Eq. 26 is a function of time and position. However, gravity number at the inlet is constant (the medium injection rate is constant due to equal pressure gradients in the two media). The flow rate in medium 1 is \( k_{\theta} A_{\theta} / (k_{\theta} A_{\theta}^2 + k_{\theta} A_{\theta}^2) \) and in medium 2 is \( k_{\theta} A_{\theta}^2 / (k_{\theta} A_{\theta}^2 + k_{\theta} A_{\theta}^2) \). At a given pressure and a fixed geometry, \( N_{g,\theta} \) is proportional to \( \sin \theta / \rho_{\theta} \); therefore, tilt angles as small as 5° to 15° could have a substantial effect on gravity number (\( \sin 5° \approx 0.09, \sin 15° \approx 0.26 \), and \( \sin 90° = 1 \)). Note that the inlet gravity number of both media are the same.

In Eqs. 23 and 26, phase viscosities can be calculated using equilibrium phase compositions from Lohrenz et al. [15] correlation, and the gas and liquid phase relative permeabilities can be obtained from

\[
k_{r,\theta} = k_{\theta} \left[ \frac{S_\theta}{1 - S_\theta} \right]^{n_\theta}
\]
and
\[ k_{it} = k_p^* \left[ 1 - S_g - S_{cr} \right]^{n_s} \]  
(28)

Relative permeabilities are a function of the gas saturation, \( S_g \). For two-phase equilibrium conditions,
\[ S_g = \frac{V}{V + (\rho_f/\rho_l)(1 - V)} \]  
(29)

where \( V \) is the mole fraction of the gas phase given by,
\[ V = \frac{Z_i - z_{ii}}{z_{ig} - z_{ii}} \]  
(30)

After establishing phase behavior and flow relations, the entries in the matrices of Eq. B-1 can be calculated. Partial derivatives are performed numerically due to implicitness of the phase behavior and flow relations. The numerical derivatives are calculated using the central difference formulation. For a generic function \( H \),
\[ \frac{\partial H}{\partial Z_i} = \frac{H (Z_i^1 + \Delta Z_i, Z_i^2, Z_i^2 + \Delta Z_i, \Delta Z_i) - H (Z_i^1 - \Delta Z_i, Z_i^1, Z_i^2, \Delta Z_i)}{2 \Delta Z_i} \]  
(31)

To illustrate the significance of crossflow in layered and fractured media, we study gas injection in the following examples.

Layered Media Examples. For the layered media, a permeability contrast of 10 is chosen. Layer 1 has a permeability of 100 md (high permeability), and the permeability of layer 2 is 10 md (low permeability). Table 1 provides other relevant layer data and the parameters of the relative permeability. The porous media are initially saturated with normal decane (C10) and methane (CH4) is injected into the system. The temperature is fixed at 100°F. Flow at two pressures, 1000 and 4000 psi, is investigated. At 1000 psi, the surface tension at the interface between the gas and liquid phases is high and the capillary pressure may not be negligible. Nevertheless, to use the model, we will assume that capillary pressure can be neglected. For a binary system according to the Gibb's phase rule, the composition in the two-phase region is fixed at specified temperature and pressure. The results for four examples are presented next.

**Displacement at 1000 psi without gravity.** We consider first horizontal displacement at 1000 psi. The fractional flow function for \( \theta = 0 \) (or \( N_r^a = 0 \)) is independent of rate (see Eq. 25). The composition profiles are shown in Fig. 1a at \( t_D = 0.25 \) PV injection. Fig. 2 is the enlarged version of the Fig. 1a around the upstream edge of the composition profiles. Variation of flow rates in each layer is shown in Fig. 3a. Note that the sum of the flow rates of the two layers is significantly less than one, especially for \( P_D > 0.5 \). Figs. 4a and 5a show the saturation profiles and the recoveries of each layer, respectively. Two different types of recovery curves are plotted in Fig. 5. One is based on the remaining in-situ amount of C10, and the other one is based on the layer effluent production. The total pore volume produced is the same for both plots. The two types of plots highlight the contribution of crossflow from one layer to the other layer. Fig. 5a shows that there is very little crossflow in the absence of gravity.

At the injection point (H), see Fig. 2, solution starts with a phase change shock into the two-phase region along the tie line. The slowest shock, trailing shock, is the one where the

<p>| TABLE 1—ROCK AND FLUID PROPERTIES—LAYERED MEDIA |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Layer 1</th>
<th>Layer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi ) (frac)</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>A (cm²)</td>
<td>7432</td>
<td>7432</td>
</tr>
<tr>
<td>k (Darcy)</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>( k_{rg} )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( k_{rt} )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( S_{cr} )</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>( n_g )</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( n_l )</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
composition of the low permeability layer (layer 2) enters into the two-phase region (H–G shock – see Fig. 2). The upstream side of this shock is a single-phase gas (point H) and the downstream side is in two-phase (point G). Solution by a finite difference simulator revealed that the crossflow is from layer 2 to layer 1 at the trailing shock. The material balance equations for the trailing shock are given by Eqs. A-4, 23, and 24. In the shock balance equations, the upstream composition (superscript I) of the trailing shock is known from the injection condition (point H). The downstream side compositions, Z_{1}^G and Z_{2}^G, and the rate, \( q_{T, D}^G \), are the only unknowns. However, the shock balances, Eqs. A-4, 23 and 24 constitute two independent equations, since \( \lambda_{D, I}^G = \lambda_{D}^G = \lambda_{D}^G \). The trailing shock is a limit of continuous variation at which the downstream side of the shock moves with the speed of downstream composition. Therefore, the three independent equations are:

\[
\lambda_{D, I}^G = \lambda_{D}^G = \lambda_{D}^G = \lambda_{D}^G.
\]  

(32)

The above system of equations are nonlinear in unknowns, \( Z_{1}^G, Z_{2}^G, \) and \( q_{T, D}^G \). They can be solved using the multidimensional Newton-Raphson method. In Eq. 32, \( \lambda_{D}^G \) is the eigenvalue associated with layer 2 at point G. For flow without gravity, \( \lambda_{D}^G \) equals to the tie-line eigenvalue in layer 2,

\[
\lambda_{D}^G = \frac{2 \rho_{D}^G d_{T, D}^G}{R_{2} d_{S}^G}.
\]  

(33)

At the downstream of trailing shock the solution path continues along the direction of the eigenvector associated with the slow eigenvalue (zone of continuous variation). The end point of this zone of continuous variation (point F) is not known a priori. Therefore, this zone of continuous variation is extended to a point that layer 1 enters into two-phase region (point E). At this stage, the location of this point is unknown. In layer 1, the shock that enters into two-phase region will be called the trailing (slow) intermediate shock. The trailing intermediate shock is defined by Eqs. A-4, 23 and 24, where point F is the upstream and point E is the downstream. From the finite difference solution, the crossflow is found to be from layer 1 to layer 2 at that shock. However, crossflow changes direction at the downstream of the trailing intermediate shock. The trailing intermediate shock is also a limit of continuous variation such that,

\[
\lambda_{D, I}^{E, F} = \lambda_{D}^{E, F} = \lambda_{D}^{E, F} = \lambda_{D}^{E}.
\]  

(34)

where \( \lambda_{D}^{E} \) is obtained from Eq. 33. The nonlinear system of equations given by Eq. 34 yield the composition of the trailing intermediate shock (E–F shock). From the intermediate shock towards the downstream, there is an intermediate zone of continuous variation which terminates at the point that the low permeability layer (layer 2) enters into the single-phase liquid region via a shock (leading intermediate shock). The location and the composition of that shock are also unknown. In other words, the intermediate zone of continuous variation (DE) is bounded between two intermediate shocks which are unknown. We cannot guess the upstream composition of the leading intermediate shock on this intermediate zone of continuous variation (DE) which comes from the guessed point on the trailing intermediate shock. Therefore, we need to change the direction in our solution algorithm. The solution could also be initiated from the initial composition point (point A, see Fig 1a). The initial composition (single-phase liquid) is connected to the two-phase region via a phase change shock (A–B shock) similar to the trailing shock. Since that shock is the fastest, it will be called the leading shock. As the leading shock and the zone of continuous variation (leading), the crossflow is from layer 1 to layer 2. Only the shock balance equation with the crossflow term will be different for the leading shock. The shock balance equations are the same as Eq. 34 except that,

\[
\lambda_{D}^{E} = \frac{q_{T, D}^{E} d_{T, D}^{E}}{R_{1} d_{S}^{E}}.
\]  

(35)

Solution of the systems in Eq. 34 with Eq. 35 for A–B shock is similar to the solution of trailing shock equations. The leading intermediate shock occurs due to change in the number of phases in layer 2. In systems with no crossflow, this shock corresponds to the leading shock in the slow layer. The details of the solution are given in Appendix C.

Figs. 1e, 6a, and 7a show the composition profiles, rate history, and recovery curves in the absence of crossflow (i.e., NC). Comparison of the composition profiles of solutions with and without crossflow (see Figs. 1a and 1e) reveals that there is a mild crossflow between the leading intermediate shock and the leading shock. The leading intermediate shock of CE solution corresponds to the fastest shock of NC solution in layer 2, and the leading shock of CE solution corresponds to the fastest shock of NC solution in layer 1. As can be seen in NC solutions (Fig. 1e), CH₄ is not present in layer 2 ahead of the leading shock, whereas due to crossflow there is some CH₄ present in the CE solution.

Figure 2—Close look at the composition profiles at t₀ = 0.25 PV for CH₄-C₁₀ displacement (with crossflow) at 1000 psi and NGR = 0 layered media.

Displacement at 1000 psi with gravity. In this example, we consider the downward injection of CH₄ at a rate less than the maximum gravity drainage rate of layer 2. The maximum gravity drainage rate of layer 2 is calculated from

\[
q_{gr} = \frac{k_{2} \mu_{2}}{\rho_{2} g} \sin \theta / \mu_{2}.
\]

We assign a gravity
number (see Eq. 26) of 17.9 to this problem (gravity number is the same for both layers) which corresponds to an injection rate of \( \frac{dP_{vz}}{dz} = 0.614 \frac{dP}{dz} \); at such a low injection rate (combination of large \( \theta \) and low injection rate), the displacement becomes efficient due to effect of gravity on the fractional flow curve (Eq. 25). The composition profile at \( t_D = 0.25 \) PV is shown in Fig. 1b. Comparison of this plot with Fig. 1a reveals a drastic influence of gravity on the displacement process. Figs. 3b, 4b, and 5b depict the rate, saturation and recoveries. The recovery data in Fig. 5b demonstrate a very pronounced effect of crossflow. The two layers contain nearly the same amount of oil (i.e., \( C_{10} \)) during the course of gas injection. However, bulk of the oil is produced from the more permeable layer (layer 1) due to crossflow from layer 2 to layer 1 (see Figs. 5a and 5b).

The features of the solution with and without gravity are quite different. The gravity dominant case has only two shocks since the two layers enter and exit the two-phase region with the same speed. One interesting feature of the solution with gravity is that there is a spreading segment in the single-phase region in layer 1 (see Fig. 1b, AB).

![Figure 3 - Flow rate profiles at \( t_D = 0.25 \) PV injection (with crossflow) at different pressures and inlet gravity numbers—layered media.](image)

The trailing shock (E -> F) is calculated using Eqs. 23 and 24 and two eigenvalues of the downstream side of the shock. Therefore, at the trailing shock,

\[
\lambda_{T,D1}^{EF} = \lambda_{T,D}^{EF} = \lambda_{B2}^{EF} \tag{35}
\]

where \( \lambda_{B1}^{EF} \) and \( \lambda_{B2}^{EF} \) are different from the definitions in Eqs. 33 and 35 due to effect of gravity. Both eigenvalues are directly obtained from Eq. B-7. Solution of the system in Eq. 36 yields downstream composition and rates of the trailing shock. In Eq. 36, the direction of crossflow is not given a priori, since both layers enter two-phase region simultaneously. The solution indicates that the crossflow is from layer 2 to layer 1 at the downstream of trailing shock and then, there is a zone of continuous variation (DE) where crossflow changes direction. At the leading edge of the continuous variation (D) crossflow is from layer 1 to 2. The change of crossflow direction shows itself as an extreme point in the rate profiles (see Fig. 3b).

The leading shock is constructed between the leading zone of continuous variation in single-phase region (AB) and the trailing zone of continuous variation (ED). From the solution of the shock balances, upstream and downstream compositions of the leading shock are obtained. Since the parametric representations of both continuous segments are known, one composition on each segment is sufficient to solve for the leading shock. Therefore, two balance equations are sufficient,

\[
\lambda_{T,D1}^{CD} = \lambda_{T,D}^{CD} = \lambda_{B2}^{EF}. \tag{37}
\]

The AB segment of the solution is not fully known, since the total rate at point B is unknown. In order to calculate AB variation, total rate at point A in layer 2 is guessed and the full solution is obtained. The guessed rate of layer 2 at point A is varied until the overall material balance is satisfied.

The results without crossflow are shown in Figs. 1f, 6b, and 7b. The rate history in Fig. 6b shows that the amount of \( CH_4 \) entering layer 2 increases until \( t_0 = 0.34 \) PV leading to a better sweep in the low permeability layer (layer 2). The total recovery and layer recovery plots (the recoveries based on in-situ amount of \( C_{10} \) for CE solution) are similar for solutions with and without crossflow (see Figs. 5b and 7b).

The main differences between the cases with and without gravity are the length of the transition zone, leading shock strength (shock height), and the solution structure. For flow with no gravity (i.e. horizontal layers), breakthrough occurs at 0.43 PV injection (Fig. 5a), whereas the breakthrough time for the case with strong gravity is 0.66 PV (Fig. 5b). The speed of the leading shock for flow with gravity is high. Furthermore, overall mole fraction of \( CH_4 \) at breakthrough at the outlet of layer 1 is 0.40 for no gravity case and zero (just at breakthrough) for the case with gravity. The leading shock for flow with gravity is stronger and slower than without gravity resulting in higher recovery. The main contribution of gravity is the enhancement of crossflow between the two layers resulting in a high recovery from the low permeability layer (Figs. 1a and 1b). The gravity effect is not very pronounced at the trailing shocks since those shocks are mainly affected by the solubility of \( C_{10} \) in \( CH_4 \). This fact will be seen more clearly in the following two examples.

Displacement at 4000 psi without gravity. In order to investigate the effects of component partitioning, the injection of \( CH_4 \) at 4000 psi is considered and the solution is obtained in the absence of gravity. Solution profiles are shown in Figs. 1c, 3c, and 4c. The recovery curves in Fig. 3c reveal that the crossflow in the absence of gravity is insignificant.

The solution profiles include four distinct shocks, each is associated with phase change in a particular layer. At the inlet, solution starts with a shock across the phase boundary in layer 2 (F -> G) shock (see Fig. 1c). This shock can be
solved using the shock balance equations for crossflow from layer 2 to layer 1 (Eqs. 32 and 33), where \( \lambda_D \) is calculated at point F. The solution proceeds with a segment of continuous variation (EF) where layer 2 contains the two-phase fluid mixture and layer 1 contains only the gas phase. The continuous variation segment ends at an unknown point E which can be obtained from the intermediate shock balances. The intermediate shocks, C–D and D–E shocks are resolved at the last stage, since they are coupled shocks due to an unknown intermediate state, D. Therefore, we start solving for the leading shock, A–B, shock with a downstream flow rate guessed similar to the previous cases. Due to volume change on mixing, total flow rate in layer 2 at the downstream of the leading shock is unknown. The leading shock, A–B, is obtained using Eqs. 34 and 35 where \( \lambda_D \) is calculated at point B. The continuous solution segment, BC, is obtained from point B towards upstream by integration along the eigenvectors. Similar to point E, point C is also not known and can be obtained from the coupled intermediate shock balances. For the intermediate shocks, the following balance equations are used. For the trailing intermediate shock, E–D, where crossflow is from layer 2 to layer 1

$$\Lambda_D^{DE} = \Lambda_D^{DE} = \Lambda_D^{DE} = \lambda_D$$  \hspace{1cm} (38)

and for the leading intermediate shock C–D, where crossflow is from layer 1 to layer 2,

$$\Lambda_D^{CD} = \Lambda_D^{CD} = \Lambda_D^{CD} = \lambda_D$$ \hspace{1cm} (39)

![Figure 4—Saturation profiles at \( \tau_p = 0.25 \) PV injection (with crossflow) at different pressures and inlet gravity numbers—layered media.](image)

The unknowns in Eqs. 38 and 39 are the compositions of layers at points C, E and D, and the rate of one of the layers at these locations. The rate of the other layer can be obtained from the crossflow equilibrium condition in Eq. 18. The unknowns of the coupled shock balances are \( Z_1^{DE}, Z_2^{DE}, Z_1^{CD}, Z_2^{CD}, \text{ and } \Lambda_D^{CD} \text{ and } \Lambda_D^{DE} \text{ and } \Lambda_D^{DE} \text{ are independent unknowns. Similarly, } Z_1^{CD} \text{ and } Z_2^{CD} \text{ can be calculated once } Z_1^{DE} \text{ is known. Therefore, we have 6 equations and 5 unknowns. One extra equation is used to obtain the value for the total rate of layer 2 at the downstream of the leading shock. The correct solution is obtained when all the shock balances and the overall material balance are satisfied. The details of the solution algorithm are given in Appendix C.}

The solution without crossflow is presented in Figs. 1g, 6c, and 7c. Comparison of composition profiles of the cases with and without crossflow (Figs. 1c and 1g) shows that in layer 2, \( CH_4 \) presence in front of the C–D shock is due to crossflow from layer 1 to layer 2 (see Fig. 1c) which is similar to the corresponding low pressure case. This feature is missing in Fig. 1g due to the no-crossflow condition. Although, the overall recoveries of the two systems, with and without crossflow, show similar behavior (Figs. 5c, and 7c), layer 2 produces more in the crossflow case due to crossflow from layer 1.

![Figure 5—Recovery plots (with crossflow) at different pressures and inlet gravity numbers—layered media.](image)

**Displacement at 4000 psi with gravity.** This case is identical to the example at 1000 psi with gravity, except that the displacement pressure is 4000 psi. The gravity number for the system is 17.9 and \( \tau_{T, \text{inj}} / \tau_p = 0.614 \). \( \tau_p \) is calculated based on the equilibrium phase compositions, phase densities and viscosities at 4000 psi. The qualitative aspect of the solution is the same as the 1000 psi case with gravity. Therefore, solution construction for this case is not repeated.
The solution profiles and the recovery curves can be seen in Figs. 1d, 3d, 4d, and 5d. The main difference between the solutions at 4000 psi and 1000 psi is due to phase behavior. The slow shock (D–E) for the 4000 psi case is better developed and much faster as compared to the low pressure case (1000 psi). The leading shock (B–C) is also stronger in both layers. Thus, almost 80 percent of the initial oil C₁₀ is produced at the bulk breakthrough (arrival of the leading shock).

The solution without crossflow is presented in Figs. 1h, 6d, and 7d. The differences in the speeds of both shocks in the two layers can be viewed clearly. The trailing shock in layer 2 is about 3 times slower than the trailing shock in layer 1, and the leading shock in layer 1 is 2.6 times faster than the leading shock in layer 2. However, the corresponding shocks (trailing and leading shocks in the two layers) in the solutions with CE have the same speed. In other words, both layers enter and exit two-phase region via shocks of the same speed due to crossflow.

Fig. 8 shows the fractional flow functions at 4000 psia for the cases with and without gravity (i.e., \( \mu \text{gr} = 0 \) and 17.9), as well as the composition path for the \( \mu \text{gr} = 17.9 \) case. The labels in Fig. 8b correspond to the labels in Fig. 1d. The effect of gravity is very pronounced as can be observed from a negative tail below the injection point (see Fig. 8b). Note that from A to B, the overall methane concentration in layer 2 changes very little, whereas from C to D, the overall composition in layer 2 changes significantly with a small change in the composition of layer 1.

Fractured Media Examples. For the study of gas injection in fractured media, the matrix and fracture permeabilities are assumed 2 md and 34 darcy, respectively. The matrix block width is 30 cm, and the fracture aperture is 20 microns. Other parameters of the fracture and matrix are shown in Table 2. The fracture and the matrix are initially saturated with normal decane (C₁₀) and methane (C₄H₁₆) is injected into the system at 4000 psi. In the displacements at 1000 psi in the layered system, the phase behavior effects were not pronounced and therefore, gas injection in fractured media will be only studied at 4000 psi.

Figure 7—Recovery plots (without crossflow) at different pressures and gravity numbers—layered media.

Figure 6—Inlet rate history (without crossflow) at different pressures and inlet gravity numbers—layered media.

Figure 8—Fractional flow functions and composition paths at 4000 psia.
A total of four cases will be studied. The first example concerns with the dominant viscous forces, the second and the third examples are intermediate viscous-gravity cases, and the last example is the dominant gravity case. From the cross-sectional area and permeability of the fracture and the matrix given in Table 2, the dimensionless flow rates at the inlet of the fracture and matrix are 0.526, and 0.474, respectively.

**TABLE 2—ROCK AND FLUID PROPERTIES—FRACTURED MEDIA**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Layer 1</th>
<th>Layer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ (fraction)</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>$A$ (cm$^2$)</td>
<td>0.12</td>
<td>1860</td>
</tr>
<tr>
<td>$k$ (Darcy)</td>
<td>34</td>
<td>0.002</td>
</tr>
<tr>
<td>$k_{or}$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$K_r$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>$n_f$</td>
<td>1.2</td>
<td>4.5</td>
</tr>
<tr>
<td>$n_i$</td>
<td>1.2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

**Dominant Viscous Crossflow.** When the gravity number is zero or very small, i.e., $N_{gr}$ is zero or small, the injected fluid, $q_{T,inj}$, mostly flows through the fracture. As a result, the recovery performance is very low. A special feature of low gravity number is a long two-phase transition zone. To study the features of low gravity number (i.e., dominant viscous crossflow) we have selected inlet $N_{gr} = 0.034$, i.e., $q_{T,inj} = 61.5 g_f$.

Composition profiles capturing the fronts in the fracture and the matrix are shown in Figs. 8a and 9a at $t_P = 0.00025$ PV and at $t_D = 0.5$ PV, respectively. The rate and saturation profiles are shown in Figs. 10a and 11a at $PV = 0.5$.

In this example, there are four shocks similar to the high pressure layered system with no gravity. We classify the shocks (fronts) arising in the solution based on their speeds relative to each other. The fastest shock which is associated with the fracture flow is called the leading shock; the slowest shock which is associated with the matrix flow is called the trailing shock; and the shock(s) between the leading and trailing shocks is called an intermediate shock(s). When there are more than one intermediate shock, they are classified according to their speeds. Each shock in the system corresponds to a phase change in either the fracture or the matrix. Although the breakthrough in the fracture is very fast compared to $t_D = 1$, the fronts in the fracture should still be resolved to obtain the full solution. The location of the shocks are especially important due to the localized nature of crossflow.

As expected, the flow with high $CH_4$ concentration in the fracture advances orders of magnitude faster than that in the matrix. The fronts in the matrix, where almost all the initial fluid ($C_{10}$) resides, move slower than those in the fracture leading to low recovery. Fig. 12a shows the recovery curves for both the crossflow and no crossflow cases. As can be seen, the case with crossflow leads to a slightly higher recovery than the case with no crossflow. However, the overall system recovery is very low due to the bypassed oil in the matrix; only about 4 percent of the total rate goes into the matrix for a long period of time as seen in Fig. 10a. Therefore, the low rate in the matrix combined with the $C_H_4$-filled fracture (in a very short time) results in such a low recovery.

Figs. 9a and 18a show the composition profile and the rate history, respectively, for the no-crossflow case. Again, most of the injected fluid passes through the fracture leading to a low overall recovery (Fig. 12a).

**Figure 9—Composition profiles at $t_P = 0.00025$ PV injection (with crossflow) for different inlet gravity numbers at 4000 psia.**

(a) $N_{gr} = 0.034$, (b) $N_{gr} = 0.86$, (c) $N_{gr} = 1.43$, and (d) $N_{gr} = 5.43$ fractured media.

**Figure 10—Composition profiles at $t = 0.00025$ PV injection (no crossflow) for different inlet gravity numbers at 4000 psia.**

(a) $N_{gr} = 0.034$, (b) $N_{gr} = 0.86$, (c) $N_{gr} = 1.43$, and (d) $N_{gr} = 5.43$ fractured media.

**Moderate Viscous Crossflow.** In order to highlight the effect of viscous and gravity crossflow, a moderate viscous crossflow is selected with $N_{gr} = 0.86$ and $q_{T,inj} = 2.45 g_f$.

The qualitative features of this case are similar to the dominant viscous crossflow case, but there is an improvement in recovery performance. Figs. 8b and 9b show the composition profiles at $t_P = 0.00025$ and $t_D = 0.5$ PV, both in the fracture and in the matrix. Breakthrough in the fracture is almost instantaneous with a narrow two-phase region as the segment between the two shocks in Fig. 8b indicates (with respect to $N_{gr} = 0.034$ - Fig. 8a). Rate and saturation profiles are shown in Figs. 10b and 11b. As compared to the previous example, the crossflow reverses direction at the second shock front from the injection end of the displacement (Fig. 11b). The leading front in the matrix (Fig. 9b) advances faster than in the previous case. The recovery performance of this case is shown in Fig. 12b. The positive effect of gravity is clearly seen in the recovery plot. In the previ-
ous case \( q_{T,\text{inj}} = 61.5 \, q_{gr}^2 \), the total recovery is less than 5 percent at 1 PV (Fig. 12a) while the corresponding total recovery is about 21 percent for this case \( q_{T,\text{inj}} = 2.46 \, q_{gr}^2 \). The contribution of crossflow to recovery is, however, low (about 2 percent at 1 PV); the total recovery with crossflow is slightly higher than the total recovery without crossflow (Fig. 12b).

The composition profile and rate history of the no-crossflow case are shown in Figs. 9c, 10c, 11c, and 12c, respectively. The total inlet flow rates for the fracture and the matrix stay almost constant. The total fracture injection rate quickly (when both fast fronts in the fracture arrive to the production end) reaches to 77 percent of the total system rate and stays about the same for at least 2 PV injection. The magnitude of the total recovery is about the same as the total rate share of the matrix (about 21 percent at 1 PV - see Fig. 12b).

As a supplement to the moderate viscous crossflow case, we lowered the total injection rate further for the third example \( N_{gr} = 1.43 \) and \( q_{T,\text{inj}} = 1.47 \, q_{gr}^2 \). The composition profile in the fracture is shown in Fig. 8c at 0.00025 PV. The transition zone between the two fronts becomes smaller as the two-phase transition shocks in the fracture approach each other (the trailing shock becomes faster and the leading shock becomes slower - see also Figs. 8a and 8b). At a certain gravity number \( N_{gr} \approx 1.5 \), both fast shocks in the fracture converge resulting in a single shock. However, due to the negligible fracture volume, this increment in the recovery mainly comes from the matrix rate enhancement and the crossflow. The composition, rate and saturation profiles are shown in Figs. 9c, 10c, 11c, and 12c, respectively. Although the breakthrough in the fracture is almost instantaneous, more than 31 percent of \( C_1 \) is recovered at 1 PV.

The recovery curve for the no-crossflow case (see Fig. 12c) starts deviating from the case with the crossflow at later times \( t_p > 0.6 \, PV \) causing lower recovery than the crossflow case (28 percent at 1 PV). Composition profile at 0.5 PV and the rate history of no crossflow case are shown in Figs. 9g and 13c, respectively. Without crossflow, the rate share of the fracture increases as displacement progresses, leading to lower recovery than the corresponding crossflow case at later times (since most of \( CH_4 \) bypasses the matrix). But, the recovery performance of the no-crossflow case is still significantly higher than the previous no-crossflow cases with lower gravity numbers \( N_{gr} = 0.034 \), and \( N_{gr} = 0.859 \).

**Figure 10** — Composition profiles at \( t_p = 0.5 \, PV \) injection (with and without crossflow) for different inlet gravity numbers at 4000 psia; the right column is without crossflow and the left column is with crossflow—fractured media.

**Figure 11** — Flow rate profiles at \( t_p = 0.5 \, PV \) injection (with crossflow) for different inlet gravity numbers at 4000 psia—fractured media.

**Dominant Gravity Crossflow.** For the dominant gravity crossflow case, the injection rate is lowered further to \( q_{T,\text{inj}} = 6.14 \, q_{gr}^2 \); \( N_{gr} = 3.43 \). The qualitative aspects of this case are significantly different from the previous cases discussed above. As Fig. 8d reveals, the fast front in the fracture has a \( CH_4 \) mole fraction of 0.08. The leading edge
of the fracture flow is in a single-phase liquid state due to a pronounced crossflow from the fracture to the matrix at the intermediate shock. Since the fracture pore volume is small, even a moderate amount of crossflow can eliminate two-phase flow in the fracture. At \( t_D = 0.5 \) PV, only single-phase gas is present in the fracture at the trailing edge of the displacement (Fig. 11d). The rate profiles of both the matrix and the fracture indicate that crossflow changes direction between the trailing shock and the intermediate shock. The change of crossflow direction shows itself as an extreme point in the rate profiles (Fig. 10d). At the intermediate shock, the fluid in the fracture changes from the single-phase gas to the single-phase liquid without entering into the two-phase region (Fig. 11d), and the leading shock is coupled with the intermediate shock. This phenomenon occurs due to strong crossflow from the fracture to the matrix.

![Graphs](image)

Figure 12—Saturation profiles at \( t_D = 0.5 \) PV injection (with crossflow) at different inlet gravity numbers at 4000 psia—fractured media.

The efficiency of the displacement can be seen in Fig. 12d. Significant amount of \( C_{10} \) is produced at the bulk breakthrough (66 percent at 0.8 PV). The case without crossflow produces considerably less \( C_{10} \) (40 percent at 0.8 PV). The composition profile and inlet rate history for the no-crossflow case are shown in Figs. 9b and 13d, respectively. As the injection proceeds, the relative rate share of the fracture increases leading to low recovery at later times. Around \( t_D = 1.25 \) PV, the total inlet rate of the fracture exceeds the total inlet rate of the matrix (Fig. 13d).

Discussion

Resolving the flow in the fracture and in the matrix is a difficult task both analytically and numerically, mainly due to the large contrast in the permeability and PV of the fracture and matrix media. The two distinct characteristic speeds (eigenvalues) of the system, one very large and the other very small cause complication. The large eigenvalue corresponds to the fracture flow and the small one corresponds to the matrix flow. Solution of the displacement problem in a

![Graphs](image)

Figure 13—Recovery plots for (with and without crossflow) for different gravity numbers at 4000 psia—fractured media.

![Graphs](image)

Figure 14—Inlet rate history (without crossflow) for different gravity numbers at 4000 psia—fractured media.
composite fracture/matrix system involves resolving both characteristic speed scales accurately. The ratio of the characteristic speeds can be as high as 10^{2.1}. This means that the speed of propagation in the fracture is about 10^6 times faster than the matrix. The solution of the shock balances is very sensitive to the initial guess. In the case of coupled intermediate shocks, the sensitivity becomes extreme, due to the inter-dependency of the two characteristic speed scales. This is basically due to the phase boundaries between CH₄ (injected gas) and C₁₀ (initial fluid) and the equilibrium gas and liquid phases. Even a small amount of crossflow from the matrix to the fracture can cause a phase change to occur in the fracture. Because the shock balances are solved using the Newton-Raphson technique, the set of unknowns may fluctuate until convergence is achieved. This type of underestimation or overestimation can have significant effect on the fracture flow, and can cause divergence. One main reason of divergence is the discontinuity of eigenvalues across phase boundaries. The unphysical phase change in the fracture, therefore, can cause discontinuities in the characteristic speeds and divergence. As a practical step, the governing parameters of the problem are changed slowly, while the answer of the previous solution is used as the initial guess for the new problem. In the examples considered here for the fractured media, the gravity number is increased gradually to obtain the solutions with pronounced gravity.

In general, some of the qualitative features of the solution are known a priori. For instance, phase boundaries are crossed via shocks, and the solution is divided into different segments by those shocks. However, due to the pronounced effect of gravity, the direction reversal in the crossflow can alter the qualitative features of the solution. For example, fracture flow can skip the two-phase region with a direct jump from the single-phase gas to the single-phase liquid. This kind of behavior along with the unknown downstream rates due to volume change on mixing can make it difficult to obtain the proper solution.

Comparison of the results of the all the cases indicates that the recovery enhancement may occur at a high gravity number (see Fig. 12). By increasing gravity, more fluid goes into the matrix yielding a higher overall sweep of the initial oil, C₁₀. This type of recovery improvement occurs in both solutions, with and without crossflow (Fig. 12). At low to moderate gravity numbers, the recovery plots for both crossflow and no-crossflow solutions exhibit similar responses. But for the dominant gravity crossflow, the difference between the crossflow and no-crossflow solutions becomes significant. At very high gravity numbers (very low rate of injection as compared to the initial free gravity drainage rate of the matrix), the crossflow and no-crossflow solutions approach each other. At the limit when the rate goes to zero, high displacement efficiency in both crossflow and no-crossflow cases is achieved.

**Conclusions**

1. An analytical one-D model based on the method of characteristics is used to study the effect of crossflow on the recovery. In this work, the effects of gravity and volume change on crossflow are taken into account on the performance of a two media system. For a two-layer media, in spite of very pronounced crossflow due to gravity, the total recovery with and without crossflow is surprisingly close for the examples that we have studied. However, with crossflow, bulk of the oil is transferred from the less permeable to the more permeable layer and then produced from the more permeable layer. In this respect, layered and fractured media behave the same.

2. For recovery performance of a fractured media comprised of a matrix block and a fracture, the crossflow between the matrix and the fracture often changes direction. In one of the examples, the recoveries at 1 PV injection are 63 percent with crossflow and 47 percent without crossflow. The effect of crossflow in a multi-block system may be even more pronounced.

### Nomenclature

- \( A \) = flow area
- \( A^k \) = flow area of layer \( k \)
- \( A_T \) = total flow area (Eq. 14)
- \( C_R \) = capacitance ratio
- \( f_j \) = fractional flow function of phase \( j \)
- \( F_i^k \) = molar fractional flow of component \( i \) in layer \( k \) (Eq. 13)
- \( g \) = acceleration of gravity
- \( G_i^k \) = number of moles of component \( i \) per unit volume in layer \( k \) (Eq. 12)
- \( k \) = absolute permeability
- \( k_{ro} \) = relative permeability of liquid phase (oil)
- \( k_{po} \) = relative permeability coefficient of liquid phase (oil)
- \( k_{sg} \) = relative permeability of vapor phase (gas)
- \( k_{pg} \) = relative permeability coefficient of vapor phase (gas)
- \( L \) = system length
- \( M_i^k \) = molecular weight of gas phase
- \( M_t^k \) = molecular weight of liquid phase
- \( n_o \) = relative permeability exponent of liquid phase (oil)
- \( n_g \) = relative permeability exponent of vapor phase (gas)
- \( n_p \) = number of phases
- \( n_c \) = number of components
- \( P \) = pressure
- \( q_{TR}^k \) = gravity drainage rate of layer \( k \)
- \( q_i^{kTR} = \frac{k^k A^k(A^k p_i^k - A_i^k p_i^k) g \sin \theta}{\rho_i^k} \)
- \( q_T \) = total rate of injection
- \( q_{TR, in}^k \) = total rate of injection in layer \( k \) at the inlet
- \( q_{TRD, in}^k \) = total dimensionless rate of injection in layer \( k \) at the inlet
- \( q_k^T \) = total rate of injection in layer \( k \)
- \( R_k \) = fractional pore volume of layer \( k \)
- \( S_j \) = saturation of phase \( j \), fraction
- \( S_o \) = liquid phase (oil) saturation, fraction
- \( S_g \) = gas phase (vapor) saturation, fraction
- \( S_{or} \) = residual saturation of liquid phase (oil), fraction
- \( t \) = time
crossflow in fractured/layersed media incorporating gravity, viscous, and phase behavior effects

t₀ = dimensionless time, (Eq. 14)

x = spatial dimension

x₀ = dimensionless distance, (Eq. 14)

xᵢj = mole fraction of component i in phase j

λ = eigenvector

Zᵢ = overall mole fraction of component i

Z = composition vector

Greek Symbols

ϕ = porosity

ϕᵢᵏ = porosity of layer k

ϕ = volume averaged porosity (Eq. 14)

μᵢ = viscosity of phase j

ρᵢ = molar density phase j

ρᵢ₀ = dimensionless molar density of phase j

θ = tilt angle

Δ = shock speed

Δ₀ = dimensionless shock speed

λ = eigenvalue (characteristic velocity)

Λ₀ = dimensionless eigenvalue (dimensionless characteristic velocity)

λᵢₖ = mobility of phase j in layer k, λᵢₖ = kᵢₖ/μᵢ

λₜ = total fluid mobility in layer k, λₜ = ∑ₖ λᵢₖ

Abbreviations

CE : crossflow equilibrium

NC : no crossflow

VE : vertical equilibrium

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References


Appendix A—Shock Balances with Crossflow

Discontinuities or shocks are required in solution construction when continuous variations are not possible (i.e., faster upstream compositions than the downstream compositions - multivaluedness). We define a shock with two sets of parameters, upstream and downstream, across which the parameters change discontinuously. In the following, Ii and Ij indicate upstream and downstream values, respectively. Consider a control volume of length $Δz$ where a shock passes across. At time $t$ (just before shock passes the control volume), the amount of component $i$ present in the control volume is $φΔz(G_i)$. At time $t+Δt$ (just after the shock passes the control volume), the amount of component $i$ present in the
control volume is \( \phi^i A^i \Delta x (G_i^s)^{i+} \). As shock passes through the control volume, inflow of component \( i \) is \( (q_{i, F_i}^s)^{i+} \Delta t \), and outflow of component \( i \) is \( (q_{i, F_i}^s)^{i+} \Delta t \). If there was no crossflow, the change in the amount of component \( i \) in the control volume would be equal to the difference between inflow and outflow. However, crossflow will emerge from the difference of change in total material in the control volume and the net total flow. Hence, total crossflow is defined by,

\[
\phi^i A^i \Delta x \sum_{i}^{} \left( [(G_i)^{i+} - (G_i)^{i-}] - \Delta t \sum_{i}^{} (q_{i, F_i}^+)^{i+} - (q_{i, F_i}^-)^{i-} \right) \tag{A-1}
\]

For component \( i \), the crossflow term becomes,

\[
\frac{F_i^+}{\sum_{i}^{} F_i} \left\{ \phi^i A^i \Delta x \sum_{i}^{} \left( [(G_i)^{i+} - (G_i)^{i-}] - \Delta t \sum_{i}^{} (q_{i, F_i}^+)^{i+} - (q_{i, F_i}^-)^{i-} \right) \right\} \tag{A-2}
\]

and the shock balance for component \( i \) including the crossflow term is,

\[
\phi^i A^i \Delta x (G_i^s)^{i+} - \phi^i A^i \Delta x (G_i^s)^{i-} = (q_{i, F_i}^s)^{i+} \Delta t - (q_{i, F_i}^s)^{i-} \Delta t + \frac{F_i}{\sum_{i}^{} F_i} \left\{ \phi^i A^i \Delta x \sum_{i}^{} \left( [(G_i)^{i+} - (G_i)^{i-}] - \Delta t \sum_{i}^{} (q_{i, F_i}^+)^{i+} - (q_{i, F_i}^-)^{i-} \right) \right\} \tag{A-3}
\]

or,

\[
\Lambda = \frac{\Delta x}{\sum_{i}^{} F_i} \left\{ \sum_{i}^{} \left( [(G_i)^{i+} - (G_i)^{i-}] - \sum_{i}^{} \sum_{i}^{} (q_{i, F_i}^s)^{i+} \right) - \sum_{i}^{} \sum_{i}^{} \sum_{i}^{} (q_{i, F_i}^s)^{i-} \right\} = A^i \phi^i \left\{ \sum_{i}^{} \left( [(G_i)^{i+} - (G_i)^{i-}] - \sum_{i}^{} \sum_{i}^{} (q_{i, F_i}^s)^{i+} \right) \right\} \tag{A-4}
\]

The main issue in Eq. A-4 is to calculate the term \( F_i^+ / \sum_{i}^{} F_i \) shock which is discontinuous across a shock. For \( F_i^+ / \sum_{i}^{} F_i \) shock, a rate-weighted form similar to Zappata and Lake [3], and Pande and Orr [4] is adopted. Thus,

\[
\frac{F_i^+}{\sum_{i}^{} F_i} \left( \frac{\sum_{i}^{} F_i^+}{\sum_{i}^{} F_i} \right) = \alpha_i^+ = \frac{F_i^+}{\sum_{i}^{} F_i} \left( \frac{\sum_{i}^{} F_i^+}{\sum_{i}^{} F_i} \right) \tag{A-5}
\]

For \( \alpha_i^+ = 0 \), Eq. A-4 is equivalent to the shock balance for the homogeneous system. The shock balances without the crossflow term are given by Eqs. 23 and 24 of the text.

**Appendix B: Eigenvalue Problem and Solution Procedure**

The eigenvalue problem for a two-component mixture based on Eqs. 11, 15 and 16 of the text is,

\[
\begin{pmatrix}
\frac{a}{\partial \xi_I^+} & \frac{b}{\partial \xi_I^+} & \frac{c}{\partial \xi_I^+} \\
\frac{a^T}{\partial \xi_I^-} & \frac{b^T}{\partial \xi_I^-} & \frac{c^T}{\partial \xi_I^-} \\
\frac{a^T}{\partial \xi_D} & \frac{b^T}{\partial \xi_D} & \frac{c^T}{\partial \xi_D}
\end{pmatrix} - \lambda D
\begin{pmatrix}
d & 0 & 0 \\
0 & d & 0 \\
0 & 0 & d
\end{pmatrix}
= 0
\]

\[
\begin{bmatrix}
\frac{dx_I}{dt} \\
\frac{dx_D}{dt} \\
\frac{dx_D}{dt}
\end{bmatrix} = 0
\]

where the eigenvectors are defined by,

\[
X = \begin{bmatrix}
x_{I1} \\
x_{I2} \\
x_{I3}
\end{bmatrix}
\]

The top row elements are,

\[
c = \frac{\partial q_{I, D} F_i}{\partial z_I^+} - \frac{\partial \sum_{i=1}^{2} q_{I, D} F_i}{\partial z_I^+} \tag{B-3}
\]

\[
b = \frac{\partial q_{I, D} F_i}{\partial z_I^-} - \frac{\partial \sum_{i=1}^{2} q_{I, D} F_i}{\partial z_I^-} \tag{B-4}
\]

\[
c = \frac{\partial q_{I, D} F_i}{\partial q_{I, D}} - \frac{\partial \sum_{i=1}^{2} q_{I, D} F_i}{\partial q_{I, D}} \tag{B-5}
\]

\[
d = R_i \left( \frac{\partial q_{I, D} F_i}{\partial z_I^+} - \frac{\partial \sum_{i=1}^{2} q_{I, D} F_i}{\partial z_I^+} \right) \tag{B-6}
\]

The eigenvalues, \( \lambda_D \), and \( \lambda_D \), are the roots of the equation,

\[
\begin{pmatrix}
(a - \lambda_D) & b & c \\
\frac{a^T}{\partial \xi_I^+} & \frac{b^T}{\partial \xi_I^+} - \lambda_D \frac{c^T}{\partial \xi_I^+} \\
\frac{a^T}{\partial \xi_D} & \frac{b^T}{\partial \xi_D} - \lambda_D \frac{c^T}{\partial \xi_D}
\end{pmatrix}
= 0
\]

Eq. B-7 has two roots, \( \lambda_D \), and \( \lambda_D \), each eigenvalue, \( \lambda_D \), has an associated eigenvector, \( \xi_n \) (Eq. B-2) which describes the direction in the composition-rate space. Each eigenvalue yields one potential composition path. The first and second rows of the eigenvector, Eq. B-2 provide the changes in the compositions of medium 1 and medium 2, respectively. The last row in Eq. B-2, \( \sum_{i=1}^{2} q_{I, D} F_i / \xi_D \), provides the changes in \( q_{I, D} \) along a selected eigenvector direction. The rate in layer 1, \( q_{I, D}^{1} \), can be updated from the crossflow equilibrium condition stated in Eq. 18 once the changes in \( Z_I \), \( \xi_D \), and \( q_{I, D}^{1} \) are known.
Solution Construction. A correct physical solution consists of compatible waves where velocity constraint is satisfied. The velocity constraint requires monotonic wave velocities which increase from upstream to downstream. Shocks are included in the solution when the condition of monotonocity is violated. In addition, the phase boundaries are crossed via the shocks.

The shocks in binary flow are always associated with phase appearance and disappearance in one or both of the media. In other words, the shocks appear in the solution when the phase boundaries are crossed in at least one of the media. In the solutions, the path switch is allowed as long as the velocity constraint is satisfied. The solution is constructed combining separate segments (zone of constant states, shocks, and zone of continuous variations) satisfying both the material balance and the velocity constraint.

Computation of Eigenvalues and Eigenvectors. Computation of eigenvalues and eigenvectors entails calculation of the entries in Eq. 10-7. Each entry of this determinant is coupled with the phase equilibrium calculations. Therefore, a phase behavior model must be used in the calculations. In this study, we have selected the Peng-Robinson [14] equation of state to perform the phase equilibrium calculations.

Appendix C: Solution Steps

Displacement at 1000 psi without gravity. There are two key composition points that correspond to both intermediate shocks and are obtained through a trial and error procedure. We have used a procedure similar to the one by Pande and Orr [4] for low permeability contrast displacements. The iteration steps are (see Fig. 2):

1. Calculate the trailing shock composition (G).
2. Compute the trailing continuous variation (GF).
3. Guess a point on the trailing continuous variation. This point constitutes the upstream composition of the trailing intermediate shock (E).
5. Compute the intermediate continuous variation (ED) based on the guess in step 3.
7. Compute the leading zone of continuous variation (BC).
8. Guess a point on the leading zone of continuous variation (C). This point will be downstream composition of the leading intermediate shock.
9. Calculate the composition of the leading intermediate shock (CD) using Eqs. 33 and 34.
10. Compute the intermediate zone of continuous variation (DE) based on the guess in step 8.
11. The intermediate zone of continuous variation segments in steps 5 and 10 must be the same if both of the guesses in steps 3 and 8 are right. Both guesses are varied until the area between the two solution curves (steps 5 and 10) is negligible.

The solution procedure becomes more complicated when the volume change on mixing is considered. The total flow velocity in medium 2 at the downstream of the leading shock must also be obtained through an iterative procedure. From the crossflow equilibrium condition, the total rate in medium 1 is determined. However, with the inclusion of volume change on mixing, the above iteration scheme is not sufficient to obtain the solution. Overall material balance with the steps above should also be used. Therefore, even when step 11 is satisfied, overall material balance error should be checked to ensure that the estimated rate value at the downstream of the leading shock is correct. If the overall material balance is not satisfied, the estimated rate in medium 2 in front of the leading shock has to be modified and steps 3 to 11 should be repeated. The convergence will be achieved when both step 11 and overall material balance are satisfied.

Displacement at 4000 psi without gravity. The solution steps are as follows:

1. Calculate the trailing shock (F—G shock) composition and rate.
2. Compute the continuous variation, FE.
3. Calculate the leading shock (A—B shock) composition and rate with a guessed total rate in medium 2 at point A.
4. Compute the continuous variation, BC.
6. Check the overall material balance to validate the estimated downstream rate in step 3. If the overall material balance and one of the remaining shock balances do not hold, then we go back to step 3 until the total material balance and all the shock balances are satisfied.

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