

# Fundamental limits to force detection using quartz tuning forks

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This paper explores the fundamental limits of the use of quartz tuning forks as force detectors in scanned probe microscopy. It is demonstrated that at room temperature, pressure, and atmosphere these force sensors have a noise floor of  $0.62 \text{ pN}/\sqrt{\text{Hz}}$  and exhibit a root mean square Brownian motion of only  $0.32 \text{ pm}$ . When operated as a shear force sensor both dissipative and reactive forces are detected on approach to the sample. These forces are sufficient to reduce the amplitude of motion of the probe nearly to zero without physically contacting the surface. It is also demonstrated that conventional proportional-integral feedback control yields closed loop responses at least 40 times faster than their open loop response. © 2000 American Institute of Physics.

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## I. INTRODUCTION

It is well established that quartz tuning forks can be used as sensors for acoustic and force microscopy.<sup>1,2</sup> Their very high mechanical quality factor  $Q(10^3-10^5)$  provides a built-in high gain and makes them very sensitive to sub-pN forces when used at or near their resonance frequency ( $10^4 \text{ Hz}$  to  $0.5 \times 10^6 \text{ Hz}$ ). Their advantage is that the measurement of their oscillation amplitude uses the piezoelectric effect native to quartz crystals, yielding an electric signal proportional to the applied forces and making them small, robust, and simple to operate compared to optical force measurement schemes. They have been used as force detectors in near-field optical microscopy,<sup>3</sup> atomic force microscopy,<sup>4</sup> magnetic force microscopy,<sup>5</sup> and magnetometry.<sup>6</sup> This paper is designed to clearly elucidate the fundamental limits associated with the use of these sensors for force microscopy.

The key to implementing tuning forks for force detection is to accurately measure the fundamental resonance of the tuning fork as a function of applied force. This can be done either by shaking the fork at its mechanical resonance and monitoring the induced voltage or by directly driving the tuning fork with a resonant voltage and measuring the induced current. We have chosen to implement the latter, as it provides for a system that is simpler mechanically at the expense of only minimal electronics.

It is well known that the equivalent circuit for the tuning fork is a series  $RLC$  resonator in parallel with package capacitance,<sup>7</sup> which is typically of order a few picofarads. When driving the tuning fork directly with a voltage source, the package capacitance yields both series and parallel resonances, dramatically distorting the line shape from that of the

bare  $RLC$  resonator. To eliminate the effect of the package capacitance, we use the bridge circuit shown in Fig. 1(a). The transformer yields two wave forms phase shifted from each other by  $180^\circ$ . By appropriately adjusting the variable capacitor, the current through the package capacitance is negated by the current through the variable capacitor.

## II. ELECTRONICS AND CALIBRATIONS

A standard operational amplifier circuit is used to convert the net current to a voltage. We have worked to miniaturize the circuitry and locate the amplifier at the base of the tuning fork so as to eliminate any complications due to cable capacitance. The current to voltage ( $I-V$ ) gain of the circuit has been calibrated from dc to  $100 \text{ kHz}$  and is found to be described as  $Z_{\text{gain}} = R_g / \sqrt{1 + (2\pi f R_g C_g)^2}$  where  $f$  is the frequency,  $R_g = 9.51 \text{ M}\Omega$ , and  $C_g = 0.260 \text{ pF}$  is stray capacitance in parallel with  $R_g$ . The resulting  $I-V$  gain at  $32.7 \text{ kHz}$  is  $Z_{\text{gain}} = 8.47 \text{ M}\Omega$ . Having calibrated the measurement system, the impedance of the tuning fork can be accurately measured. We measure a bare fork at room temperature, pressure, and atmosphere. White voltage noise is applied to the fork and the resulting output is recorded. The ratio of the output to input is shown as the closed circles in Fig. 1(b). The response fits well to the Lorentzian line shape

$$A \frac{f_0}{Q} f / \sqrt{(f_0^2 - f^2)^2 + \left(\frac{f_0}{Q} f\right)^2} \quad (1)$$

with  $A = 17.14$ ,  $f_0 = 32\,773.3 \text{ kHz}$ , and  $Q = 8552$ , shown as the solid line. This result demonstrates that the effect of the package capacitance has been negated and the harmonic oscillator is an excellent model for the response of the tuning fork. Values for the  $RLC$  circuit model can be determined by comparing the above-mentioned line shape to the formula for the gain of the amplifier with the  $RLC$  resonator as the input impedance, yielding  $R = Z_{\text{gain}}/A = 0.494 \text{ M}\Omega$ ,

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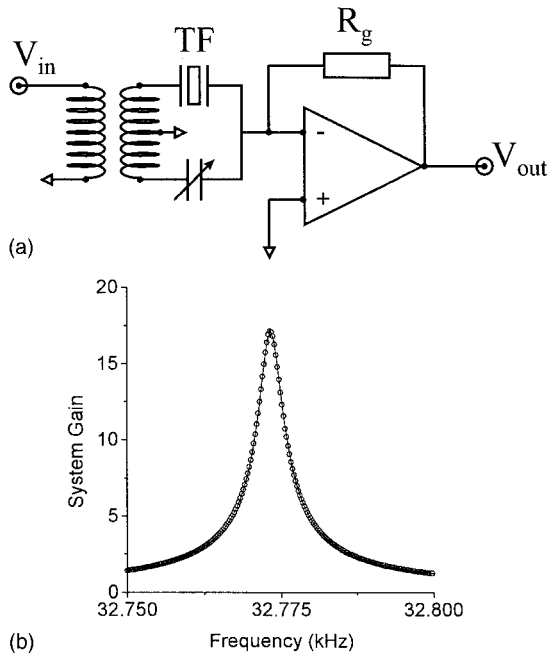


FIG. 1. The measurement system (a) and system response (b). The tuning fork, TF, is driven by a voltage source,  $V_{in}$ , which is coupled to TF through a transformer. The center tapped transformer also supplies a  $180^\circ$  phase shifted wave form to the variable capacitor so as to cancel the current due to stray parallel capacitance of the tuning fork. The net response is that of a series  $RLC$  oscillator and is due only to the motion of the quartz resonator. The resulting current is measured as a voltage,  $V_{out}$ , across  $R_g$ . The response of the circuit  $V_{out}/V_{in}$  is shown as a function of frequency in (b). The solid line is a fit to Eq. (1), demonstrating that the tuning fork responds like a series  $RLC$  resonator and that the bridge circuit nulled the stray parallel capacitance of the tuning fork.

$L = RQ/2\pi f_0 = 20.5$  kH, and  $C = 1/(4\pi^2 f_0^2 L) = 1.14$  fF.  $R$  is large enough that we attribute it completely to mechanical dissipation associated with motion of the quartz.

Another important calibration is the amplitude of oscillation of the tuning fork as a function of the output voltage, which we denote by the parameter  $\alpha$ . This has been determined by interferometrically measuring the physical amplitude of oscillation of one arm of the tuning fork while simultaneously measuring the output voltage of the system. This interferometric technique is described in detail elsewhere.<sup>8</sup> To interpret this measurement, it is necessary to define the relationship between the measured output voltage and the oscillation amplitude of one arm of the tuning fork. The output voltage is sensitive only to the antisymmetric mode of the tuning fork,  $V_{out} = c(x_1 - x_2)$ , where  $c$  is a constant and  $x_1$  and  $x_2$  are the amplitude of motion of the two arms of the fork. When driving the fork with an external voltage, as in our experiment, only the antisymmetric mode is excited, yielding  $x_1 = -x_2$ . Thus, we define  $V_{out} = 2cx_1 = x_1/\alpha$ . Our calibration yields  $\alpha = 59.6 \pm 0.1$  pm/mV. Viewing the tuning fork as a current source, it is convenient to write the above-mentioned equation in terms of the current to voltage converting resistor,  $\alpha = \beta/Z_{gain}$ , yielding  $\beta = 0.505$  m/A. Because the charge separation in the tuning fork is amplitude dependent, calibration in terms of current yields an accurate measure of the amplitude of motion of the fork.

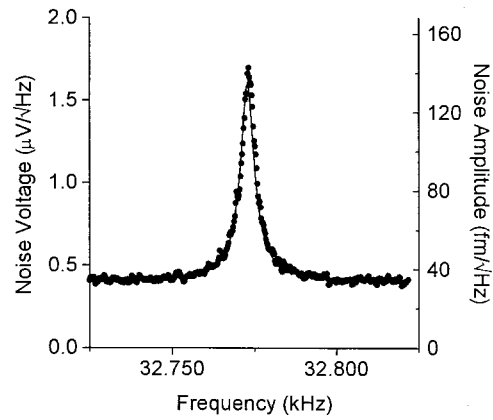


FIG. 2. The noise spectrum of the circuit in Fig. 1(a). These data were taken by grounding  $V_{in}$ . The solid line is a parameterless fit, as described in the text, using the circuit parameters determined from Fig. 1(b). The agreement between data and theory demonstrate that the origin of the noise is both the Johnson noise associated with  $R_g$  and the thermal noise of the resonator.

The fundamental limits of our ability to measure the resonance are defined by the intrinsic system noise. We can determine these limits experimentally by measuring the output noise with the input voltage grounded. The resulting output is shown in Fig. 2 as the closed circles. A noise analysis of the circuit shows the two primary noise sources are the Johnson noise of the feedback resistor,  $\sqrt{4k_B TR_g}$  V/ $\sqrt{Hz}$ , and the Johnson noise associated with mechanical dissipation in the fork, as manifested by the  $R$  in the series  $RLC$  equivalent circuit,

$$\sqrt{4k_B TR(Z_{gain}/R)} \times (ff_0/Q/\sqrt{(f_0 - f^2)^2 + (ff_0/Q)^2}) V/\sqrt{Hz}. \tag{2}$$

$k_B$  is the Boltzmann constant,  $T$  is temperature, and all the other parameters have already been determined experimentally. These two noise terms add in quadrature. *This model has no adjustable parameters* and is shown as a solid line in Fig. 2. Based on the agreement between the model and the data, we assert that enough about the performance of the system is understood to make definitive statements about the fundamental limits to the use of the quartz tuning fork for force measurements. In particular, we address issues related to the minimum detectable displacement, minimum measurable force, and maximum measurement speed.

### III. DISPLACEMENT AND FORCE DETECTION

The first issue relates to the minimum detectable displacement of the tuning fork. The power spectrum of the noise associated with the tuning fork [i.e., the square of Eq. (2)] can be integrated so as to obtain the root mean square (rms) voltage noise,  $V_{rms}^2 = 4k_B TR(Z_{gain}/R)^2 (\pi f_0/2Q)$ , which evaluates to  $V_{rms} = 3.81$   $\mu$ V. To relate this to the thermal motion of the arms of the tuning fork, we take the time average of our previous definition relating output voltage to motion of the arm of the fork:

$$V_{\text{rms}}^2 = \langle c(x_1 - x_2)^2 \rangle = c^2(\langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle) = 2c^2\langle x_1^2 \rangle,$$

where we have made the approximation that the arms of the tuning forks are only very weakly coupled and their thermal motion is therefore uncorrelated. We thus arrive at the relation  $x_{\text{rms}} = \sqrt{2}\alpha V_{\text{rms}} \approx 0.321$  pm. This is the random motion of one arm of the fork due to thermal fluctuation.

With this value for  $x_{\text{rms}}$ , we use the equipartition theorem to calculate an effective spring constant,  $K = k_B T / x_{\text{rms}}^2 \approx 40.3$  kN/m.<sup>9</sup> We can compare this experimentally determined value of  $K$  with the calculated spring constant for one arm of the tuning fork. The theoretical spring constant is obtained from the formula  $K = Ewt^3 / (4l^3)$ ,<sup>10</sup> where  $E = 7.87 \times 10^{10}$  N/m<sup>2</sup> is the Young modulus of quartz. We have measured for our tuning fork  $w = 0.50$  mm is the width of the fork,  $t = 0.65$  mm is the thickness of the fork, and  $l = 3.85$  is the length of one arm of the fork. Using these parameters we obtain  $K \approx 47$  kN/m, which agrees reasonably well with our experimental result.

The thermal energy can be thought of in terms of an effective force acting on the tuning fork. This force has a flat power spectrum,  $S_F$ , in units of square Newtons per hertz. One can calculate the power spectrum from<sup>11</sup>

$$x_{\text{rms}}^2 = \int_0^\infty S_F \left| \frac{f_0^2/k}{f_0^2 - f^2 - i(ff_0Q)} \right|^2 df.$$

Evaluating the integral and again using the equipartition theorem, one obtains

$$S_F^{1/2} = \sqrt{2/\pi f_0 Q} (k_B T / x_{\text{rms}}) = 0.615 pN / \sqrt{\text{Hz}}.$$

$$S/N = \frac{\frac{1}{\beta} \frac{Q}{k} F_s}{\sqrt{4k_B T \frac{1}{R_g} \Delta f + 4k_B T \frac{Q}{Z_r} \int_{\Delta f} df \frac{(ff_0/Q)^2}{(f_0^2 - f^2)^2 + (ff_0/Q)^2}}}, \tag{3}$$

where all terms are written as currents. The numerator is just the response of the system to a resonant force and the denominator is the quadrature sum of the two noise terms. We have written  $R$  as  $Z_r/Q$  where  $Z_r = \sqrt{L/C}$  is the resonant impedance of the inductive and capacitive terms in the  $RLC$  resonator. This equation is useful because it shows directly how  $S/N$  scales with  $Q$ ,  $R_g$ , and  $\Delta f$ . The noise is dominated by the resonant impedance of the tuning fork as long as the resonant impedance is significantly less than  $R_g$ . This criterion is good out to frequencies of order

$$|f - f_0| \leq \frac{f_0}{2Q} \sqrt{\frac{R_g}{Z_r/Q} - 1}.$$

This is clearly seen to be the case in Fig. 2. One would prefer that the noise always be dominated by the thermal noise of the fork; however, as one works at larger bandwidths, the

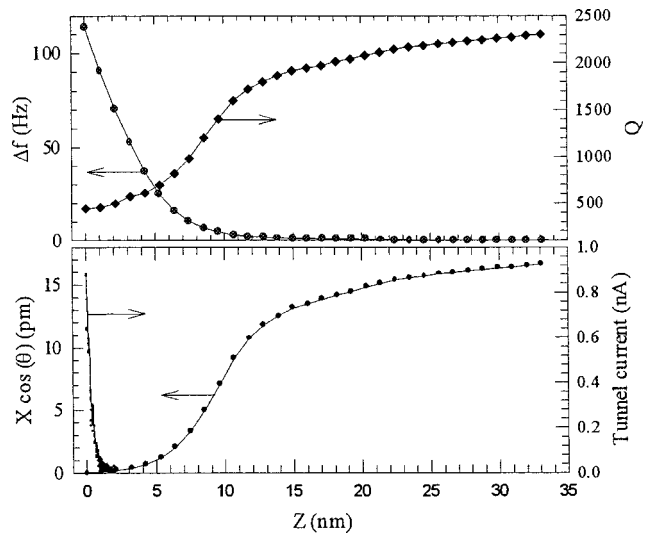


FIG. 3. The behavior of the resonator as a function of the height of the probe above the surface taken in atmospheric conditions. (a) The output of the “X” channel of the lock-in amplifier as a function of the height of the probe above the surface. Note that this goes to zero as the probe reaches the surface. Also shown is the tunneling current, which we use as an indication of when the probe makes contact with the surface. (b)  $f_0$  and  $Q$  of the resonator are shown as a function of the height of the probe above the surface. These data were taken by positioning the probe open loop at a fixed height above the surface and measuring the response function of the resonator, as in Fig. 1(b). Note that frequency shifting only occurs in the last few nanometers of the approach.

As will be discussed in the following, it is important to understand the sensitivity to force as a function of bandwidth. The ratio of signal voltage to noise voltage,  $S/N$ , as a function of bandwidth,  $\Delta f$ , is given by

noise associated with  $R_g$  becomes dominant. This can be remedied by making  $R_g$  larger; however, that degrades the time response of the amplifier due to the stray capacitance,  $C_g$ . Ultimately, the choice of  $R_g$  is a tradeoff between fast response and large  $S/N$  ratio. We believe that  $R_g \sim 10$  M $\Omega$  is a reasonable compromise for our system.

Tuning forks are usually used as force sensors by attaching an appropriate probe to one arm of the fork. The measurement is made by resonantly driving the fork with a constant amplitude force and monitoring the response as a function of the height of the probe above the sample. The oscillatory motion of the probe can be either parallel or perpendicular to the surface. In our experiments, the probe oscillates parallel to the surface. As the probe gets closer to the sample both dissipative and reactive forces are experienced by the probe, resulting in a decrease in the amplitude and a change in phase of the tuning fork. It is important that the

probe remain stiff at the resonant frequency of the fork to ensure maximum damping of the tuning fork. Shown in Fig. 3(a) is an approach curve for  $A \cos \theta$ , where  $A$  is the amplitude of oscillation and  $\theta$  is the phase of the oscillation referenced to the phase of the unperturbed (i.e., fully retracted) oscillator. In practice this corresponds to the “X” output of a lock-in amplifier. These data are taken at room temperature, pressure, and atmosphere with a peak amplitude of 16 pm. The probe is a sharpened gold scanning tunneling microscopy probe and the sample is freshly peeled, highly oriented, pyrolytic graphite. This arrangement allows simultaneous tunneling between the probe and the sample so as to indicate when the probe contacts the surface. The tunneling current is also shown in Fig. 3(a). The height of the probe above the surface is calibrated using the onset of tunneling as the indicator of contact between the surface. These data completely rule out the notion<sup>12</sup> that approach curve involves contact between the probe and the surface.

Shown in Fig. 3(b) is  $Q$  and  $f_0$  as a function of the height of the probe above the surface. These data were taken in open loop, parking the positioning system a fixed distance above the surface and probing the response of the fork. Note that dissipative forces dominate most of the approach while shifts in resonant frequency become important only within a few nanometers of the surface. This fact is crucially important for the analysis that follows. A future paper will discuss the nature of the forces involved in this experiment.<sup>13</sup>

**IV. MEASUREMENT SPEED**

The final issue is the speed with which the measurement can follow changes in force. Force images of surfaces are made by fixing the height of the probe above the surface using a closed loop control system. Some combination of the amplitude, and/or phase of the tuning fork signal is used as the set point in the control loop. When using conventional Si cantilevers it is well known<sup>14</sup> that one must resort to phase sensitive techniques of system control in order to make the system respond faster than of the open loop response time of the resonator,  $1/\tau = \pi f_0/Q$ . These techniques have also been successfully implemented with tuning fork systems.<sup>15,16,5</sup> However, we show in the following that conventional proportional and integral (PI) feedback control is sufficient for most uses of tuning forks. We justify this experimental observation with a short theoretical discussion.

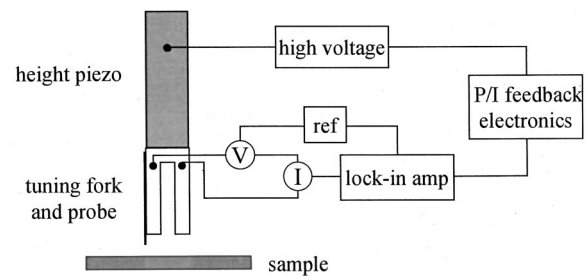


FIG. 4. A schematic representation of the control system. The tuning fork is driven by a voltage source. The resulting current is demodulated using a lock-in amplifier and input to the control electronics, which drives a high voltage amplifier. The loop is closed by driving a piezoelectric tube to adjust the height of the probe above the surface which controls the amount of current generated by the tuning fork.

The experiment is done at room temperature, pressure, and atmosphere. For clarity, the experiment is depicted schematically in Fig. 4. A tapered fiber probe is mounted on the fork consistent with operation in a near-field scanning optical microscope. The open loop response of the loaded tuning fork corresponds to the line shape in Eq. (1) with  $f_0/2Q = 8$  Hz and correspondingly  $\tau = 20$  ms. The tuning fork is driven at resonance by a voltage source of fixed amplitude and frequency and the resulting current is measured with the circuit of Fig. 1(a). This output is detected with a lock-in amplifier whose phase is referenced to the phase of the resonantly driven tuning fork. The “X” output of the lock-in amplifier is fed to conventional PI feedback electronics which then adjusts the position of the probe above the surface. The set point of the feedback is adjusted so that the probe is fixed approximately 10 nm above the surface. To test the response of the tracking system, the vertical position of the sample is intentionally dithered at 100 Hz using a 1 nm square wave. The output of the control electronics, shown in Fig. 5, clearly follows the 100 Hz signal. The rise time of the response is of order 0.5 ms, 40 times faster than the open loop response. We are unable to go faster and believe the reason is due to resonances in the mechanical positioning system; however, the point is clear: simple PI control electronics yield a stable tracking system that can operate at speeds much faster than the response of the tuning fork resonator.

A simple theoretical justification of this experimental result follows. Consider the tuning fork and the lock-in ampli-

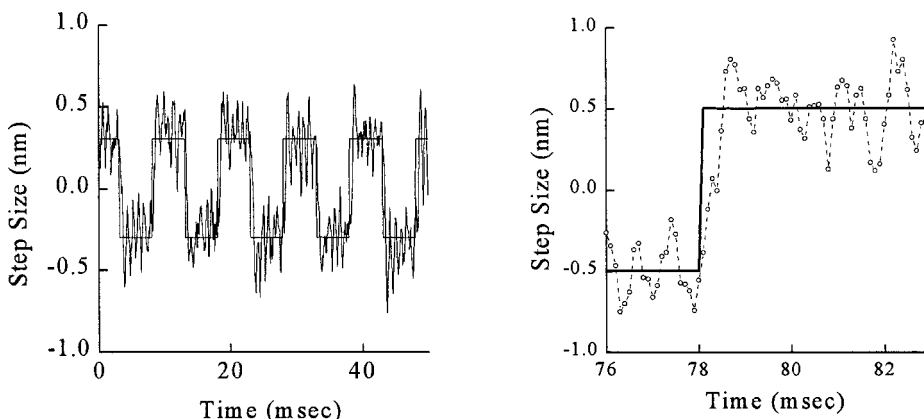


FIG. 5. The response of the feedback loop to a 1 nm, 100 Hz square wave modulation of the sample-tip height is shown. The input is shown as the solid line and the response is shown as the circles. (right) Expanded view of (left). The response time of the system is of order 0.5 ms, which is 40 times faster than the open loop resonator.

fier as a single unit. The lock-in amplifier demodulates the signal from the fork, beating the ac signal to dc. The response function of a harmonic oscillator demodulated at its resonant frequency is a single pole low pass filter with the response  $(i/\tau)/(2\pi f + i/\tau)$ , where  $\tau$  is given above. The control system is now that of a classic tracking system where the system to be controlled responds as a single pole low pass filter. This problem is well documented in standard undergraduate texts.<sup>17</sup> The optimum control electronics for this system uses *PI* feedback electronics. The response function for the closed loop system is proportional to  $e^{-\Gamma t} e^{\pm i\Omega t}$ . When the relative magnitudes of  $P$  and  $I$  are optimized to  $I = (1 + P)^2/2\tau$ , the relations  $\Gamma = \Omega = (1 + P)/2\tau$  are obtained. Note that as  $P$  and  $I$  are increased the system responds faster and remains stable.

Complications to this simple analysis occur because  $Q$  and  $f_0$  are functions of the height of the oscillator above the surface. These complications degrade the maximum achievable response speed only when they introduce additional phase lag in the feedback path. If only the  $Q$  of the resonator changes, the tuning fork still demodulates as a single pole low pass filter and no additional phase lag is introduced. As the resonant frequency shifts to  $f_0 + \epsilon$ , the demodulated response becomes

$$\frac{1}{2} \left[ \frac{i/\tau}{2\pi(f - \epsilon) + i/\tau} + \frac{i/\tau}{2\pi(f + \epsilon) + i/\tau} \right].$$

The closed loop response of this system retains the same functional form,  $e^{-\Gamma t} e^{\pm i\Omega t}$ , with the modification

$$\Gamma = \frac{(1 + P)}{2\tau}, \quad \Omega \approx \frac{(1 + P)}{2\tau} \sqrt{1 + \left( 2\pi\epsilon \frac{2\tau}{(1 + P)} \right)^2}.$$

This simple analysis shows clearly the origin of the ringing that normally accompanies frequency shifts of the resonator. This ringing is analogous to increasing the integral gain such that the control loop is out of balance. This effect is not important as long as the shift in resonant frequency remains small,  $\epsilon < (1 + P)/(4\pi\tau)$ . This condition is  $\epsilon < 300$  Hz in the above experiment, which is satisfied even for the maximum frequency shifts shown in Fig. 3(b). The situation is different for very high sensitivity Si cantilever applications where the dither amplitudes and frequency shifts are considerably larger. Clearly, shifts in resonant frequency become a serious problem for these Si cantilevers and solutions involving phase sensitive detection are necessary.<sup>14</sup> However, as is documented previously, tuning fork sensors do not suffer these problems and thus simple *PI* control is sufficient.

It is clear that the advantage of having a probe with very high  $Q$  is that it allows for measuring very small forces. From the above-mentioned noise analysis, the signal-to-noise ratio scales as  $Q/\sqrt{\Delta f}$  when measuring with a closed loop

response significantly faster than the open loop response. This strongly suggests that one should not aim to reduce the  $Q$  of the system in an attempt to scan fast. It is far more efficient to maximize  $Q$  and correctly implement the control system.

## V. CONCLUSIONS

In summary, we have documented some fundamental limits to the use of quartz tuning fork resonators as force detectors in scanned probe microscopy. At room temperature, pressure, and atmosphere these force sensors have a noise floor of  $0.62$  pN/ $\sqrt{\text{Hz}}$  and experience a root mean square thermal motion of only  $0.32$  pm. We show the  $Q$  should always be maximized to obtain the highest force sensitivity and that this in no way degrades the response time of the measurement when implemented within a standard closed loop configuration. Finally, proportional-integral feedback is a stable control system for making measurements faster than the open loop response time of these resonators because their dither amplitudes and force induced frequency shifts are relatively small.

## ACKNOWLEDGMENTS

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