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The Design of Exactly Constrained Walking Robots

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Abstract— **This paper discusses the design of legged walking robots that are exactly constrained during the stance phase of locomotion. Legged robots with a large number of actuated degrees of freedom, while allowing for the widest range of controllable foot placements, often end up with overconstrained kinematics when in contact with the ground, requiring complex redundant control schemes for effective locomotion. Exactly-constrained robots would be capable of full body mobility while avoiding the weight and complexity costs of fully actuating each joint and would also allow for simpler control schemes. We discuss the constraints and degrees of freedom of a common legged robot kinematic structure and describe strategies for removing redundant constraints. Two major design considerations – architectural singularities and the uniqueness of the ground reaction forces – are discussed along with potential solutions. Finally, a prototype exactly-constrained walking robot is presented as a validation of this design strategy.**

I. INTRODUCTION

Perhaps the biggest advantage of legged robots over their wheeled peers is the potential for locomotion over rough terrain, including uneven ground in human environments, such as stairs and curbs. Through the ability to pick their legs up over obstacles and place them in arbitrary locations in their workspace, such robots can have multiple, controllable stance points for stabilizing the body. Kinematic legged walkers (that is, walkers that rely on kinetostatic/ZMP stability rather than a dynamic gait) are often fully actuated to enable arbitrary reconfiguration of each leg relative to the body. However, once such robots are placed on the ground, they are often overconstrained due to the fact that the ground contacts add more restraints on movement than the passive degrees of freedom in the robot.

The problem of overconstraint presents a design tradeoff between control authority and control complexity. Articulated/active degrees of freedom, while theoretically allowing a larger range of stably controllable configurations, require a correspondingly large number of highly-responsive closed-loop degrees of freedom. In the common scenario when the active control is not instantaneous, these controlled degrees of freedom present as physical constraints on motion. While this overconstraint is not bad *per se*, it makes effective control difficult. Many control laws have been proposed to address this problem, either by avoiding overconstrained motions (e.g. [1]), by using impedance control (e.g. [2]), or by learning very low-impedance force profiles specific to the tasks performed (e.g. [3]). Although these methods have been shown to work, they all rely on actuators having very low output impedance, or the use motors with high-impedance gear transmissions with high-fidelity output sensing, and end up being imperfectly implemented due to shortcomings in either/both.

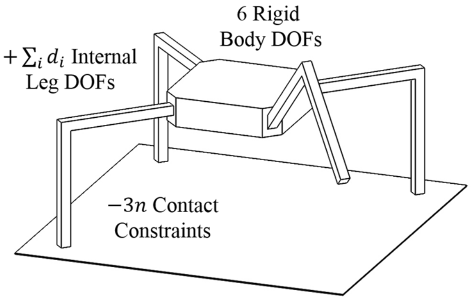

Figure 1. General mobility of an n-legged robot in contact with the ground. The robot begins with 6 rigid body DOFs, gains any unconstrained interal leg DOFs, but loses 3 DOFs per ground contact.

Legged robots can be designed to allow full body mobility in any phase of the gait cycle without control redundancy if the kinematic and actuation structure is carefully chosen. Such a robot would not be as dependent as a fully actuated robot on a suspension to guarantee full ground contact on all feet, and could also be made lighter and cheaper through a reduction in the number of actuators used. Furthermore, and perhaps most importantly, exactly constrained actuation eliminates the need for complicated control schemes to accommodate kinematically redundant actuation.

The large number of multi-legged robots described in the literature to date might be placed in two major categories: “full mobility” designs (ranging from four to six controllable body degrees of freedom) with a large number of actuated degrees of freedom, and minimalistic designs with few actuators that lack fully-controllable mobility. The majority of early multi-legged robots fall into the former category [4-6], having the ability to exert arbitrary forces on the ground, but in doing so may impose internal motions on the body of the robot in a way that violates the kinematic constraints with the ground. For example, the legs can be driven into the surface of the ground, producing large, unpredictable forces capable of disrupting the body stability or causing foot slippage.

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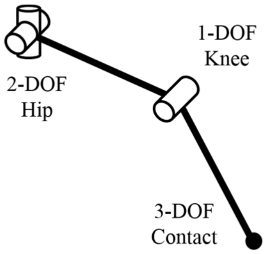

Figure 2. Schematic of a URS leg in contact. The hip and knee joints can be actuated; the foot contact acts as a spherical joint.

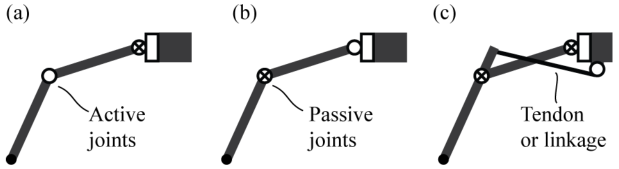

Figure 3. Three potential actuator reductions for URS legs: (a) passive planar hip joint, (b) passive knee joint, and (c) coupled planar hip and knee joints. The crossed joints in (a) and (b) are passive, while the actuated tendon in (c) couples both passive joints.

In terms of the latter category of legged platform design with a smaller number of actuators, there have been a number of more simplistic multi-legged robot designs proposed in the past decade or so. In order to address the incompatibility of the mechanical structure to uneven terrain, a number of these robots have included elastic elements in the leg to act as a suspension and ensure ground contact [7-10]. Alternatively, others have implemented series-elastic actuators to similar effect [11-13]. However, as far as we are aware, no one has systematically approached the design of multi-legged walking robots with exact constraint as a major functional goal.

In this paper, we show that it is possible to design exactly-constrained kinetostatic walkers by considering the nature of the foot contacts with the ground as well as the degrees of freedom and design of the legs of the robot. We lay out several performance factors for such walkers that heavily influence the space of feasible designs, and discuss an example 4-legged walking robot that is fully actuated and exactly constrained when supported on any three legs. We begin the paper (Section II) with a general overview of legged robot mobility along with an example to illustrate how actuators can be selectively removed to result in exact constraint. Section III describes two major design considerations that must be taken into account when dealing with exactly-constrained walking robots, specifically, architectural singularities and the uniqueness of the ground reaction forces. Finally, Section IV presents experimental validation of the feasibility of exactly-constrained robot locomotion.

II. LEGGED ROBOT MOBILITY

In this section we step through the analysis of the mobility of legged robots to explore how exactly-constrained walking robots can be designed by removing actuated degrees of freedom (rather than adding compliant elements to mitigate overconstraint). One common leg architectures, the Universal-Revolute-Spherical (URS) leg, is analyzed and candidate actuation schemes are presented that leave the robot exactly constrained when three legs are in contact with the ground (with four and more legs discussed in section IV).

A. Basic Mobility Analysis

The issue of overconstraint in legged robots can be illustrated by looking at an n-legged robot, each leg having d_i unconstrained (e.g. unactuated) internal degrees of freedom (Fig. 1). In the absence of contact, the mechanism has a total of 6 rigid body degrees of freedom (DOFs) plus any internal DOFs of the legs; in a fully actuated robot, $d_i = 0$ so the total mobility of the floating robot is $m = 6$. Once the robot is in contact with the ground, each foot exerts 3 contact constraints based on its approximation as a point contact [14]. As such, a fully actuated robot would have a total mobility of $m = 6 - 3n$; for exact constraint, one must remove $(3n - 6)$ actuators from the robot. For example, a robot standing on 3 fully-actuated legs (the minimum for statically stable stance) would have a mobility of $m = -3$, yet if 3 actuators are removed $(\sum_i d_i = 3)$ our robot would be exactly constrained.

Another way to look at this is by using the Chebychev–Grübler–Kutzbach (CGK) criterion [15], a traditional mobility metric for mechanisms. It can take a number of equivalent forms, but for a mechanism in 3D space the CGK mobility is given as:

$$m_{CGK} = 6(N - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

where N is the number of bodies (leg links + platform + ground), j is the number of joints, and f_i is the number of DOFs of the i^{th} joint. The CGK criterion gives the total number of actuation constraints necessary to exactly define the configuration of a robot; the degree of over-, under-, or exact constraint can be determined by subtracting the total number of actuators in the robot. For example, if each of the legs of a tripod stance set had k links and k 1-DOF joints (not including the ground contact), all actuated, $m_{CGK} = 6([nk + 2] - [nk + n] - 1) + n(k + 3) = 6 - 3n + nk$, there are a total of nk actuators, and a net mobility of $m = 6 - 3n$ as before.

This leads to an interesting result – if one wanted to design a robot with a stance tripod of identical legs they would have to remove one actuated DOF per leg (since $3n - 6 = 3$ for $n = 3$). Additionally, the addition of more joints in the legs, assuming that they are accompanied an equal number of actuators, do not change the overall degree of overconstraint. However, the addition of more legs requires more complicated actuation schemes to meet the requirements for exact constraint; for example, a 4-legged stance set would require the removal of 6 actuators over 4 legs, a much trickier prospect.

B. Example – 3-URS Robots

One common leg design used both in legged robots and in parallel mechanisms is the URS leg (Fig. 2) [16,17], where the universal and revolute joints can be actuated and the foot contact is treated as a spherical joint. These legs allow for full body mobility in space and have two links, a 2-DOF joint at the hip, and a 1-DOF joint at the knee. Based on the conclusions in the previous discussion, an exactly constrained 3-URS stance set of URS legs would require the removal of 1 actuator per leg for a total of 6 actuators. This can be accomplished in a number of ways; we will examine 3 possibilities.

One simple way to remove an actuator from a URS leg would be to leave the planar hip joint passive (Fig. 3a). In this case the remaining hip joint serves to select the plane of operation of the leg while the knee joint determines the distance from the foot to the hip. An alternative underactuated URS leg would leave the knee joint passive (Fig. 3b). In this case the planar hip joint determines the angle of the proximal link while the overall distance from the foot to the knee is left undetermined. Finally, the planar hip and knee joints can be coupled, either through a tendon or linkage as in Fig. 3c, to remove an actuator. In this case the mechanism has some instant center of rotation based on its kinematic parameters but no joint is fully passive.

C. *Summary*

In this section we have presented a method for determining the degree of overconstraint of a legged robot in contact with the ground as well as a strategy for removing some of its actuators to ensure that a tripod (or any other set) of legs allow for full mobility without overconstraint. We also showed an examples of a leg design along with some potential underactuation schemes.

III. DESIGN CONSIDERATIONS

When delegating more of a walking robot's function to the passive mechanics of the limbs, the particulars of the mechanical design become much more important. In this section we will present two major design considerations that must be taken into account when designing exactly constrained walking robots.

A. *Architectural Singularities*

The first, and most important, design consideration is that any set of legs supporting a statically stable walking robot are equivalent to a parallel manipulator, and can therefore be analyzed within a parallel mechanisms framework [18]. As with parallel mechanisms, kinematic features such as architectural singularities must be considered during both mechanical design and gait synthesis.

An architectural singularity in an exactly constrained walking robot is defined as a configuration in which the matrix of actuator, joint, and contact constraints becomes rank-insufficient [19]; a simple case would be where one or more of the kinematic constraints are redundant such that the robot gains additional uncontrollable DOFs. Methods for identifying such configurations have been well researched in the parallel mechanisms literature [19], but in the case of an overconstrained robot they tend to be less relevant, as even with a redundant constraint the robot often remains fully constrained.

One method for identifying architectural singularities has been presented by Dai et al. [20] using screws, represented as \$, which are elements in a six-dimensional vector space used to represent motions of (twists) and forces acting on (wrenches) a body [21,22]. For a revolute joint, the screw representing its motion is defined as $\$ = \{v, r \times v\}$ where r is the position vector of the screw and v is a unit vector in the direction of the joint axis. For a prismatic joint the screw representing its motions is $\$ = \{0, v\}$.

In order to fully describe the state of constraint of a parallel mechanism or legged robot with n legs, a number of screw systems, or vector subspaces, are defined. For each of the legs, we first construct the branch motion-screw system, S_{bi}, which is simply the subspace spanned by the motion screws of each of the joints. This describes the body motion permitted by each leg; we can then find the reciprocal screw system, also known as the branch constraint-screw system, S_{bi}^r, which spans the constraints imposed on the body by each leg. In this case, the definition of a reciprocal system is:

$$S^r \equiv \{\$_1 | \$_1 \circ \$_2 = 0, \forall \$_2 \in S\} \tag{2}$$

where the reciprocal relationship is defined as:

$$\$_1 \circ \$_2 = \$_1^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \$_2 \tag{3}$$

and the matrix is partitioned into 3×3 blocks with I being the identity matrix [23]. In practical terms, two screws are reciprocal when the wrench / constraint represented by one performs no virtual work while the body undergoes an infinitesimal twist / motion represented by the second screw. In other words, the reciprocal relationship allows for the constraints that correspond to a set of DOFs to be found and vice versa.

Given S_{bi}^r for each leg, the body constraint-screw system, S^r, can then be defined as the union of all the S_{bi}^r's, or:

$$S^r = S_{b1}^r \vee S_{b2}^r \cdots \vee S_{bn}^r \tag{4}$$

This system spans all of the constraints acting between the ground and the body. It represents the fact that any constraint acting on the body can be applied by any of the legs. Incidentally, the reciprocal system to S^r, the body motion-screw system, S_f, spans all of the permissible relative motion between the ground and the body. This can be used to study the instantaneous DOFs of the robot body.

A robot can be determined to be at an architectural singularity based on whether the rank of S^r is less than the sum of the dimensions of S_{bi}^r. In a system with no redundant constraints, $\operatorname{rank}(S^r) = \sum_i \dim(S_{bi}^r)$, as each leg constraint imposes a distinct constraint on the body. However, if $\operatorname{rank}(S^r) < \sum_i \dim(S_{bi}^r)$, at least one of the constraints is redundant and the robot may be underconstrained. In the case of an exactly constrained walking robot, the architectural singularities of the overall robot are much less important than the architectural singularities of the *actuated* robot. This means that after the removal of actuators each remaining actuated joint is treated as fixed (for the purposes of analysis) and its constraints can then be analyzed.

In a 3-URS robot, an exactly constrained walker could have passive knee joints as in Fig. 3b. In this case, S_{bi}^r contains two pure force wrenches acting at the foot; one parallel to the passive joint axis and one collinear with the distal leg link, stemming from the fact that members of a reciprocal system must intersect or be parallel to *all* of the screws in the base

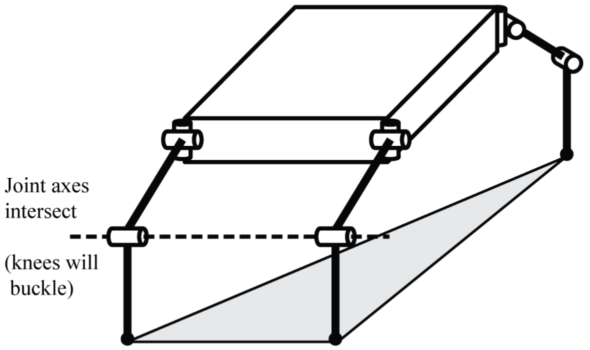

Figure 4. One architectural singularity of a 3-URS stance set – two coaxial passive joints. Due to the loss of constraint the knees will simply be able to buckle.

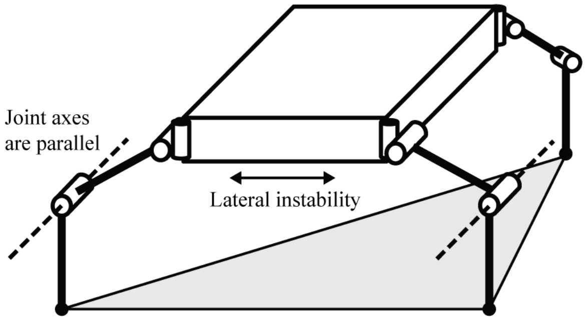

Figure 5. An ill-conditioned robot configuration. The two legs with parallel knees are unable to resist a lateral force; the third leg can resist this force but is poorly positioned to do so with any significant stiffness.

system. Combined, the three legs provide the six constraints needed to define the body pose, but there are certain configurations that result in constraint redundancy.

The most significant architectural singularity is the configuration where two adjacent legs operate in parallel planes (assuming identical legs) with both feet at the same ground height (e.g. both the planar hip and knee joint angles are the same, see Fig. 4). In this case, the constraint screws passive to the knee joint axis are collinear and the robot gains an additional DOF about the passive joint axis. This case could also be identified by noting that the two passive joints would be collinear as well (see [24]), but that simply reflects the complementary relationship between constraint and motion.

In addition to truly singular configurations, one would also want to avoid any other marginally stable configuration; one example would be configurations where the robot's constraint Jacobian is of sufficient rank to define the robot's pose but ill-conditioned such that any slop or compliance in the mechanism would allow it to move. In the case of our 3-URS robot, an example of such a configuration is one where any two legs are aligned / coplanar (Fig. 5). With very stiff joints, the walker would be exactly constrained, but due to the fact that the coplanar legs allow for planar motion of the body and the remaining leg provides a pivot fairly far from that plane, the robot would in practice move uncontrollably.

Both of these classes of instability would result in an under-constrained and uncontrollable system. When designing exactly-constrained walking robots, care must be taken to avoid such configurations, both during mechanical design as well as when synthesizing gaits for locomotion.

B. Ground Reaction Forces

A second major design consideration for exactly constrained walking robots is the fact that the ground reaction forces are uniquely determined by the robot's configuration. Such a robot is only exactly constrained under the assumption that its contacts are valid; in most cases, this means ensuring that the reaction forces fall within the friction cone at the foot. It is important to examine the nature of the ground reaction forces *a priori* to ensure that these constraints will remain stable during locomotion. Overconstrained walking robots, on the other hand, avoid this issue through the use of redundant DOFs in the legs to allow for control over the reaction forces.

The uniqueness of the ground reaction forces stems from the fact that there is a complementary relationship between the motion of the robot and its force balance. In other words, if all the actuator velocities uniquely determine the body's net twist, e.g. the system:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = A\dot{\theta} \tag{5}$$

is exactly determined, it is likewise true that the joint torques uniquely determine the body's net wrench:

$$\begin{bmatrix} F \\ \tau_b \end{bmatrix} = A^T \tau \tag{6}$$

For many mechanisms, the uniqueness of the ground reaction forces can be explained geometrically as well. As we described in Section II, designing an exactly constrained robot involves removing actuator constraints from each of the robot's legs; each leg therefore has some instant center of rotation, about which the leg is free to pivot. Consequently, in static balance the leg cannot resist a net torque about that point If we simply leave one joint passive, it would just be at the passive joint; any joint coupling would present some configuration-dependent center of rotation. In order for the leg to be in equilibrium, the reaction forces at the foot *must* pass through that center of rotation; in other words, the leg cannot resist a net torque about that point.

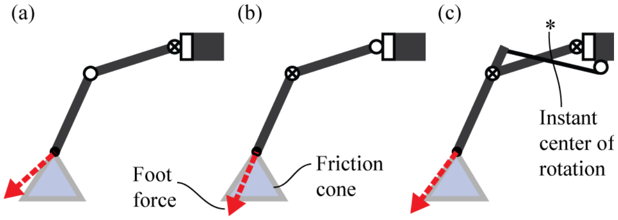

Figure 6. Reaction forces for underactuated URS legs: (a) passive planar hip joint, (b) passive knee joint, and (c) coupled planar hip and knee joints. Note how each force passes through the leg's instant center of rotation.

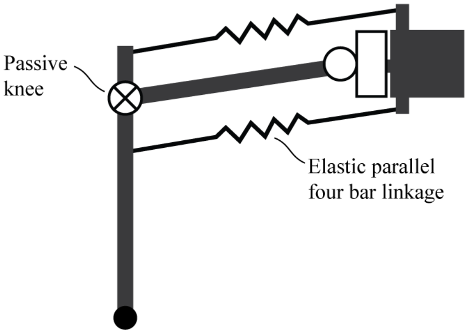

Figure 7. Potential URS leg design for nominally vertical distal link – sprung parallel 4-bar linkage between planar joints. The springs allow the foot to resist torques about the passive knee joint as well.

For example, in the 3-URS robot three potential strategies for removing actuators from the robot's legs were detailed (Fig. 3). Fig. 6 shows the resulting reaction forces for each of the strategies. The passive hip joint (Fig. 6a) requires the contact point to be near the hip to prevent the foot from slipping, undesirably reducing our static stability margin [25]. The coupled joints (Fig. 6c) are better, but synthesizing a leg with coupled joints whose instant center is generally above the contact point is non-trivial. A potentially easier solution would be to leave the knee passive (Fig. 6b) and, since the force is aligned with the distal leg link, simply keep that link as close to vertical as possible.

One strategy for achieving a nominally vertical distal leg link is through the use of springs to create a virtual 4-bar parallel linkage between the two planar joints (Fig. 7). In the absence of torques about the knee, the springs provide a kinematic suggestion to the leg keeping the distal link vertical throughout its workspace. The springs also allow for the leg to resist some net torque about the knee depending on the specific geometry and spring stiffness used.

C. Summary

When designing a robot for exactly-constrained locomotion, several stability criteria specific to this case emerge, namely, architectural singularities, ill-conditioned configurations, and configurations in which the ground reaction forces implied by the choice of passive DOFs would tend to cause instability. We have shown that simple passive structures such as a sprung parallel 4-bar linkage could be used to improve robot stability.

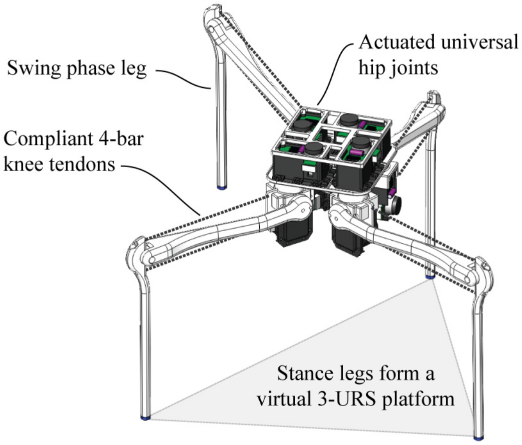

Figure 8. Illustration of the prototype 4-URS exactly constrained walking robot. Note the passive knee with a sprung virtual parallel 4-bar linkage for stability.

IV. EXPERIMENTAL CONCEPT DEMONSTRATION

A. Prototype Design

In order to verify the feasibility of exactly constrained robot walking, a prototype robot was constructed based on the design considerations and principles outlined above as a proof-of-concept, shown in Fig. 8. The simplest robot with a 3-legged stance set would have a total of 4 legs, with one leg being transferred per gait step to shift the weight of the robot to a new stance tripod. A four-legged walker is not particularly efficient at walking, but it is capable of demonstrating the stability and mobility of a tripod stance, and the stability of support transitions.

The prototype was built with 4 URS legs constructed using 3D-printed parts (Stratasys ABS*plus*). The universal hip joints of each leg were arranged in a square pattern 30 mm on a side, with the yaw axis of each hip oriented vertically. Both hip joints were actuated with Robotis Dynamixel RX-28 servo motors. The knee joints are passive, and were connected to the body with a four-bar parallel elastic linkage, as shown in Fig. 8, to ensure that the robot's legs stay parallel to the z axis of the robot body while in swing phase. The proximal leg links are 150 mm long, and the distal links are 160 mm long. At the end of each leg, a molded rubber foot improves the frictional contact with the ground, so that the assumption of no-slip point contact with the ground is reasonable.

B. Gait Design

Because it is easy to maneuver the walker into a configuration where the constraints from the passive knee joints are singular (as in Fig. 4) or ill-conditioned (vis. Fig. 5), and because only one leg of the walker could be repositioned at a time, the choice of gait for this robot was highly constrained. Figure 9 shows the tripod gait, with a step length of roughly ½ body length, that was used to control the robot. Three legs were held widely apart, and one of the two rear legs was used during each step to support the body while the front legs were repositioned. Body motion was achieved by using the tripod as a parallel platform to reposition the body in between stance changes. A robot with more legs would allow

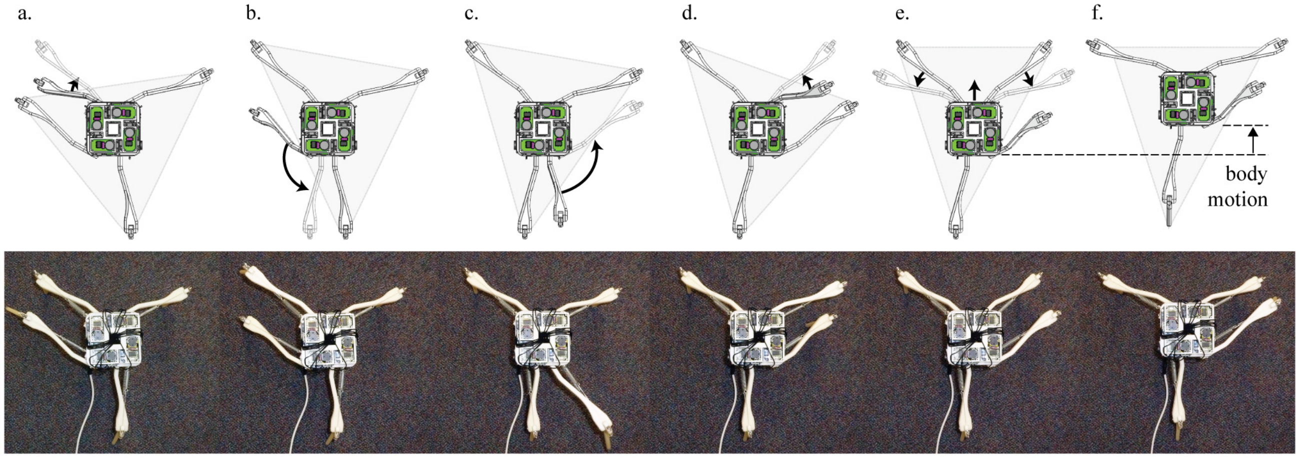

Figure 9. Locomotive gait of prototype 4-URS walker. The number of distinct motions was due to the fact that only one leg could be moved at a time as well as the fact that architectural singularities had to be avoided (see Figs. 4-5). The robot's configuration in (f) mirrors its configuration in (a).

for more flexibility when synthesizing gaits, both in terms of simultaneously repositioning several legs as well as avoiding architectural singularities.

C. *Results and Discussion*

The first part of the video attachment to this paper demonstrates the ability of three stance phase legs to reposition the body with three translational and three rotational degrees of freedom. The distal leg links remained mostly vertical during these motions, keeping the ground reaction forces from leaving the friction cone of the contact with the floor. In experiments, little slippage was observed. The problem of singular or ill-conditioned passive joint constraints, on the other hand, was a noticeable limitation to the range of motion. Two legs could not be brought into a position 180 degrees from each other as in Fig. 5 without buckling at the knees. Because the walker's body center of mass is already close to the hip joint, these regions of the workspace were also close to the edge of the static support polygon, and so did not reduce the size of the workspace relative to the support polygon area. However, in a wider-bodied robot having the same legs, it may be possible to see a more substantial reduction in body motion relative to the support area.

The locomotive gait diagrammed in Fig. 9 is also demonstrated in the second part of the video attachment, playing at 1x speed. Frames from an overhead view of the walking experiments are shown for comparison at the bottom of Fig. 9. Walking was observed to be stable up to the limitations imposed by the maximum speed of the actuators. Some static friction was observed in the elastic parallel four-bar mechanism keeping the distal links of each leg vertical, but this did not degrade stance stability significantly. Because the legs had to be repositioned one by one, the rate of body motion per step was small. This was anticipated, as the stance tripod can only be altered one at a time. A larger, six-legged walker capable of moving multiple legs per step is planned, that will enable a more in-depth exploration of gait synthesis with exactly-constrained support legs.

D. *Summary*

These preliminary results show that standing, body repositioning, and walking are possible with exactly-constrained walking robots. The main purpose of these experiments was to explore the stance stability, and to verify that the elastic four-bar mechanisms positioning the passive leg joints in swing phase enabled repeatable placement of the feet. Future iterations of this walking platform will explore increasing the number of legs and the size of the robot body, to improve gait length, static and dynamic stability, and the richness of locomotion primitives available.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have demonstrated that fully actuated legged robots often employ designs that result in overconstrained kinematics when in ground contact. This property results in increased weight, complexity, and cost due to the number of actuators used, and often requires the use of a suspension and complex control schemes to ensure stable ground contact. As an alternative, we have presented a general strategy for the development of exactly constrained kinetostatic walking robots. Such robots would be lighter, cheaper, and simpler to control than overconstrained legged robots at the cost of control over the ground forces.

Using basic metrics for the mobility of mechanisms, we can analyze the degree of over-constraint of a particular robot or leg design and determine the number of actuators that must be removed for an exactly constrained design. Based on the number of legs, different actuation and coupling schemes will provide exact constraint; however, selecting a preferential design requires the consideration of several important factors. First, the kinematic design and gait must avoid architectural singularities, and we presented one tool from the parallel mechanisms community for identifying such singularities. Additionally, since an exactly constrained walking robot will have uniquely determined ground reaction forces, the selected design must ensure stable ground contact.

A simple 4-URS exactly constrained walking robot was designed and constructed as a proof-of-concept for such walkers. The robot demonstrated full mobility and exact constraint while in stance and was able to walk using a basic gait. This provided strong support for the feasibility of similar walkers as well as a wide range of future directions for investigation.

Looking forward, we aim to design and analyze highly capable exactly constrained walkers. This will involve the investigation of alternative leg architectures, the consequences of differently sized stance sets, and the analysis of a wide range of different actuation schemes to ensure exact constraint. We would also like to take a more rigorous look into the use of passive elements such as springs to increase the effective workspace of such robots and their effect on the performance of the robot. Additionally, we will perform a study of different gait and control scheme to maximize the robot's stability and performance over a variety of terrains and while using both static and dynamic gaits. Finally, we intend to study the feasibility of lower-mobility robots; reducing the number of independent DOFs of the robot's body's workspace in the stance phase would allow us to further reduce the number of actuators used, and it is not clear that effective locomotion requires all 6 spatial DOFs to be independently controlled. Through all this, we hope to develop much more efficient high-performance legged robots for locomotion over a variety of terrains and in applications such as extra-terrestrial exploration or disaster response.

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