

Asymptotically Tight Delay Guarantees Through Wireless Link Scheduling

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Dynamic Scheduling in Wireless networks

- Network consisting of multiple nodes
- Certain sets of links can be simultaneously active
- Each link generates packets as per an exogenous arrival process
- Goal:
 - Need to *stabilize* the network (schedule the links so that the expected queue lengths are bounded at each link)
 - **Low Delay**

Throughput Vs Delay

- Characterization of maximum stability region well-understood
 - Tassiulas and Ephremides, TAC 1992
- Delay minimization more challenging than throughput maximization
 - Not enough to empty the network in finite expected time but need to lower this expected time as well
 - Families of policies maximize throughput but do not minimize delay

Related Work

- Neely, Infocom 2008
 - Maximal scheduling attains $O(\log L)$ delay in its stability region in a network with L links
- Jung and Shah, ISIT 2007, Sarkar and Ray, TAC 2008
 - Throughput optimal policies that attain $O(1)$ expected queue length per link in special classes of networks

Delay properties of Throughput Optimal Scheduling

- Throughput optimal scheduling
 - Maximum weight independent set scheduling (MWIS)
 - Randomized scheduling (RS(\mathbf{p}))
- Assumptions
 - IID arrivals
 - Ratio between the second and first moments of the number of arrivals in each link is upper bounded by a constant γ
- Terminologies
 - Arrival slack β
 - Chromatic number of interference graph of network N $C(N)$

Upper bound on expected delay

- MWIS attains an expected delay of at most $(\gamma + 1)/2 * (C(N)/\beta)$ in network N .
- There exists a probability vector \mathbf{p} such that $RS(\mathbf{p})$ attains an expected delay of at most $(C(N)/\beta)$ in network N .
 - Bernoulli arrivals
 - Constructive proof
 - Design of \mathbf{p} needs knowledge of the arrival rate vector, link independent sets, chromatic number, coloring sets.

Upper bound on expected delay

- Chromatic number of interference graph can be significantly smaller than the number of links in the network
- n nodes randomly deployed with connectivity radius $O(\log n)$
 - Chromatic number $O(\log^2 n)$
- MWIS and RS(p) attain delays that are within constant factors of each other.

Lower Bound on the Expected Delay

- For any real numbers β in $(0, 1)$, ε in $(0, 1)$, any positive integer \underline{C} there exists a network N with $C(N) = \underline{C}$ and an arrival rate vector with slack β such that the expected delay attained by any scheduling policy in N is at least $((1 - \varepsilon)^2 / (1 + \varepsilon)) \underline{C} / 2$
- For constant β MWIS and RS(\mathbf{p}) (for some \mathbf{p}) attain delays that are within constant factors of the minimum delay.

Lower Bound on the Expected Delay: Proof

- Consider a network with \underline{C} sets of K links each
- Any two links in the same set can transmit simultaneously.
- Any two links in different sets interfere.
- Chromatic number is \underline{C}
- Arrival rate in each link is $(1 - \beta)/\underline{C}$
- For large K , each independent set receives approximately the same number of packets every slot.

Lower Bound on the Expected Delay: Proof

- Consider the packets that arrive in an arbitrary slot t
- Expected delay of these packets is the overall expected delay.
- Schedule so as to minimize the expected delay of these packets.
- When each independent set receives equal number of packets in slot t , expected delay for these packets is at least $\underline{C}/2$.
- Expected delay of these packets under any scheduling is at least the delay attained by the above schedule.

Lower Bound on the Expected Delay

- Assumptions on arrival distribution mild
 - Lower bound holds for a large class of arrival distributions
- Good topologies where this lower bound does not hold exist.
- Characterize topologies that allow some scheduling policies to attain lower delays?
- Can a tighter bound be obtained by exploiting the relation between the expected delay and β ?

Tighter lower bound for randomized scheduling

- For any real numbers β in $(0, 1)$, any probability vector \mathbf{p} , any positive integer \underline{C} there exists a network N with $C(N) = \underline{C}$ and an arrival rate vector with slack β such that the expected delay attained by $RS(\mathbf{p})$ in N is at least $(\underline{C}-1)/\beta + 1$
 - Consider a link interference graph that is complete and has C links.
 - Arrival rates are equal in each link and the vector has a slack β .
 - Delay is minimized in the RS family when all links are scheduled with equal probability.
 - Above minimum delay is $(\underline{C}-1)/\beta + 1$