ENERGY LEVEL SPECTROSCOPY OF A BOUND VORTEX-ANTIVORTEX PAIR

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Abstract We report microwave spectroscopy of energy levels of a bound Josephson vortex-antivortex (VAV) state in an annular Josephson junction. The bound VAV pair is formed in the narrow long junction by applying an in-plane magnetic field. The dissociation of the bound state, which is associated with a switching of the junction from the superconducting to the resistive state, is induced by ramping up a bias current applied to the junction. This process can be mapped to the escape of a particle from a potential well. Above the crossover temperature, the pair dissociation is thermally activated, below it is induced by quantum tunnelling, which is inferred from switching current measurements. In the quantum regime, the dependence of the energy level spectrum on field and bias current is probed using microwave spectroscopy.

Keywords: Macroscopic quantum effects, long Josephson junctions, vortex-antivortex nucleation, microwave spectroscopy, thermal-quantum crossover.

Macroscopic quantum phenomena in Josephson coupled systems have attracted considerable attention both theoretically and experimentally [1–9]. Most of the studied systems, such as dc-biased Josephson junctions (JJ), superconducting quantum interference devices (SQUIDs) and small Josephson-junction arrays, are based on a small number of point-like Josephson junctions. A few degrees of freedom, namely the Josephson phases, suffice to describe the dynamics of these systems. At low temperatures, quantum-mechanical effects such as macroscopic quantum tunnelling, energy level quantization [2–5], and coherent oscillations [6–9] of the Josephson phase have been observed.

In the case of spatially extended Josephson systems, such as large parallel arrays or Josephson-junction ladders, the analysis and observation of quantum dynamics is more complex. A particularly interesting system in that respect is the *quasi-one-dimensional long Josephson junction* in which the phase continuously varies in space. In the classical limit the phase dynamics in the long JJ is described by the sine-Gordon equation. This system supports well-known nonlinear excitations [10], for example Josephson vortices (magnetic fluxons) and vortex-antivortex pairs, that can interact with inhomogeneities or linear (Josephson-plasma) modes. The classical dynamics of such excitations are well established [10]. In particular, the escape induced by thermal fluctuations of the Josephson phase from a metastable state has been investigated both experimentally and theoretically [11, 12]. Recently, the ac-induced escape of a vortex in the presence of thermal fluctuations has been studied in detail by numerical simulations [13]. A number of macroscopic quantum-mechanical effects have been predicted for Josephson vortices [14–19], however, only few have been observed experimentally until now. In particular, macroscopic quantum tunnelling of many vortices [20, 21] and, more recently, also tunnelling and level quantization of an individual vortex [22] have been observed.

A next natural step in the study of *macroscopic quantum effects* in long JJs is to investigate the dissociation of a *single vortex-antivortex pair*. The crossover of the dissociation process from the thermal to the quantum regime has already been theoretically analyzed and experimentally observed [23]. A characteristic of the dynamics of the bound VAV pair is its small oscillation frequency in the effective binding potential, a property which has been theoretically analyzed in Ref. [23]. In the quantum limit, the small oscillation frequency determines the energy level separation. As shown for the single-vortex case [22], resonant microwave absorption can be used to map the level structure of the system by observing the enhancement of the escape rate. In this paper we report microwave spectroscopic measurements of the energy levels of a VAV pair.

A *single* VAV pair appears naturally in a long *annular* JJ subject to an external magnetic field H [24, 25] applied in the plane of the junction, see Fig. 1a-c. For experiments, we have fabricated annular junctions of diameter $d = 100\,\mu\mathrm{m}$ and widths $w = 0.5\,\mu\mathrm{m}$ and $w = 1\,\mu\mathrm{m}$ by etching a sputtered Nb/AlO$_x$/Nb trilayer patterned using electron-beam lithography [26]. The critical current density of the junctions is $220\,\mathrm{A/cm^2}$ at $20\,\mathrm{mK}$. The Josephson length is $\lambda_J \approx 30\,\mu\mathrm{m}$, resulting in a normalized junction length of $L \equiv \pi d/\lambda_J \approx 10.5$. The $0.5\,\mu$m-wide junction was used to observe the thermal to quantum crossover [23], and the $1\,\mu$m-wide junction was used for spectroscopic measurements. A measurement of the magnetic field dependence of the switching current for the $0.5\,\mu$m-wide annular junction is shown in Fig. 1d. In the field range $|H| < 1.5\,\mathrm{Oe}$ (main central lobe), the switching of the JJ from the superconducting to the resistive state occurs through the *dissociation* of a single field-induced VAV pair confined in the potential well formed by external magnetic field and dc bias current. This process is confirmed by direct numerical simulations of the perturbed sine-Gordon model for an annular long

JJ [24, 25] subject to an in-plane dc magnetic field H. The numerically found magnetic-field dependence of the switching current is in excellent agreement with measurements, see the solid line in Fig. 1d. Simulations of the magnetic field distribution in the junction clearly show the nucleation and subsequent dissociation of the VAV pair, see Fig. 1c. Fluctuations, thermal and quantum, induce oscillations of the confined pair. At high temperatures, the dissociation takes place by thermal activation over the confinement barrier. At low temperatures, *macroscopic quantum tunnelling* through the barrier occurs. At fields $|H| > 1.5\,\mathrm{Oe}$ the system becomes bistable as a well-separated VAV pair is spontaneously created in the junction. This state is perfectly reproduced by our numerical calculations, see the first side lobes in Fig. 1c.

According to the theoretical derivation in Ref. [23], the phase distribution in the junction can be written as $\varphi(x,t) = \pi/2 + \xi(x,t)$, where $\xi(x,t)$ is a perturbation describing a small-amplitude VAV pair,

$$\xi(|x-x_1|,A) = \sqrt{2\delta}\left[\frac{3}{\cosh^2\left(\frac{|x-x_1|+A}{2}(2\delta)^{1/4}\right)} - 1\right] . \tag{1}$$

Here, $\delta \equiv (1 - I/I_{c0}) \ll 1$ and $x_1(t)$ is the center-of-mass coordinate of the bound pair. The distance between the vortex and the antivortex $A(t)$ is allowed to vary in time. The magnetic field and dc bias create the pinning potential for such a state. Since we consider dc magnetic fields only, we neglect the dynamics of the center-of-mass coordinate $x_1(t)$. Hence the effective energy of the JJ,

$$E(A) = \frac{m_{\text{eff}}(A)}{2}\dot{A}^2 + U_{\text{pot}}(A) , \tag{2}$$

is a function of A and $\dot{A} \equiv dA/dt$. Expressions for $U_{\text{pot}}(A)$ and $m_{\text{eff}}(A)$ are derived in [23].

In the absence of fluctuations, the critical current, $\delta_c(h) = 2h/3$, is found by minimization of the energy $E(A)$ in A. The corresponding critical amplitude A_0 is determined by the condition $\sinh\left((2\delta)^{1/4}A_0/2\right) = 1$. Note that the critical current decreases linearly with magnetic field, in contrast to the case of *linear* long JJs, where similar considerations yield $\delta_c^{\text{lin}}(h) \propto h^{4/3}$. Both results are valid in the case of uniform bias-current distribution and for $L/2\pi \gg 1$.

In the presence of thermal or quantum fluctuations, the dissociation of the pinned VAV pair occurs at a random value of bias δ exceeding $\delta_c(h)$. Assuming weak fluctuations, $\delta - \delta_c(h) \ll \delta_c(h)$, we expand the energy about $A = A_0$ ($\delta A \equiv A - A_0$) as

$$E(A) = \frac{\chi h^{5/4}(\delta\dot{A})^2}{2} + \frac{3^{3/2}\sqrt{h}}{2}\left(\delta - \delta_c(h)\right)(\delta A) - \frac{h^2}{6}(\delta A)^3 , \tag{3}$$

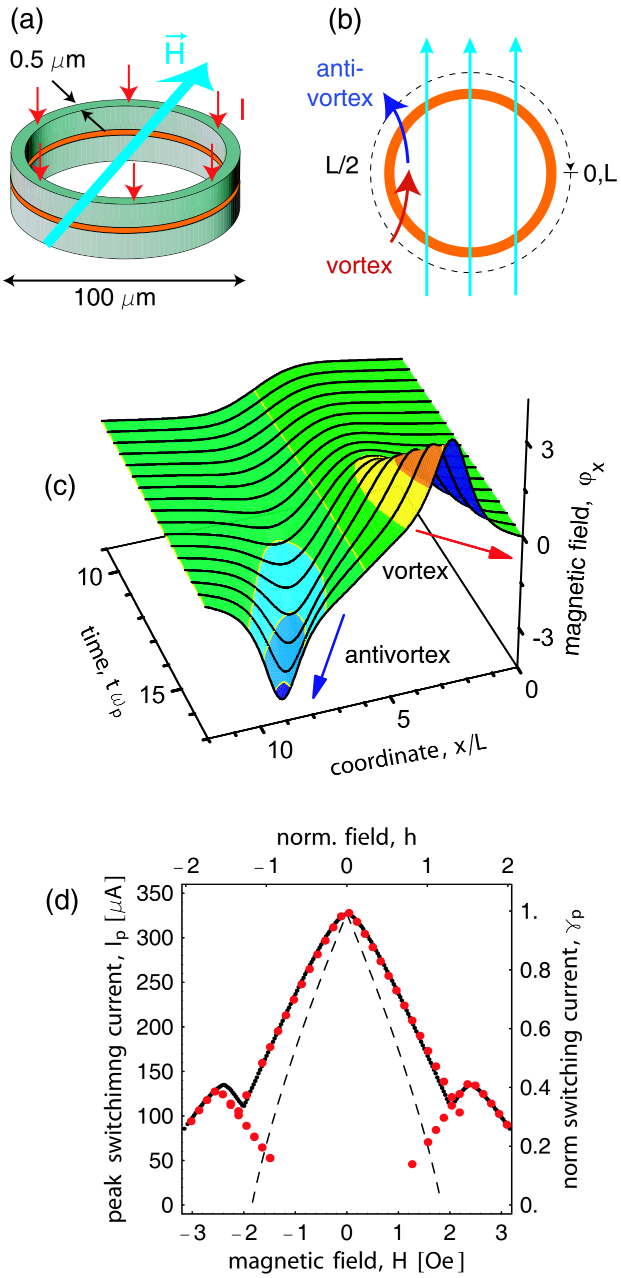

Figure 1 (a) Schematic view of a long annular Josephson junction of diameter 100 μm *without trapped vortices* in an in-plane external magnetic field H with uniform bias current I. (b) Generation of a confined vortex-antivortex pair with center coordinate at $x = L/2$. (c) Numerically simulated evolution of the magnetic field distribution, as the Josephson junction switches to the resistive state. The emerging vortex and antivortex move in opposite directions. (d) Magnetic field dependence of switching currents: experimental data at $T = 100$ mK (circles), numerical calculation for $L = 10.5$ (solid line), and theoretical prediction [23] for $L \gg 2\pi$ (dashed line).

where χ is a numerical coefficient of order unity. Thus, the problem of the fluctuation-induced dissociation of a bound VAV pair is mapped to the well-known problem of particle escape from a cubic potential. The dissociation rate depends on the height of the effective potential barrier,

$$U_{\text{eff}}(\delta) = 2 \cdot 3^{5/4} h^{-1/4} (\delta - \delta_c(h))^{3/2} \ . \tag{4}$$

At high temperatures, the dissociation is driven by thermal activation over this barrier. Using well known theory describing the particle escape from such a potential well [1–4], we find the switching rate of a long Josephson junction from the superconducting to the resistive state to be

$$\Gamma_T(I) \propto \exp\left[-U_{\text{eff}}/k_B T\right]. \tag{5}$$

Thus, at high temperatures the standard deviation σ of the critical current distribution should increase with temperature and weakly depend on magnetic field as $\sigma_T \propto T^{2/3} h^{1/6}$. Notice that σ_T *increases* with H, in contrast to small Josephson junctions [1, 2] where $\sigma_T \propto (I_c(H))^{1/3}$ decreases with H.

We experimentally investigated the fluctuation-induced dissociation of the VAV pair by measuring the temperature and magnetic field dependence of the statistical distribution P of the switching currents $I < I_{c0}$, using techniques described in [11, 16, 27]. In Fig. 2a, the temperature dependence of the switching current distribution measured at $H = 0$ is shown. At high temperatures the $P(I)$ distribution is temperature-dependent. At low temperatures a saturation of the distribution width is observed. In Fig. 2b, the standard deviation σ of $P(I)$ is plotted versus bath temperature T for two values of magnetic field. The standard deviation σ is to a good approximation proportional to $T^{2/3}$, and is larger for the higher field, as predicted above. As clearly seen in Fig. 2b, σ decreases with temperature and saturates below a crossover temperature of $T^\star \approx 100\,\text{mK}$. At $T < T^\star$ the dissociation of the VAV pair occurs through *macroscopic quantum tunnelling*. The crossover temperature $T^\star \simeq \hbar\omega(\delta)/(2\pi k_B)$ is determined by the small oscillation frequency $\omega(\delta)$ of the VAV pair. In the quantum regime, this frequency $\omega(\delta) = 3^{3/8}/\sqrt{\chi}\,(\delta - \delta_c(h))^{1/4}$ determines the oscillatory energy levels $E_n \simeq \hbar\omega(\delta)(n + 1/2)$ of the pinned VAV state. A further discussion of the thermal and quantum dissociation rates and their specific scaling with H is given in [23]. Using the techniques demonstrated in Ref. [22] for single vortices, we have performed microwave spectroscopic measurements of the VAV pair energy levels. The measurements were performed at temperatures between 15mK and 20mK by exciting the VAV pair into the first excited state by means of microwave radiation. A Lorentzian peak in the decay rate Γ centered around the resonant current for a given spectroscopy frequency is observed. Rewriting

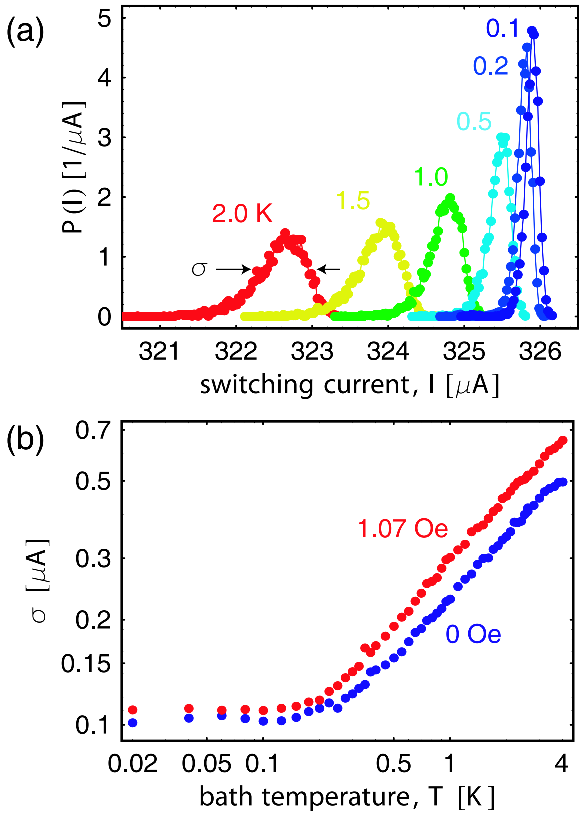

Figure 2 (a) Switching-current distributions $P(I)$ at $H = 0$ for the indicated bath temperatures T. (b) Distribution of the standard deviation σ of $P(I)$ versus the bath temperature for the two indicated values of magnetic field.

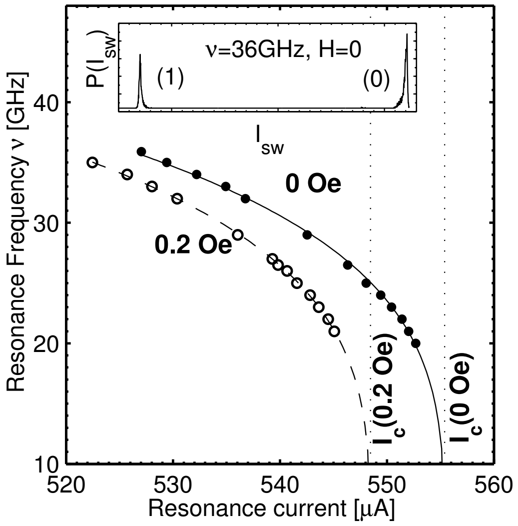

Figure 3 Resonance current versus resonance frequency. Experimental data for zero magnetic field (filled circles) and $0.2\,\mathrm{Oe}$ (open circles) are compared to the model (solid line and dashed line, respectively). The respective critical currents are indicated by the dotted vertical lines. In the inset, the original switching-current probability distribution $P(I_{\mathrm{sw}})$ is indicated with the same current scale as the main figure for a single data point. The peak (1) corresponds to the escape from the resonant level, the peak (0) to the escape from the ground state.

the small oscillation-frequency dependence as

$$\nu(I,H) = \nu_0(H)\left(1 - \frac{I}{I_c(H)}\right)^{1/4} \tag{6}$$

highlights the small-junction-like scaling of $\nu(I,H)$ with $I/I_c(H)$ at $H = \mathrm{const}$ and $I/I_c(H)$ close to unity. The factor $\nu_0(H)$ is the magnetic-field-dependent frequency of internal oscillations of the VAV pair at zero bias current. For a number of different values of magnetic field, we have measured the dependence of the resonance current on microwave frequency. Experimental data for zero magnetic field and $H = 0.2\,\mathrm{Oe}$, which compare well to Eq. (6), are displayed in Fig. 3. Least-squares fits to the data for different magnetic fields yield both $I_c(H)$ and $\nu_0(H)$.

The theoretical dependence of $\nu_0(H)$ on $I_c(H)$ for VAV dissociation is given by

$$\nu_0(H) = \frac{\omega_0}{2\pi}\frac{3^{3/8}}{\sqrt{\chi}}\left(\frac{I_c(H)}{I_{c0}}\right)^{1/4}, \tag{7}$$

where $\omega_0/2\pi$ is the plasma frequency of the junction at $H = 0$. This expression differs from the one expected for the phase escape in a small Josephson junction $\nu_0(H) \propto \sqrt{I_c(H)/I_{c0}}$. In Fig. 4 we fit the experimentally determined magnetic field dependence of the zero bias oscillation frequency $\nu_0(H)$ to Eq. (7). We determine a characteristic frequency $3^{3/8}\omega_0/(2\pi\sqrt{\chi})$ of $74.7\,\mathrm{GHz}$, from the least-squares fits. We note that the field dependence of the zero bias oscillation frequency extracted from experimental data is in good agreement with the predictions based on our model for VAV dissociation. For comparison, the expected oscillation frequency for homogeneous escape of the phase

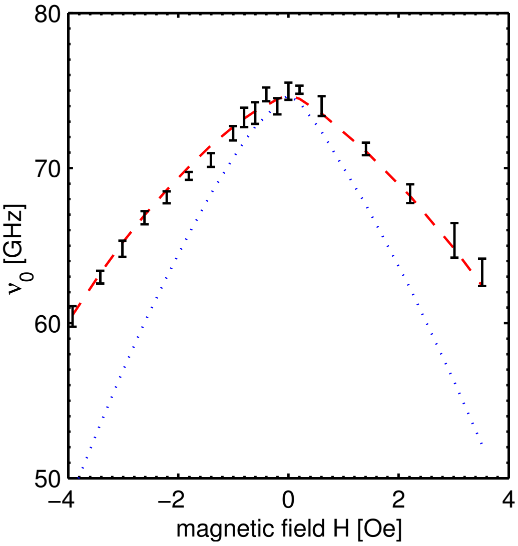

Figure 4 Field dependence of zero bias oscillation frequency ν_0 extracted form measured data (solid error bars), $\nu_0(H)$ calculated from $I_c(h)$ using the long-junction model (dashed line) and the small-junction model (dotted line).

is shown by the dotted line in the same plot, see Fig. 4. The prediction for homogeneous escape of the phase clearly disagrees with the experimental data.

Summary

We have shown that the switching of an annular Josephson junction from the superconducting to the resistive state in the presence of magnetic field occurs through the nucleation and subsequent dissociation of a single vortex-antivortex pair. At low temperatures we observe the pair dissociation by quantum tunnelling. Using microwave spectroscopy, we have shown in this work that the resonantly stimulated dissociation process agrees with the model derived in [23]. The resonant enhancement of the dissociation rate is a manifestation of the energy-level quantization.

Acknowledgments

We thank A. Price for critical reading of the manuscript, A. Abdumalikov for useful discussions and J. Lisenfeld and C. Coqui for technical help. We acknowledge the partial financial support of this project by the Deutsche Forschungsgemeinschaft (DFG).

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