

QUANTIZATION OF THE HALL EFFECT IN A 3-DIMENSIONAL QUASIPERIODIC SYSTEM

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The observation of the quantum Hall effect in an electronic system does not depend on the strict two-dimensionality of the system, but rather that the conductivity in the direction of the magnetic field (σ_{zz}) vanishes. This condition can be achieved in a superlattice when the Landau level spacing exceeds the zero-field miniband width of the superlattice. It is now possible to test this criterion experimentally on a quasiperiodic system to examine if the presence of localized and singular states of the quasicrystal affects this condition. We have experimentally realized this system with a modulation-doped GaAs/AlGaAs Fibonacci superlattice that exhibits a spectra corresponding to a Cantor set in X-ray diffraction. No evidence for the affect of these states on the magnetotransport is seen.

Quantized Hall resistance and the simultaneous zero diagonal resistance state of two-dimensional carriers is now a well-documented phenomena in a number of two-dimensional electron (or hole) systems.¹⁻³ In these structures, at low temperature and high magnetic field, the diagonal component of the resistivity tensor ρ_{xx} nearly vanishes, while the off-diagonal component ρ_{xy} is quantized in units of h/e^2 ($\rho_{xy} = h/ie^2$, where $i = 1, 2, \dots$). An assumed criterion for the observation of the quantized Hall effect has been two-dimensionality of the electronic system, though experiments have also been performed on more complex topologies; specifically, layered though isolated two-dimensional regions.⁴ These systems demonstrate quantized Hall effect with $\rho_{xy} = h/ije^2$, where j is the number of two dimensional layers. This observation is intuitively understood by considering these systems as merely a stack of quantized Hall resistors in parallel.

The advent of molecular-beam epitaxy (MBE) has allowed the fabrication of superlattices that have a truly 3-D, though anisotropic, dispersion. This allows an investigation of the quantized Hall effect in an electronic system where the carriers are no longer strictly confined to two dimensions. Utilizing such a structure, it has been demonstrated⁵ that the assumed two-dimensional criteria of the quantized Hall effect can be relaxed as long as the conductivity of the system in the magnetic field direction (σ_{zz}) vanishes. This occurs when the separation of the Landau levels exceeds the superlattice miniband width. To this date, however, there has been little investigation along this direction for more complex superlattice miniband structures.

A seminal work by Merlin *et al*⁶ has demonstrated an entire new class of superlattice structures, experimentally possible by MBE, that exhibit quasiperiodicity, e.g. "Fibonacci superlattices". These superlattices exhibit spectra that correspond to Cantor sets, which invalidate the Bloch theorem in the superlattice direction. Here we report observations of the quantum Hall effect in such a Fibonacci superlattice and compare the observations to a similar periodic superlattice.

A periodic superlattice similar to that of Ref. 5 was grown by MBE on a semi-insulating Cr-doped GaAs substrate with a 1200Å GaAs buffer. It consisted of 30 periods of the following sequence: 10Å undoped Al_{0.3}Ga_{0.7}As, 18Å Al_{0.3}Ga_{0.7}As Si-doped at $1 \times 10^{18} \text{ cm}^{-3}$, 10Å undoped Al_{0.3}Ga_{0.7}As, and 188Å undoped GaAs. The Hall mobility and carrier densities at 1.0K were measured to be 3100 $\text{cm}^2/\text{V}\cdot\text{s}$ and $4.6 \times 10^{16} \text{ cm}^{-3}$, assuming a thickness of $30 \times (188\text{Å} + 38\text{Å})$. Figure 1 shows a X-ray scan of the (002) reflection from the periodic superlattice structure. The x-ray data were recorded using CuK α_1 radiation from a sealed x-ray tube and a grooved Ge (220,220) monochromator. The measured period of the superlattice is 223Å, a value that is in good agreement with the nominal 226Å period expected from the MBE growth conditions.

Figure 2 shows the low temperature magnetotransport data of this periodic superlattice structure. The magnetoresistance periodicity gives a two-dimensional carrier density of $6.3 \times 10^{11} \text{ cm}^{-2}$, implying that the minimum at approximately 67 KG corresponds to the $i=4$ plateau. Though this plateau is not fully developed, the Hall resistance value at this value corresponds to $\rho_{xy} = h/84e^2$.

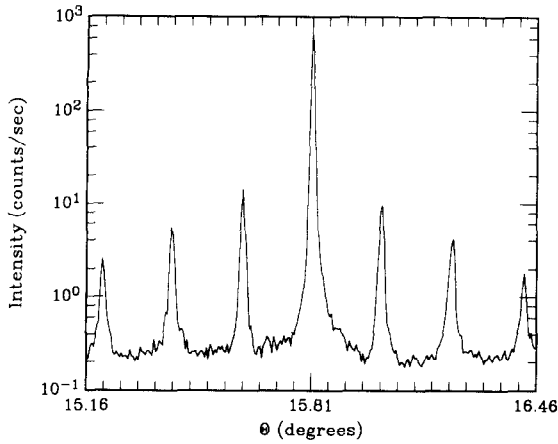


Figure 1. (002) X-ray diffraction scan of the 30 period GaAs/Al₃Ga₇As modulation-doped periodic superlattice.

This is different than the expected value of $\rho_{xy} = h/120e^2$. This effect, attributed to depletion of the edges of the superlattice due to Fermi level pinning, has been seen in previous work.⁵ However, such an interpretation should be viewed cautiously due to the 3-D character of the electronic states.

The observations are consistent with the criteria that the Landau level separation must exceed the superlattice miniband width for quantization of the Hall effect. The separation of the Landau levels becomes equal to the superlattice miniband width of 0.4 meV (calculated in the envelope function approximation) at a magnetic field of 2.3 KG, and is 11.6 meV by 67 KG. However, in contrast to the previous work there is no evidence of two characteristic oscillations ("beating") in the Shubnikov-de Haas data due to the existence of separated neck and belly orbits of the superlattice.⁷ This beating may not be observable in a superlattice with such a small miniband width due to the small separation of the orbits.

We now consider a similar system in which the superlattice is now replaced by a Fibonacci superlattice to study the effect of introducing a Cantor set of localized and singular eigenstates in the superlattice direction. In this case, the Landau levels would continuously move through a spectrum of localized and nonlocalized states, implying a field-varying condition on the requirement σ_{zz} vanishing. The requirement for vanishing z-conductivity (which now varies as the spectrum of the quasicrystal) for the quantized Hall effect should then give rise to field regions in which the quantized Hall effect is satisfied and regions in which it is not, again varying as the spectrum of the quasicrystal.

The quasiperiodic superlattice consisted of a 10-generation Fibonacci sequence of two building blocks, each a GaAs/AlGaAs component, with a ratio of the thicknesses equal to the golden mean τ

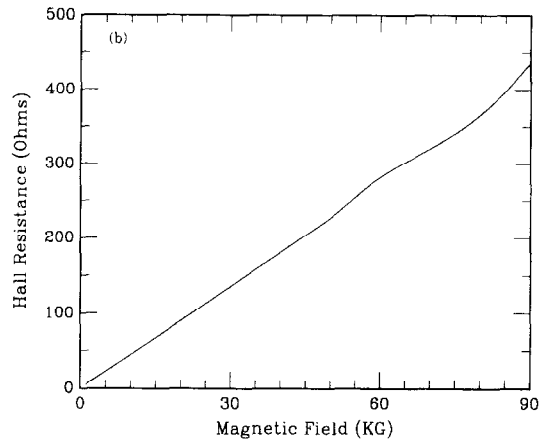
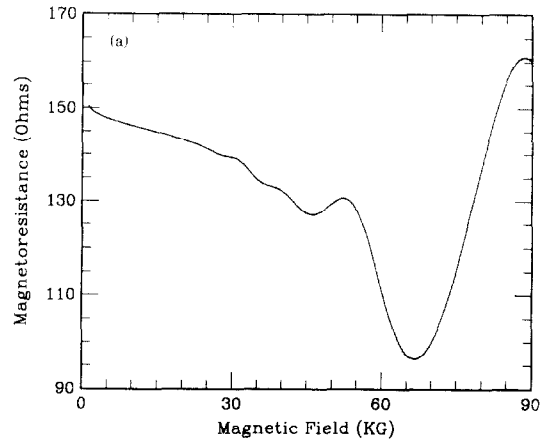


Figure 2. (a) Magnetoresistance at T=1.0K of the 30 period GaAs/Al₃Ga₇As modulation-doped periodic superlattice. (b) Hall resistance at T=1.0K of the 30 period GaAs/Al₃Ga₇As modulation-doped periodic superlattice.

$= (1 + \sqrt{5})/2$. The first block consisted of an undoped 188Å GaAs layer, a 10Å undoped Al₃Ga₇As layer, a 18Å Al₃Ga₇As layer Si-doped at $1 \times 10^{18} \text{ cm}^{-3}$, and a 10Å undoped Al₃Ga₇As layer. The second block consisted of an undoped 101Å GaAs layer, a 10Å undoped Al₃Ga₇As layer, a 18Å Al₃Ga₇As layer Si-doped at $1 \times 10^{18} \text{ cm}^{-3}$, and a 10Å undoped Al₃Ga₇As layer.

Figure 3 shows a cross-section TEM micrograph from the Fibonacci superlattice. The micrograph clearly exhibits the aperiodic nature of the superlattice. It was recorded from the region of the sample that was immediately adjacent to the GaAs buffer layer that was grown prior to the superlattice and hence represents the initial portion of the quasiperiodic structure. The sequencing of GaAs/AlGaAs blocks (ABAABABA...) in the figure is indicative of the

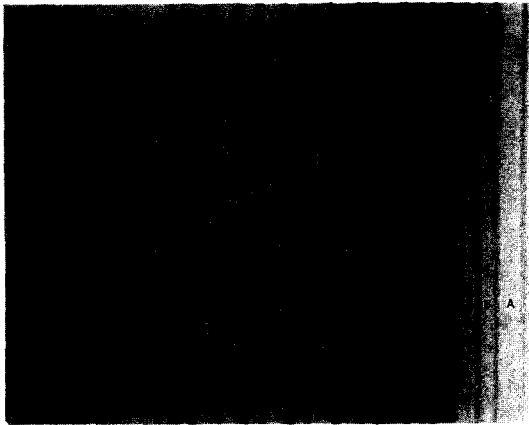


Figure 3. Cross-sectional TEM of the 10 generation Fibonacci superlattice illustrating the sequencing of the incommensurate A (188Å GaAs / 38Å AlGaAs) and B (188Å GaAs / 38Å AlGaAs) blocks.

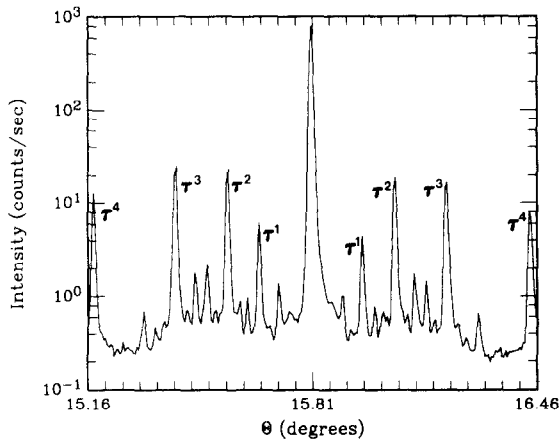


Figure 4. (002) X-ray diffraction scan of the modulation-doped 10 generation Fibonacci superlattice. The most intense peaks correspond to integral powers of τ .

beginning of the Fibonacci sequence. It should be noted that the initial A block is on top of the GaAs buffer.

Figure 4 shows a X-ray scan of the Fibonacci superlattice structure that was recorded using the same diffraction conditions employed for the periodic superlattice in Figure 1. The high resolution diffraction scan displays the dense set of diffraction peaks that are characteristic of a Fibonacci superlattice.⁸ The presence of this dense set is graphically illustrated by comparing the diffraction scans from the periodic (Figure 1) and aperiodic (Figure 4) superlattices. As shown in earlier work,⁷⁻⁸ the most intense of the peaks in

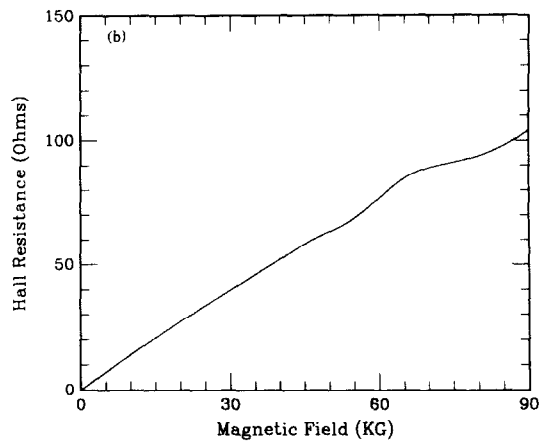
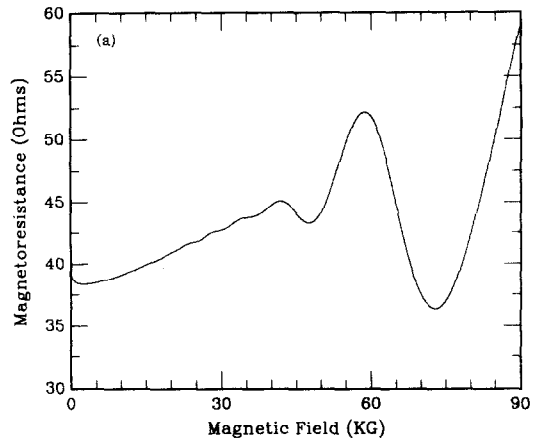


Figure 5. (a) Magnetoresistance at $T=1.0K$ of the modulation-doped Fibonacci superlattice. (b) Hall resistance at $T=1.0K$ of the modulation-doped Fibonacci superlattice.

the x-ray pattern correspond to wave vectors given by $k = 2\pi(m+n\tau)/[\tau d_a + d_b]$, where m and n are successive Fibonacci numbers. These peaks, which are equal⁹ to τ^1, τ^2, \dots and labeled the figure, have been used to calculate the quantity $(\tau d_a + d_b)$. The value of 496\AA determined from the x-ray scan is in reasonable agreement with the nominal 505\AA period that was anticipated from the MBE growth parameters. This difference (also seen in the conventional periodic superlattice) can be attributed to a $\sim 1.5\%$ discrepancy in the true versus expected GaAs growth rate.

Figure 5 shows the low temperature magnetotransport data of this structure. The Hall mobility and carrier densities at $1.0K$ were measured to be $3800\text{ cm}^2/V\text{-s}$ and $2.7 \times 10^{17}\text{ cm}^{-3}$, assuming a thickness of the entire Fibonacci superlattice ($\sim 1.72\text{ }\mu\text{m}$). Figure 6 shows the mobility as a function of temperature for this

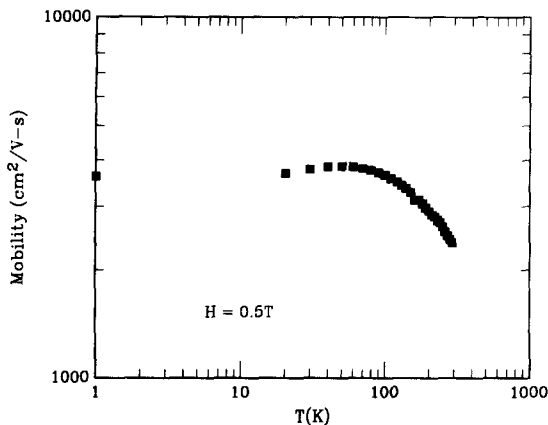


Figure 6. Mobility vs temperature for the modulation-doped Fibonacci superlattice ($H = 0.5T$).

sample, exhibiting some reduction of neutral impurity scattering at low temperatures but clearly verifying the 3-dimensional nature of the Fibonacci superlattice.

The magnetoresistance periodicity gives a two-dimensional carrier density of $6.7 \times 10^{11} \text{ cm}^{-2}$, implying that the minimum at approximately 73 KG corresponds to the $i=4$ plateau. Again, derivative spectra of the magnetoresistance gave only a single periodicity. The Hall resistance value at the $i=4$ value corresponds to $\rho_{xy} = h/284e^2$, again different from the expected value of $\rho_{xy} = h/356e^2$. However, it is not plausible that a depletion layer mechanism is responsible for the observed discrepancy. In the previous periodic superlattice (in the depletion layer model) the total depletion depth corresponds to a total of $\sim 2000 \text{ \AA}$ into the superlattice. In the Fibonacci superlattice, this depth corresponds to $\sim 9100 \text{ \AA}$, in a structure with identical termination layers and for a bulk carrier density almost 6 times greater. Clearly a naïve counting argument is invalid, especially in the light of the observation of a single periodicity. This clearly demonstrates that the naïve "digital-counting" scheme in a superlattice structure is invalid.

It should be noted that the factor of 6 may be an underestimate; at low magnetic fields ρ_{xy} is curved rather than linear in B and is not related to the positions of the $i=6$ or $i=4$ minima. This has been seen in the previous demonstration of a 3-D structure,⁵ but the second derivative $d^2\rho_{xy}/dH^2$ is negative in the present case (versus positive in a periodic structure). This phenomenon is contrary to the behavior of "traditional" 2-D GaAs/AlGaAs modulation-doped structures, and is not understood.

The magnetotransport data, examined in high resolution and repeatable, surprisingly shows no evidence of Landau levels passing through localized states created by the quasiperiodic structure.¹⁰ Again, magnetic fields are easily

reached that satisfy the criterion that the Landau level separation exceeds the "miniband width". It should be stressed that a strict "miniband width" does not exist in a Fibonacci superlattice; however, the limits of the available conduction band states (a "quasi-miniband width") can be estimated from the positions of the main quasi-gaps, and equals 2.14 meV in the present case. Thus the Landau level spacing equals the quasi-miniband width at a magnetic field of 12.4 KG, and is almost 6 times the width at 73 KG. The non-observation of the breakdown of this criteria at fields higher than this implies that the electronic states are not coherent over the length of the sample, presumably due to scattering of the conduction band electrons from the ionized impurity cores in the modulation-doped regions. This demonstrates that, although the crystal can be quasicrystalline, the electronic conduction band states do not exhibit the same fractal nature due to incoherency (in the z -direction) through the structure. This observation implies that the direct observation of quasicrystalline states in such systems via electronic transport is probably unfeasible.

In summary, we have fabricated and measured magnetotransport through an anisotropic 3-D system that exhibits quasicrystalline order along the axis of the magnetic field. There is no apparent effect on the quantum Hall effect criterion of vanishing conductivity in this direction, even though the system exhibits clear quasicrystalline order. The absence of singular and localized states in the electronic transport through this system is probably due to scattering.

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