

# Asymptotically Tight Delay Guarantees Through Wireless Link Scheduling

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# Dynamic Scheduling in Wireless networks

- Network consisting of multiple nodes
- Certain sets of links can be simultaneously active
- Each link generates packets as per an exogenous arrival process
- Goal:
  - Need to *stabilize* the network (schedule the links so that the expected queue lengths are bounded at each link)
  - **Low Delay**

# Throughput Vs Delay

- Characterization of maximum stability region well-understood
  - Tassiulas and Ephremides, TAC 1992
- Delay minimization more challenging than throughput maximization
  - Not enough to empty the network in finite expected time but need to lower this expected time as well
  - Families of policies maximize throughput but do not minimize delay

# Related Work

- Neely, Infocom 2008
  - Maximal scheduling attains  $O(\log L)$  delay in its stability region in a network with  $L$  links
- Jung and Shah, ISIT 2007, Sarkar and Ray, TAC 2008
  - Throughput optimal policies that attain  $O(1)$  expected queue length per link in special classes of networks

# Delay properties of Throughput Optimal Scheduling

- Throughput optimal scheduling
  - Maximum weight independent set scheduling (MWIS)
  - Randomized scheduling (RS( $\mathbf{p}$ ))
- Assumptions
  - IID arrivals
  - Ratio between the second and first moments of the number of arrivals in each link is upper bounded by a constant  $\gamma$
- Terminologies
  - Arrival slack  $\beta$
  - Chromatic number of interference graph of network  $N$   $C(N)$

# Upper bound on expected delay

- MWIS attains an expected delay of at most  $(\gamma + 1)/2 * (C(N)/\beta)$  in network  $N$ .
- There exists a probability vector  $\mathbf{p}$  such that  $RS(\mathbf{p})$  attains an expected delay of at most  $(C(N)/\beta)$  in network  $N$ .
  - Bernoulli arrivals
  - Constructive proof
  - Design of  $\mathbf{p}$  needs knowledge of the arrival rate vector, link independent sets, chromatic number, coloring sets.

# Upper bound on expected delay

- Chromatic number of interference graph can be significantly smaller than the number of links in the network
- $n$  nodes randomly deployed with connectivity radius  $O(\log n)$ 
  - Chromatic number  $O(\log^2 n)$
- MWIS and RS( $p$ ) attain delays that are within constant factors of each other.

# Lower Bound on the Expected Delay

- For any real numbers  $\beta$  in  $(0, 1)$ ,  $\varepsilon$  in  $(0, 1)$ , any positive integer  $\underline{C}$  there exists a network  $N$  with  $C(N) = \underline{C}$  and an arrival rate vector with slack  $\beta$  such that the expected delay attained by any scheduling policy in  $N$  is at least  $((1 - \varepsilon)^2 / (1 + \varepsilon)) \underline{C} / 2$
- For constant  $\beta$  MWIS and RS( $\mathbf{p}$ ) (for some  $\mathbf{p}$ ) attain delays that are within constant factors of the minimum delay.

# Lower Bound on the Expected Delay: Proof

- Consider a network with  $\underline{C}$  sets of  $K$  links each
- Any two links in the same set can transmit simultaneously.
- Any two links in different sets interfere.
- Chromatic number is  $\underline{C}$
- Arrival rate in each link is  $(1 - \beta)/\underline{C}$
- For large  $K$ , each independent set receives approximately the same number of packets every slot.

# Lower Bound on the Expected Delay: Proof

- Consider the packets that arrive in an arbitrary slot  $t$
- Expected delay of these packets is the overall expected delay.
- Schedule so as to minimize the expected delay of these packets.
- When each independent set receives equal number of packets in slot  $t$ , expected delay for these packets is at least  $\underline{C}/2$ .
- Expected delay of these packets under any scheduling is at least the delay attained by the above schedule.

# Lower Bound on the Expected Delay

- Assumptions on arrival distribution mild
  - Lower bound holds for a large class of arrival distributions
- Good topologies where this lower bound does not hold exist.
- Characterize topologies that allow some scheduling policies to attain lower delays?
- Can a tighter bound be obtained by exploiting the relation between the expected delay and  $\beta$ ?

# Tighter lower bound for randomized scheduling

- For any real numbers  $\beta$  in  $(0, 1)$ , any probability vector  $\mathbf{p}$ , any positive integer  $\underline{C}$  there exists a network  $N$  with  $C(N) = \underline{C}$  and an arrival rate vector with slack  $\beta$  such that the expected delay attained by  $RS(\mathbf{p})$  in  $N$  is at least  $(\underline{C}-1)/\beta + 1$ 
  - Consider a link interference graph that is complete and has  $C$  links.
  - Arrival rates are equal in each link and the vector has a slack  $\beta$ .
  - Delay is minimized in the RS family when all links are scheduled with equal probability.
  - Above minimum delay is  $(\underline{C}-1)/\beta + 1$