Control-Volume Model for Simulation of Water Injection in Fractured Media: Incorporating Matrix Heterogeneity and Reservoir Wettability Effects

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Summary
The control-volume discrete-fracture (CVDF) model is extended to incorporate heterogeneity in rock and in rock-fluid properties. A novel algorithm is proposed to model strong water-wetting with zero capillary pressure in the fractures. The extended method is used to simulate: (1) oil production in a layered faulted reservoir, (2) laboratory displacement tests in a stack of matrix blocks with a large contrast in fracture and matrix capillary pressure functions, and (3) water injection in 2D and 3D fractured media with mixed-wettability state. Our results show that the algorithm is suitable for the simulation of water injection in heterogeneous porous media both in water-wet and mixed-wettability states. The novel approach with zero fracture capillary and nonzero matrix capillary pressure allows the proper prediction of sharp fronts in the fractures.

Introduction
This work is focused on the numerical treatment of two main physical aspects of multiphase flow in fractured porous media: heterogeneity in rock-fluid properties and reservoir wettability.

In a previous work (Monteagudo and Firoozabadi 2004), a CVDF method was used to discretize the system of equations governing water injection in fractured media with strong-water-wettability state and homogeneous matrix and rock-fluid properties. The method was restricted to a finite contrast in matrix-fracture capillary pressure. In this work, we extend the CVDF model for simulation of water injection in fractured media comprised of heterogeneous rocks and wettability conditions from strong-water-wetting to mixed-wetting conditions. We also present a formulation for infinite contrast in capillary pressures of matrix and fractures (zero capillary pressure in the fracture and finite capillary pressure in the matrix).

The control volume (CV) method, first proposed by Baliga and Patankar (1980), is a finite-volume formulation over dual cells (CVs) of a Delaunay mesh. It is locally conservative and suited for unstructured grids. It has been widely employed for the simulation of multiphase flow in porous media (Monteagudo and Firoozabadi 2004; Verma 1996; Helmig 1997; Helmig and Huber 1998; Bastian et al. 2000; Geiger et al. 2003) and the convergence of the method for two-phase immiscible flow in porous medium has already been proved (Michel 2003).

Numerical treatment of heterogeneity in the framework of the CV method has been extensively studied in the past (Edwards 2002; Edwards and Rogers 1998; Prevost 2000; Aavatsmark et al. 1998a, b). Nevertheless, those works have focused on absolute permeability heterogeneity and anisotropy in single-phase flow. The main concern in those works is the use of full tensor permeability and the accurate generation of streamlines (required by the streamline numerical method). It is well known that the standard CV method produces inaccurate velocity fields around the interfaces of heterogeneous media as the contrast in permeability is increased (Durlofsky 1994). In the standard CV method, Delaunay triangles are locally homogeneous and the polygonal CV cell may be heterogeneous (see Fig. 1a). For accurate streamlines, several authors (Verma 1996; Edwards 2002; Edwards and Rogers 1998; Prevost 2000; Aavatsmark et al. 1998a) have proposed that the polygonal CV cell must be locally homogeneous, implying heterogeneous Delaunay triangles (see Fig. 1b). The latter configuration, however, generates additional problems in the simulation of multiphase flow in porous media. Basically, from mesh generation standpoint, it may not be possible to generate an unstructured mesh where the boundaries of the CV median-dual cell conform to heterogeneous interfaces in the domain. Conforming mesh is important for the discrete-fracture approach. Therefore, it would be necessary to first generate a standard CV cell mesh, and later a homogenization procedure would be required to obtain CV cells with constant permeability. The homogenization or upscaling of permeability is somehow possible, but the same is not true for rock-fluid properties; most challenging is capillary pressure with different endpoints. Therefore, the approach with the homogeneous CV cell may be suitable for single-phase simulation where rock-fluid interactions are not part of the problem. However, rock-fluid interactions have to be taken into account for simulation of multiphase flow in fractured porous medium. Frequently, capillary pressure is disregarded in two-phase flow simulations; however, capillary pressure is of importance for simulation of multiphase flow in fractured porous media (Monteagudo and Firoozabadi 2004; Karimi-Fard and Firoozabadi 2003). Predictions of flow pattern and oil recovery may be severely affected if capillary pressure effect is neglected.

With these considerations, we have opted for the use of the standard CV method with a novel approach for rock-fluid heterogeneity based on the capillary pressure continuity concept. The numerical inaccuracies of the velocity field at the interface of heterogeneous media may not be as relevant as the ones generated by homogenization of rock-fluid properties or the neglect of capillary effects in multiphase flow in fractured media.

The second focus of this work is proper accounting for reservoir wettability. In some reservoirs, wettability covers a wide range from strong water-wetting to strong oil-wetting (Rao et al. 1992; Morrow 1990; Robin 2001). Even in a single reservoir, wettability can change from strong water-wetting at the bottom to oil-wetting at the top of the oil column.

The paper is outlined as follows: We first present the governing equations for two-phase immiscible and incompressible flow for fractured media. Next, we briefly describe the CV discretization of the governing equations with the numerical treatment for heterogeneity in rock-fluid properties and different wettability states. Then, we provide numerical simulation of field-scale and lab-scale examples and numerical tests for mixed-wettability conditions in 2- and 3D fractured media. Finally, concluding remarks are made.

Governing Equations

Two-Phase Incompressible Flow in Porous Media. The two-phase incompressible, immiscible-flow displacement in porous media can be described by two equations (Monteagudo and Firoozabadi 2004; Aziz and Settari 1979):

\[-\mathbf{\nabla} \cdot (\lambda_u + \lambda_d) \nabla \Phi_w - \mathbf{\nabla} \cdot \lambda_u \nabla \Phi_t - (q_w + q_o) = 0 \]
where $\phi$ is the porosity, $\lambda_i$ is the mobility of phase $i$ defined by $\lambda_i=kk_i/\mu_i$, and $k$ is the absolute permeability; $k_i$, $\mu_i$, and $q_i$ are the relative permeability, viscosity and source/sink term of phase $i$, respectively, $\Phi_w$ is the water flow potential, $\Phi_f$ is the capillary flow potential which is defined (Karimi-Fard and Firoozabadi 2003) by

$$\Phi_i = \Phi_w - \Phi_f = P_i + (\rho_i - \rho_w)gz. \quad \text{.................. (3)}$$

In Eq. 3, $P_i$ is the capillary pressure, $\rho_i$ is the density of phase $i$, $g$ is the acceleration of gravity, and $z$ is the vertical coordinate, positive in the upward direction.

Eq. 1 is referred to as the flow-potential equation, which is elliptic in nature, while Eq. 2 is referred to as the saturation equation, which can be seen as a diffusion-convection equation (Peaceman 1977). The main variables are the wetting-phase flow potential, $\Phi_w$, and saturation, $S_w$. In most of our numerical simulations, the boundary conditions are assumed to be impervious: $v_i=\lambda_i\nabla \Phi_i = 0$ for both phases $i=\{n,w\}$, where $v_i$ is the velocity vector of phase $i$. We have also incorporated the Dirichlet boundary condition to include an aquifer in a field-scale problem that will be simulated in this work. In that case, $\Phi_w=\Phi_{w,a}$ and $S_w=1$ at the bottom of the reservoir.

Formulation for Fractured Media. Details of the formulation for fractured media are provided in Montegudo and Firoozabadi (2004). The most remarkable feature of the formulation is that no computation of matrix-fracture transfer term is required. This is possible because of the integration in the CV cell and the use of the crossflow equilibrium concept, which leads to capillary continuity. In the CV method in terms of matrix variables only.

$$\frac{\partial \Phi_i}{\partial t} - \nabla \cdot \lambda_i \nabla \Phi_i - q_i = 0, \quad \text{.................. (2)}$$

where $\lambda_i$ is the mobility of phase $i$ defined by $\lambda_i=kk_i/\mu_i$, and $k$ is the absolute permeability; $k_i$, $\mu_i$, and $q_i$ are the relative permeability, viscosity and source/sink term of phase $i$, respectively, $\Phi_i$ is the water flow potential, $\Phi_f$ is the capillary flow potential which is defined (Karimi-Fard and Firoozabadi 2003) by

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CV Discretization

We use the CV methodology for the spatial discretization of both the flow potential and saturation equations. In this method, the integration is performed over the CV cells, which are constructed as the median dual of a Delaunay triangulation in 2D or tetrahedrization in 3D. Fig. 1a shows the 2D CV cell formed by joining the baricenters of Delaunay triangles with their edge midpoints for unfractured media. In Fig. 2, we show the CV cell for fractured media with homogeneous matrix (Fig. 2a) and with heterogeneous matrix (Fig. 2b) media.

Eqs. 1 and 2 have three types of terms:

1. Time derivative, $\partial \Phi_i/\partial t$.
2. Source/sink, $q_i$ for $i=\{n,w\}$.
3. Divergent term $\nabla \cdot F$, where the flux vector $F=F(S_w)$ may be either $(\lambda_n+\lambda_w)\nabla \Phi_w$ or $\lambda_w\nabla \Phi_f$ in Eq. 1, or $\lambda_w\nabla \Phi_f$ in Eq. 2.

All mobilities are computed at the boundaries of the CV cell with the proper upwinding, based on the flow direction at the boundary. Next, we provide the discretization of all these terms for 2D fractured media with homogeneous and heterogeneous matrix. Extension to 3D media is straightforward.

Discretization for Fractured Media With Homogeneous Matrix. In the following, we first add the matrix and fracture terms and then approximate the integration in the finite-volume.

Time Derivative.

$$\int_A \frac{\partial}{\partial t} (\Phi_i S_w) dA = \left( A^m \frac{\partial \Phi_i}{\partial t} + A^f \frac{\partial \Phi_f}{\partial t} \right) S_{w}^m, \quad \text{.................. (7)}$$

where superscripts $m$ and $f$ denote matrix and fracture variables. Eq. 7 represents the change in the volume of the wetting phase in the volume element per unit time. It consists of two terms, one from the matrix contribution and the other from the fracture contribution. If there is no fracture in the volume elements, then the contribution is from the matrix only. Note that in 2D, the product $A^m \Phi_i$ ($A^m$ is the area of the matrix in the element; in 3D it should be replaced by the volume of the matrix in the element) and $A^f \Phi_f$ ($A^f$ is the area of the fracture in the element) provide the volume of the matrix and fracture in the volume element, respectively. We can express Eq. 7 in terms of $S_{w}^m$ by using Eq. 6 and the chain rule:

$$\int_A \frac{\partial}{\partial t} (\Phi_i S_w) dA = \left( A^m \phi_i + A^f \phi_f \frac{dS_w^m}{dS_w} \frac{\partial S_{w}^m}{\partial t} \right) S_{w}^m, \quad \text{.................. (8)}$$

Source/Sink Term.

$$\int_A q_i dA = A^m q_i^m + A^f q_i^f. \quad \text{.................. (9)}$$
approximated by extending the idea of Eqs. 7 and 8. Therefore, we can relate the saturation of all rocks at the interface matrices in its interior. Similar to the matrix-fracture interface, we Fig. 2b shows a CV cell with a fracture and two different rock

Discretization for Fractured Media With Heterogeneous Media. Fig. 2b shows a CV cell with a fracture and two different rock matrices in its interior. Similar to the matrix-fracture interface, we invoke capillary continuity at the interface between different rocks. Therefore, we can relate the saturation of all rocks at the interface of a heterogeneous medium.

**Time Derivative.** The integration of the time-derivative term is approximated by extending the idea of Eqs. 7 and 8.

\[
\int_A \frac{\partial}{\partial t} (\phi S_w) dA = \sum_{k=1}^{n_m} \frac{\partial}{\partial t} S_w^k \phi A^k, \quad \text{.................................. (11)}
\]

where \(n_m\) is the number of different media inside the CV cell. Eq. 11 may be expressed in terms of the saturation of the reference medium \(S_w^0\). The reference medium can be chosen on the basis of parameter \(B\) in the capillary pressure model of the form

\[
P_i^k = -B^k \ln S_w^k, \quad \text{.................................. (12)}
\]

where superscript \(k = 1 \ldots n_m\) is an index for the different media inside the control volume. Note that by using the capillary continuity concept, a relationship can be established between the different media saturations. For example, in the case of fractured medium: \(S_w^m/S_w^0 = \exp(-B^m/B^0)\).

In this work, we select the reference medium with the highest value of \(B^k\) in Eq. 12.

From Eq. 6 and the chain rule we derive:

\[
\int_A \frac{\partial}{\partial t} (\phi S_w) dA \approx \left[ \sum_{k=1}^{n_m} \frac{dS_w^k}{dS_w^0} \phi A^k \right] \frac{\partial}{\partial t} S_w^0, \quad \text{.................................. (13)}
\]

**Source/Sink Term.** This term can be written as

\[
\int_A q_w dA = \sum_{k=1}^{n_m} A^k q_w^k, \quad \text{.................................. (14)}
\]

**Divergence Term.** Integration of the divergence term is presented in Eq. 10. Each triangle contained in the CV cell is locally homogeneous, and the saturation of different media can be related through Eq. 6.

**Formulation Using Normalized Saturations.** Normalized saturation is defined as

\[
\overline{S} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}, \quad \text{.................................. (15)}
\]

where \(S_{wi}\) is the irreducible-water saturation and \(S_{or}\) is the residual-oil saturation.

By using the chain rule, Eq. 2 can be expressed as:

\[
\alpha \frac{\partial \overline{S}_w}{\partial t} - \nabla \cdot \lambda_S(\overline{S}_w) \nabla \Phi_w - q_w = 0, \quad \text{.................................. (16)}
\]

where \(\alpha = 1 - S_{wi} - S_{or}\)

**Numerical Treatment for Large Contrast Between Matrix And Fracture Capillary Pressures.** In our previous work (Monteagudo and Firoozabadi 2004), the contrast between matrix and fracture capillary pressure was expressed by the ratio \(B_m/B_f\) using the capillary pressure model in Eq. 12. All the simulations in that work were carried out for a finite ratio, that is, \(B_m/B_f > 0\). When \(B_m/B_f = 0\), the model of Monteagudo and Firoozabadi (2004) is applicable because \(S_w^m = S_w^f\) and \(dS_w^m/dS_w^f = 1\). There is a numerical problem when \(B_m/B_f > 0\) and \(B_m/B_f = 0\) when \(B_m/B_f = 1\); with strong water wetting and relatively thick fractures, \(B_m/B_f\) would be large. Below we present a solution to this problem.

Fig. 3 illustrates the imbibition process for the fracture and the matrix media. Fig. 3a shows the fracture and matrix capillary pressure curves, for \(B_f = 0.6\) atm and \(B_m = 1.0\) atm, respectively (\(B_m/B_f = 1.67\)). Fig. 3b depicts the fracture and matrix capillary pressure model for \(B_f = 0.1\) atm and \(B_m = 1.0\) atm, respectively (\(B_m/B_f = 10\)). Imbibition of a CV cell containing a matrix-fracture interface with these capillary pressure curves will follow the path through the points A, B, and C as indicated by the arrows, provided the initial water saturation is zero. Because capillary continuity expressed by Eq. 6 must be satisfied at the interface during the imbibition, \(S_w^f = 0\) and \(dS_w^m/dS_w^f = 0\) along the path from A to B for capillary pressures depicted in Figs 3a and 3b. On the other hand,
along the path from B to C the derivative \( \frac{dS_m}{dS_w} \) will be much larger when \( B_f = 0.1 \) atm (Fig. 3b) than when \( B_f = 0.6 \) atm (Fig. 3a). From this analysis, one can observe that in the event of \( B_f = 0 \) then \( B_m / B_f \to \infty \) and there is apparently no way to relate \( S_m \) and \( S_w \) along the path B to C with the formulation outlined previously because \( dS_m / dS_w \to \infty \). To avoid this numerical problem, the left term in Eq. 8 may be expressed in an alternative way:

\[
\left( A_m \phi + A_f \phi \right) \frac{dS_w}{dS_w} \frac{dS_m}{dt} = \left( A_m \phi + A_f \phi \right) \frac{dS_m}{dt} \cdot \ldots \cdot (17)
\]

Then we have two equivalent expressions for the left term in Eq. 8 that can be distinctly used along different sections of the imbibition path shown in Fig 3b. If chosen correctly, the numerical problem described here is avoided. Eq. 17 also provides a physical interpretation of the imbibition process inside the CV cell; for the path A-B, \( dS_m / dS_w = 0 \), which implies imbibition in the matrix alone, and for the path B-C \( dS_m / dS_w = 0 \), mainly only the fracture saturation changes. We have implemented this criterion in our model to switch between the two equivalent expressions in Eq. 17. That is,

If \( P > \epsilon \), then

\[
\int_A \frac{\partial}{\partial t} (\phi S_m) dA = (A_m \phi) \frac{\partial S_m}{\partial t} \cdot \ldots \cdot (18)
\]

Otherwise,

\[
\int_A \frac{\partial}{\partial t} (\phi S_m) dA = (A_f \phi) \frac{\partial S_f}{\partial t} \cdot \ldots \cdot (19)
\]

where \( \epsilon \) is a tolerance.

**Numerical Examples**

In this section, we present several numerical examples. First, we simulate oil production from data extracted from a large fractured reservoir. The second example covers data from a laboratory test of water injection in a stack of matrix blocks at different injection rates. Next, we present the results of numerical simulation for water injection in 2D and 3D for mixed-wettability state. The 2D and 3D meshes were generated with the packages triangle (Shewchuk 1996) and tetgen (Si 2002), respectively. All simulations were run with a Pentium PC 2.66 GHz.

Fig. 3—Imbibition paths in fracture (left column) and in matrix (right column) at the interface. (a) \( B_f = 0.6 \) atm and \( B_m = 1.0 \) atm; (b) \( B_f = 0.1 \) atm and \( B_m = 1.0 \) atm.
Field-Scale Example With Horizontal Production Well. A horizontal view of the reservoir simulated is shown in Fig. 4a. A region with an areal dimension of 1200×500m (shown as a rectangle with dashed lines) was selected for simulation. The region contains two vertical wells, 13V and 6V, and one horizontal well, 6H. Only the horizontal well is producing. The region contains several faults (the dark solid line). In Fig. 4a, only the three main faults (the grey solid lines) that cross the region are shown. The horizontal well is contained in one of the main faults. Fig. 4b shows the fault plane XZ joining wells 13V and 6V with details of the location of the horizontal well. We use a simple approach to represent wells by a sink term and assign a triangular flow distribution along the horizontal well (Joshi 1991) (see Fig. 5). The reservoir has a thickness of 424 m with an aquifer at the bottom.

Fig. 4—Cross-sectional study. (a) Horizontal view; (b) XZ cross section between Wells 13V and 6V showing horizontal Well 6H.

Fig. 5—Triangular flow distribution in a horizontal well.
The aquifer is treated as the Dirichlet boundary, where water flow potential and saturation are specified.

Fig. 6 shows the conforming tetrahedrization in the fractured region. The horizontal well 6H is shown as a dashed thick line. We have incorporated three main and seven minor fractures as quadrilaterals (see Table 1 for vertex coordinates). Fracture thickness was set to 0.1 cm. The mesh contains 4,000 nodes. Preliminary sensitivity study showed that with this refinement, there is very little change in oil-recovery predictions. Fluid and rock properties are shown in Tables 2 and 3, respectively. The reservoir is layered. Fig. 7 shows the permeability distribution. Rock-fluid interactions are shown in Fig. 8. Note that the relative permeability and capillary pressure curves may not show a consistent wettability state.

The initial distribution of fluids is determined using the vertical equilibrium criterion (Pooladi-Darvish and Firoozabadi 2001):

\[ P_c(z) = P_{cw} - g(z/H_0 - z/H), \]  

\[ P_c = P_{cw} - g(\rho_n - \rho_w)z, \]  

where \( P_c \) is the capillary pressure at the bottom of the reservoir \((z=0)\) and \( z \) is the vertical coordinate (positive in the upward direction). Once \( P_c \) is known, \( S_w \) is computed from Eq. 12.

The simulation was carried out to 140 years of production, keeping a constant total flow rate of 2400 B/D. The total CPU time for this simulation was 12 hours. Water saturation contours at time = 70 years are shown in Fig. 9. The corresponding simulated oil recovery curve is shown in Fig. 10. The initial reservoir pressure was 2,500 psia at \( z=0 \). The predicted average pressure across the horizontal well after 17 years of production was approximately 1,500 psia. Our preliminary results are on the order of the reported data.

### Lab-Scale Example for a Stack of Matrix Blocks

We simulate the displacement of n-decane by water from a stack of chalk matrix blocks. Description of the equipment and experimental data are provided by Pooladi-Darvish and Firoozabadi (2001). Previous attempts to model the waterflooding have not been successful when one focuses on the water/oil front in the fractures (Pooladi-Darvish and Firoozabadi 2001; Terez and Firoozabadi 1999).

In the setup, 12 matrix blocks were assembled; fractures are the space between blocks and between blocks and the container walls. The 3D conforming mesh for the porous media with 1,800 nodes is presented in Fig. 11 (note that there are three matrix blocks per row). Tables 4 and 5 present fluid and rock properties, respectively. Rock-fluid interactions parameters are shown in Table 6. The capillary pressure model used is defined by Eq. 12, and the relative permeabilities are modeled with power laws:

\[ k_{rw} = k_{r0w} (S_w)^{n_w}, \]  

\[ k_{rn} = k_{r0n}/(1 - S_w)^{n_n}, \]  

where \( S_w \) is the normalized saturation defined by Eq. 15. All the rock-fluid interaction parameters are reported in Pooladi-Darvish and Firoozabadi (2001). Fig. 12 depicts the capillary pressure and relative permeability. Note that capillary pressure in the fracture is considered zero, as was evidenced by observation of a flat oil/water interface in the fractures. The relative permeability for the matrix shows a strong-water-wetting state.

Using the previously described data (without any adjustment), we simulated the experiments. There is good agreement between the simulation results and measured recovery data for all three

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### Table 1—Field-Scale Simulation (Fracture Vertex Coordinates in M)

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<th>Vertex 4</th>
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injection rates. Results are shown in Fig. 13. Our simulation results also show the correct shape of the front in the fractures (see Fig. 14). The front based on our simulation is in better agreement with measured data than in the previous works (Pooladi-Darvish and Firoozabadi 2001; Terez and Firoozabadi 1999). One could obtain a better match by adjusting the matrix capillary pressure curve slightly. The CPU time required for a displacement of 2 PV at each flow rate was 27 hours. Flow through fractures for \( P_c < c \) requires a small timestep to avoid instabilities. This results in the increased CPU time.

**Mixed-Wettability Examples.** We present 2- and 3D numerical simulations to evaluate the performance of the CVDF method to simulate waterflooding in porous media with mixed wettability state. Because of the way the problem has been formulated, the mixed-wet state has not been implemented for two-phase immiscible flow in fractured media (Bogdanov et al. 2003).

The 2D example consists of a square of 25x25 m containing 6 fractures. The 2D mesh with 2,700 nodes is shown in Fig. 15, with the fractures marked as thick lines. Water is injected at a flow rate of \( 3.96 \times 10^{-7} \text{ m}^3/\text{s} \) (2.7x10^{-4} PV/day) at the lower left corner, and the producing well is at the opposite corner. The initial water saturation is zero in the whole domain. The 3D example consists of a cube of 20x20x20 m with one fracture plane inside. A plot of the mesh for the fracture plane and the matrix is shown in Fig. 16. Water is injected at a flow rate of 3.7x10^{-4} m^3/s (0.02 PV/day) at coordinates [0 m, 0 m, 0 m], and the producing well is placed at coordinates [20 m, 20 m, 20 m].

**Tables 7 and 8** present fluid and rock properties used for the examples. Rock-fluid interactions are shown in Fig. 17. Capillary pressure has two branches, positive and negative, indicating mixed wettability state. These rock-fluid properties are used for both 2D and 3D simulations. Because the capillary pressure model in Eq. 12 is only suitable for a water-wet media, we compute \( d\) in numerically for the mixed-wetting state. Water saturation is initialized using the positive \( P_c \) relationship shown in Fig. 17.

The CPU time for the 2D geometry was 24 hours for a 1-PV displacement. This represents a speed-up of three times when compared with a previous code (Karimi-Fard and Firoozabadi 2003). The idea is tested in the field-scale simulation of a layered reservoir.

3. A novel numerical procedure has been proposed when there is large contrast in capillary pressure at the matrix-fracture interface \( (P_c = 0) \). The procedure was tested in the simulation of the waterflooding in a stack of matrix blocks. The method not only provides proper prediction of oil recovery at different water injection flow rates, but it also predicts a sharp water front in the fractures.

4. We also present the results of the numerical simulation for water injection in a mixed-wet medium using our CVDF model. In the past, the discrete fracture approach had not been used for water injection in fractured media with a mixed-wet state that has a higher nonlinearity than a water-wet state. Results show that our formulation performs well in the mixed-wet state.

**Nomenclature**

- \( A \) = area, \( L^2, \text{m}^2 \)
- \( B \) = parameter for \( P_c \) model Eq. 12, \( m/L^2, \text{Pa [atm]} \)
- \( c \) = measure of a boundary segment of a CV cell
- \( F \) = generic flux crossing the boundaries of a CV cell
- \( k \) = absolute permeability, \( L^2, \text{m}^2 \)
- \( k_i \) = relative permeability of phase \( i = \{n, w\} \)
- \( n \) = number of boundary segments of a CV cell
- \( n_m \) = number of different media inside a CV cell
- \( n_r \) = exponent for the nonwetting-phase relative permeability model Eq. 21
- \( n_w \) = exponent for the wetting-phase relative permeability model Eq. 22
- \( n_o \) = residual oil saturation
- \( q \) = sink/source term of phase \( i \), \( t^{-1}, \text{s}^{-1} \)
- \( S_i \) = saturation of phase \( i = \{n, w\} \)

**Fig. 7—Permeability field of the layered reservoir.**

**Table 2—Field-scale Simulation (Fluid Properties)**

<table>
<thead>
<tr>
<th>Property</th>
<th>Water (kg/m³)</th>
<th>Oil (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>950</td>
<td>750</td>
</tr>
<tr>
<td>Viscosity (cp)</td>
<td>0.7</td>
<td>2.7</td>
</tr>
<tr>
<td>Bubblepoint pressure</td>
<td>390 psi</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3—Field-scale Simulation (Rock Properties)**

<table>
<thead>
<tr>
<th>Rock ID</th>
<th>Porosity (%)</th>
<th>Permeability (md)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRT3</td>
<td>10</td>
<td>2.5</td>
</tr>
<tr>
<td>DRT5</td>
<td>20</td>
<td>5.0</td>
</tr>
<tr>
<td>DRT8</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>Fracture</td>
<td>100</td>
<td>20x10¹</td>
</tr>
</tbody>
</table>

that may have very different rock-fluid properties with different endpoints. The idea is tested in the field-scale simulation of a layered reservoir.

**Conclusions**

1. We have extended the control-volume discrete-fracture method (Monteagudo and Firoozabadi 2004) to account for important physical aspects in reservoir simulation, such as heterogeneity in matrix rock and rock-fluid properties, different wettability conditions, and large contrast in the matrix-fracture capillary pressures.

2. We propose the use of capillary continuity concept in order to solve the problem of having a heterogeneous CV cell with rocks
Swi
irreducible water saturation

Si
normalized saturation of phase \(i\) (Eq. 15)

t
parameter in Eq. 16

A
boundary of CV cell A

tolerance

\(i\)
mobility of phase \(i\), \(L^3t/m, m^2/Pa\cdot s\)

\(i\)
total mobility, \(L^3t/m, m^2/Pa\cdot s\)

\(i\)
viscosity of phase \(i\), \(m/Lt, Pa\cdot s\)

\(i\)
density of phase \(i\), \(m/L^3, kg/m^3\)

\(c\)
capillary potential, \(m/Lt^2, Pa\)

\(D\)
flow potential specified at Dirichlet boundary, \(m/Lt^2, Pa\)

\(f\)
flow potential of phase \(i\), \(m/Lt^2, Pa\)

Subscripts

\(n\) = nonwetting phase

\(w\) = wetting phase

Fig. 8—Field-scale simulation (rock-fluid properties). (a) Capillary pressure; (b) relative permeability.

Fig. 9—Water saturation contours after 70 years of production. (a) Fracture contours; (b) matrix contours.

Fig. 10—Oil recovery for the field-scale simulation.
Acknowledgments

This work was supported by the member companies of the Reservoir Engineering Research Institute (RERI).

References


<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Oil (nC₁₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>950</td>
<td>800</td>
</tr>
<tr>
<td>Viscosity (cp)*</td>
<td>1.0</td>
<td>0.94</td>
</tr>
</tbody>
</table>

*1 cp = 1x10⁻³ Pa.s

**TABLE 6—CHALK ENSEMBLE (ROCK-FLUID PROPERTY PARAMETERS)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Matrix</th>
<th>Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_w$</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>$S_o$</td>
<td>0.35</td>
<td>0.0</td>
</tr>
<tr>
<td>$B$</td>
<td>1.7</td>
<td>0.0</td>
</tr>
<tr>
<td>$k_{rw}$</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>$n_w$</td>
<td>4.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$k_{ro}$</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>$n_o$</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**TABLE 5—CHALK ENSEMBLE (ROCK PROPERTIES)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Matrix</th>
<th>Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total pore volume (m³)</td>
<td>13900x10⁻⁶</td>
<td>700x10⁻⁶</td>
</tr>
<tr>
<td>Permeability (md)</td>
<td>2.5</td>
<td>1.35x10⁶</td>
</tr>
<tr>
<td>Porosity (%)</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Fracture aperture (μm)</td>
<td>–</td>
<td>194</td>
</tr>
</tbody>
</table>
Fig. 12—Lab-scale waterflooding (rock-fluid properties). (a) Capillary pressure; (b) relative permeability.

Fig. 13—Oil recovery for different water injection rates in the stack of matrix blocks.

Fig. 14—Simulated and experimental fracture waterfront at different PV displacement in the stack of matrix blocks. (a) Front level in the fracture; (b) water saturation in the fractures.


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Fig. 15—Conforming triangulation mesh for 2D mixed-wet fractured media.

Fig. 16—Conforming tetrahedrization mesh for 3D mixed-wet fractured media. (a) Fracture; (b) matrix.

Fig. 17—Mixed-wet fractured system (rock-fluid properties). (a) Capillary pressure; (b) relative permeability.
Fig. 18—Water saturation contours for injection into a 2D mixed-wet fractured media with initial $S_w=0$. (a) 25% PV displacement; (b) 50% PV displacement.

Fig. 19—Water saturation contours at 20% PV displacement for injection into a 3D mixed-wet system where initial $S_w$ field is computed with vertical equilibrium concept. (a) Fracture contours; (b) matrix contours.