TOWARDS a UNIFIED FRAMEWORK for NONLINEAR CONTROL with LIMITED INFORMATION

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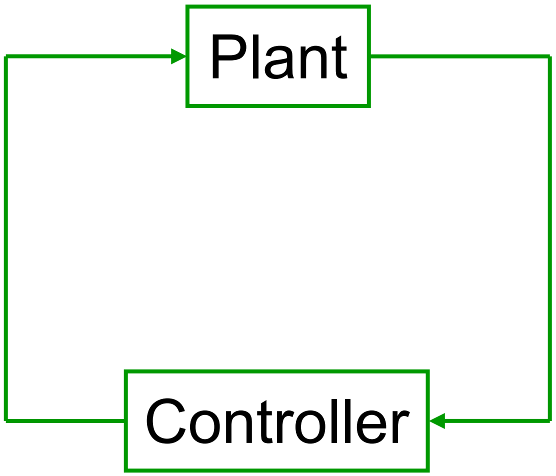
INFORMATION FLOW in CONTROL SYSTEMS



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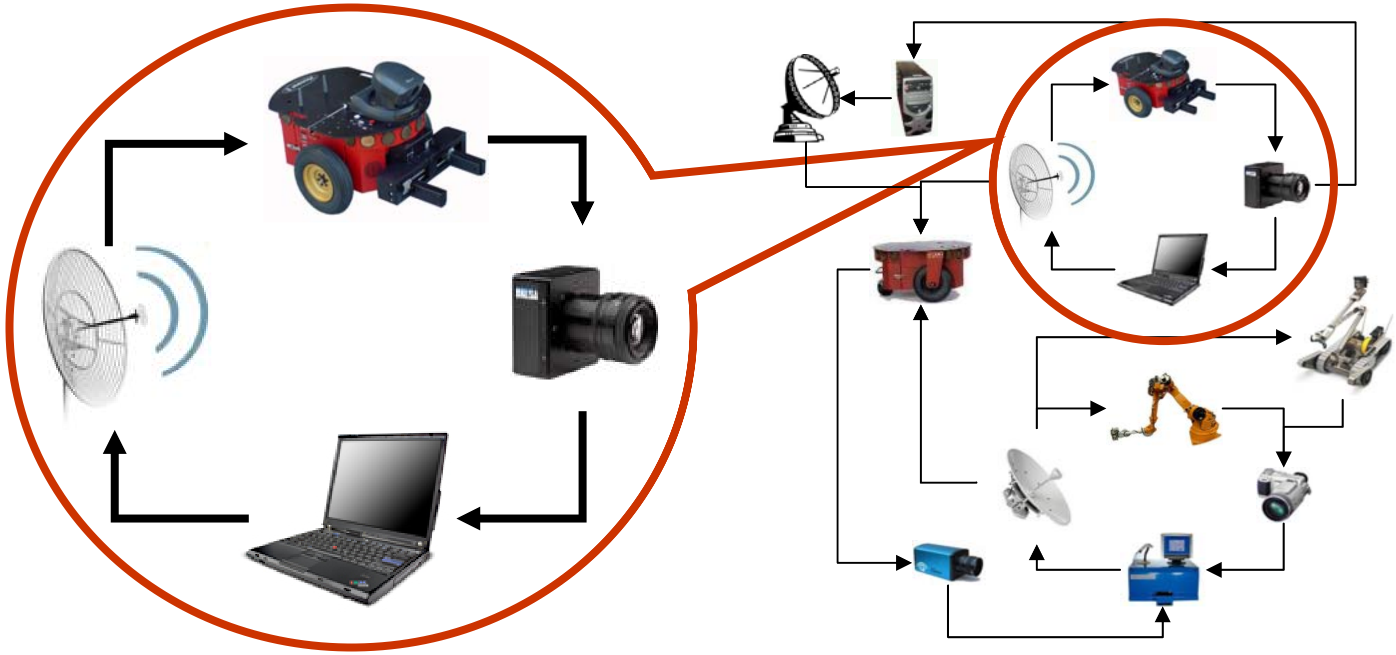


- Coarse sensing
- Limited communication capacity
- Need to minimize information transmission
- Event-driven actuators
- Theoretical interest

BACKGROUND

Previous work:

[Brockett, Delchamps, Elia, Mitter, Nair, Savkin, Tatikonda, Wong,…]

- Deterministic & stochastic models
- Tools from information theory
- Mostly for linear plant dynamics

Our goals:

- Handle nonlinear dynamics
- Unified framework for
  - quantization
  - time delays
  - disturbances

OUR APPROACH

(Goal: treat nonlinear systems; handle quantization, delays, etc.)

- Model these effects as deterministic additive error signals, e
- Design a control law ignoring these errors, $u = k(x)$
- "Certainty equivalence": apply control $u = k(x + e)$ combined with estimation to reduce e to zero

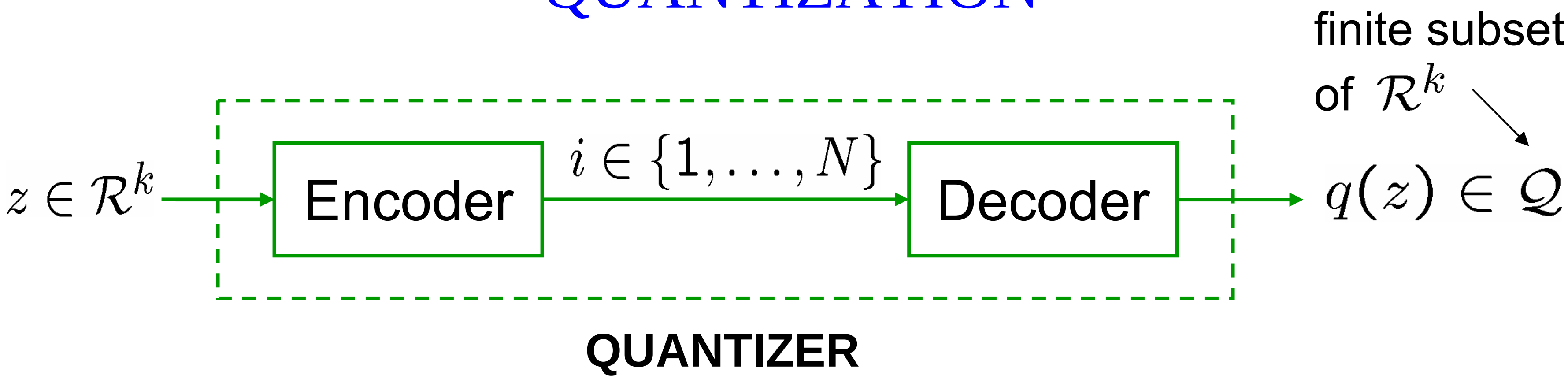
Caveat:

This doesn't work in general, need robustness from controller

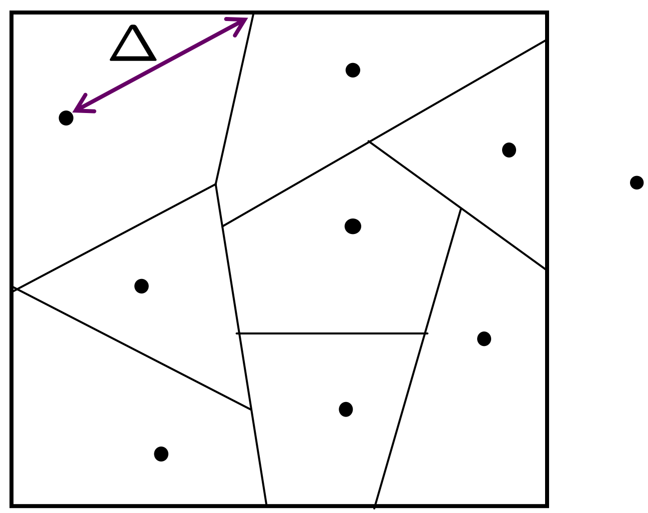
Technical tools:

- Input-to-state stability (ISS)
- Lyapunov functions
- Small-gain theorems
- Hybrid systems

QUANTIZATION



$\mathcal{R}^k$ is partitioned into quantization regions



QUANTIZATION and INPUT-to-STATE STABILITY

$$\dot{x} = f(x, u)$$

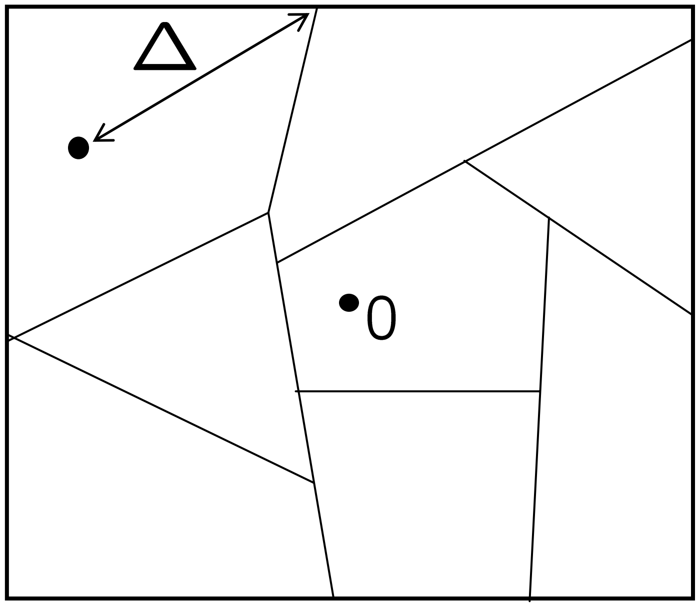
QUANTIZATION and INPUT-to-STATE STABILITY

$\dot{x} = f(x, k(x))$ – assume glob. asymp. stable (GAS)

QUANTIZATION and INPUT-to-STATE STABILITY

$$\dot{x} = f(x, k(q(x)))$$

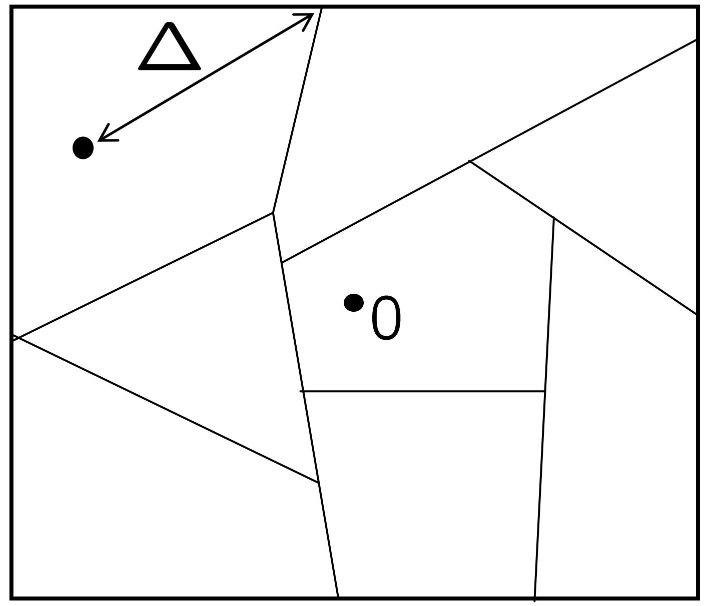
no longer GAS



QUANTIZATION and INPUT-to-STATE STABILITY

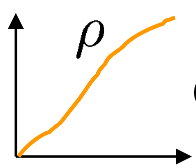
$$\dot{x} = f(x, k(q(x)))$$
$$= f(x, k(x + e))$$

quantization error



Assume $\exists V$:

$$|x| \geq \rho(|e|)$$
$$\Downarrow$$
$$\frac{\partial V}{\partial x} f(x, k(x+e)) < 0$$



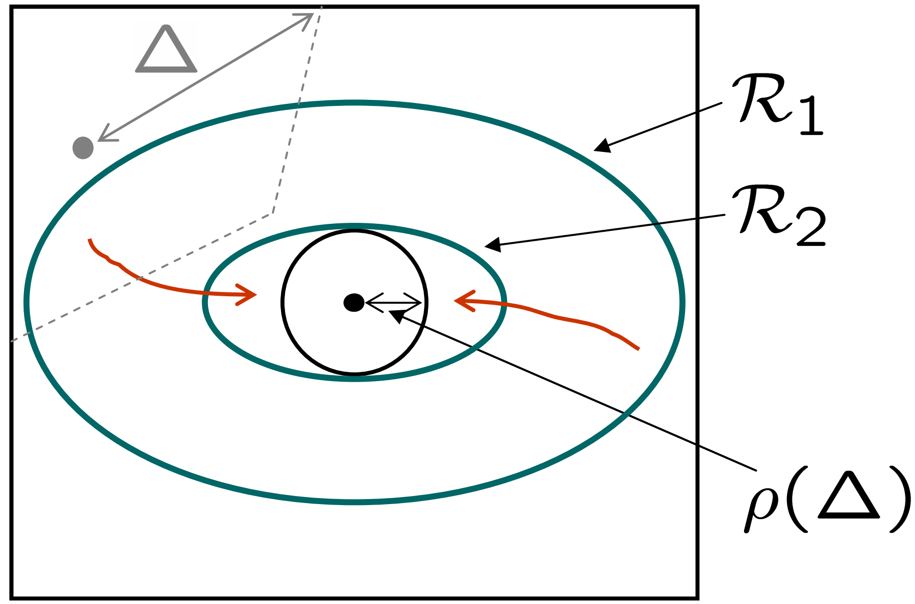
class $\mathcal{K}_\infty$

QUANTIZATION and INPUT-to-STATE STABILITY

$$\dot{x} = f(x, k(q(x)))$$
$$= f(x, k(x + \underbrace{e}_{\text{quantization error}}))$$

Assume $\exists V$:

$$\boxed{|x| \geq \rho(|e|) \;\Downarrow\; \frac{\partial V}{\partial x} f(x, k(x+e)) < 0}$$



Solutions that start in $\mathcal{R}_1$ enter $\mathcal{R}_2$ and remain there

This is input-to-state stability (ISS) w.r.t. measurement errors

In time domain: $|x(t)| \leq \beta(|x_0|, t) + \gamma\left(\|e\|_{[0,t]}\right)$ [Sontag '89]

class $\mathcal{KL}$, e.g. $ce^{-\lambda t}|x_0|$ class $\mathcal{K}_\infty$

LINEAR SYSTEMS

$$\dot{x} = Ax + Bu$$

$\exists$ feedback gain K & Lyapunov function $V = x^T P x$:

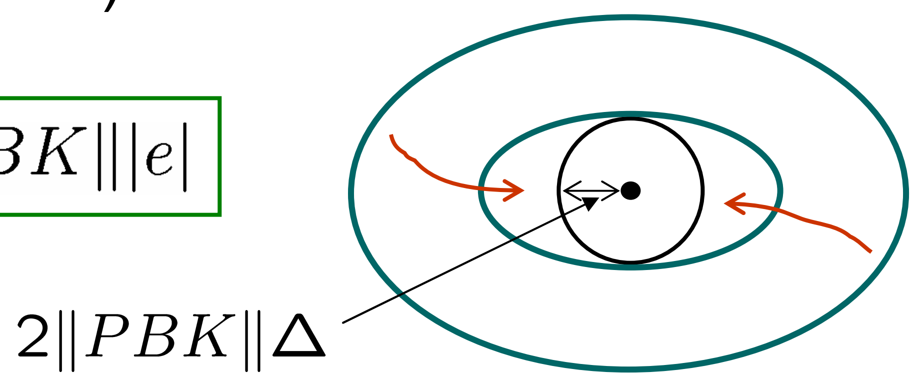
$$(A+BK)^T P + P(A+BK) = -I$$

Quantized control law: $u = Kq(x) = K(x+e)$

Closed-loop: $\dot{x} = (A+BK)x + BKe$

(automatically ISS w.r.t. e)

$\dot{V} < 0$ if $\boxed{|x| > 2\|PBK\||e|}$



DYNAMIC QUANTIZATION

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$q(x/\mu), \quad \mu$ – zooming variable

Hybrid quantized control: μ is discrete state

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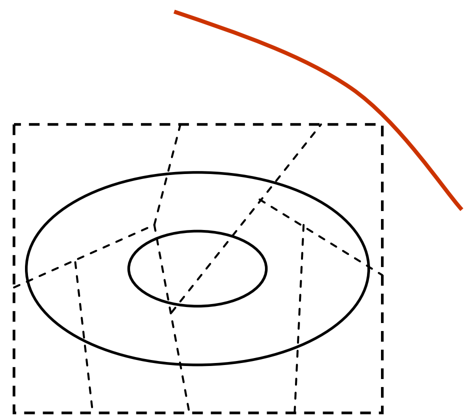
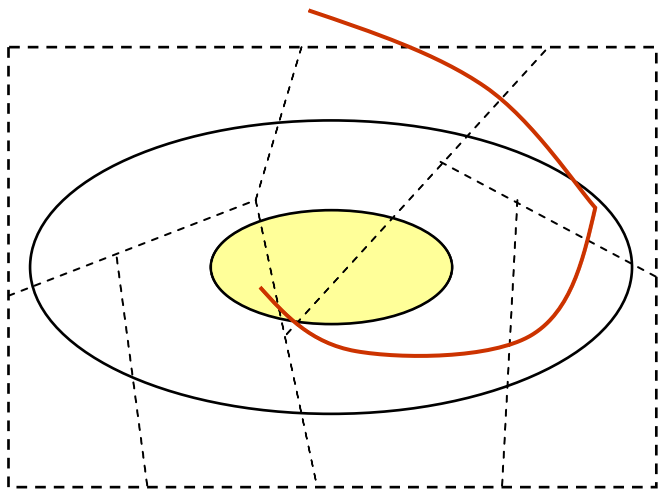
Hybrid quantized control: μ is discrete state



DYNAMIC QUANTIZATION

$q(x/\mu)$, μ – zooming variable

Hybrid quantized control: μ is discrete state

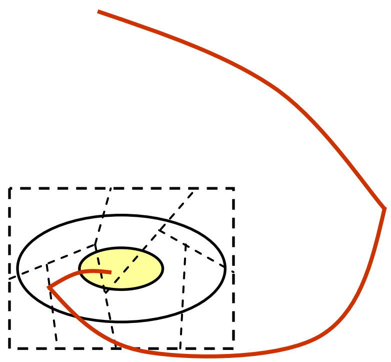


Zoom out to overcome saturation

DYNAMIC QUANTIZATION

$q(x/\mu)$, μ – zooming variable

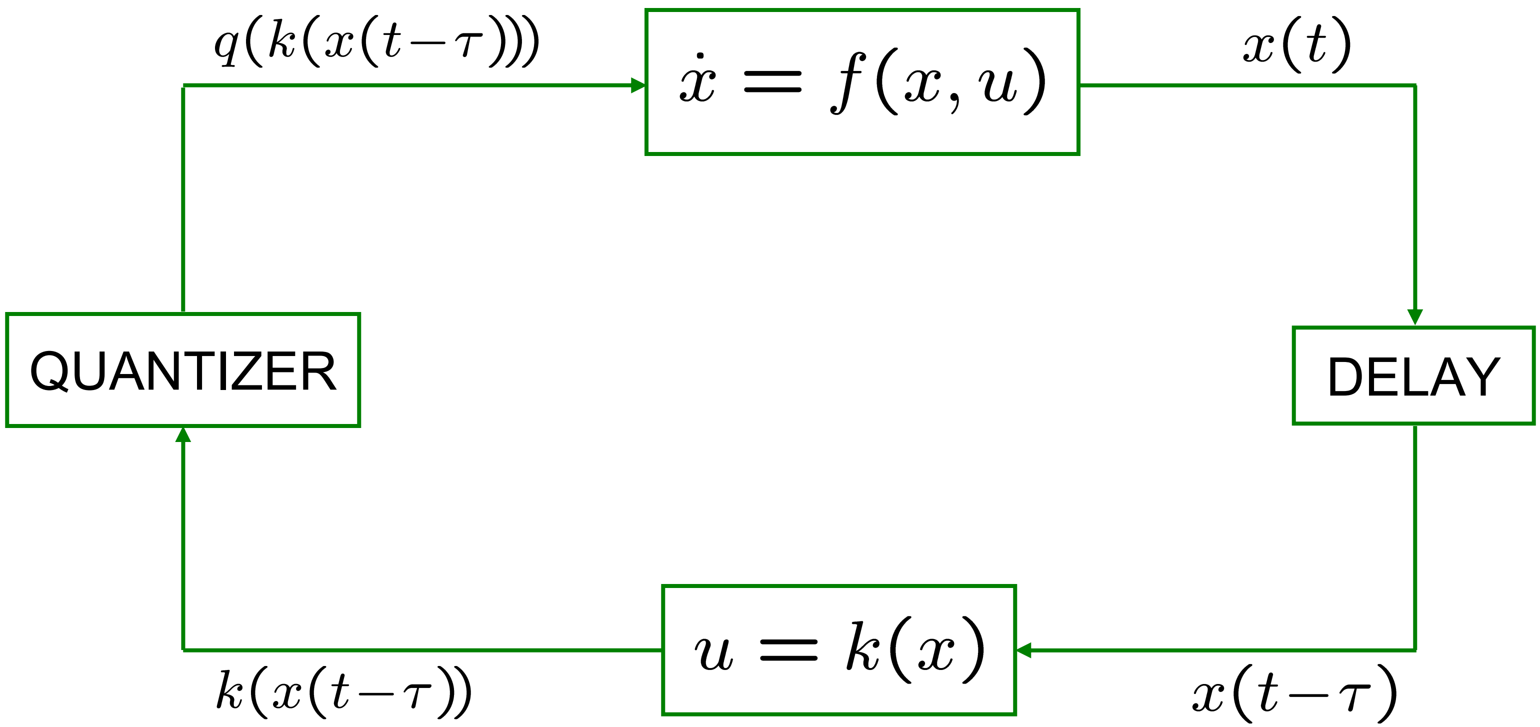
Hybrid quantized control: μ is discrete state



After the ultimate bound is achieved, recompute partition for smaller region

Can recover global asymptotic stability

QUANTIZATION and DELAY



Architecture-independent approach

Delays possibly large

Based on the work of Teel

QUANTIZATION and DELAY

$$\begin{aligned}\dot{x} &= f(x, q(k(x(t-\tau)))) \\ &= f(x, k(x) + \theta + e)\end{aligned}$$

where

$$\begin{aligned}\theta(t) &:= k(x(t-\tau)) - k(x(t)) \\ e(t) &:= q(k(x(t-\tau))) - k(x(t-\tau))\end{aligned}$$

Can write

$$\theta(t) = -\int_{t-\tau}^{t} \frac{d}{ds} k(x(s))\, ds$$

hence

$$|\theta(t)| \le \tau\gamma\Big(\|(x,e)\|_{[t-2\tau,\tau]}\Big)$$

SMALL – GAIN ARGUMENT

Assuming ISS w.r.t. actuator errors:

$$|x| \geq \rho(|\theta + e|) \;\Rightarrow\; \frac{\partial V}{\partial x} f(x, k(x) + \theta + e) < 0$$

In time domain:

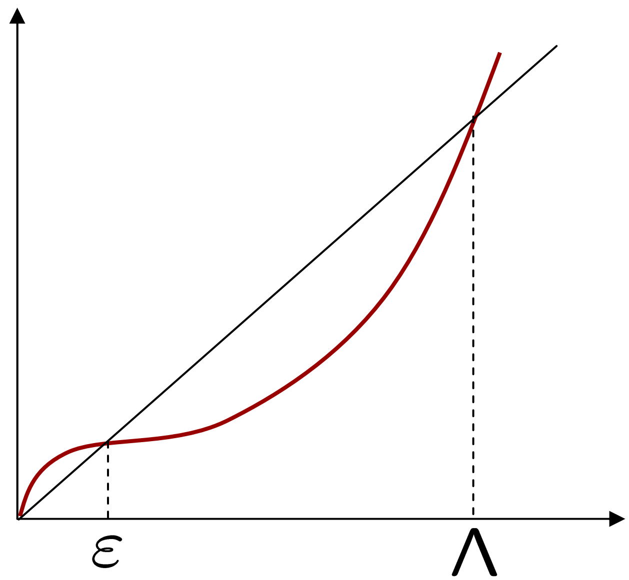
$$|x(t)| \leq \beta(|x_0|, t) + \gamma_\theta\Big(\|\theta\|_{[0,t]}\Big) + \gamma_e\Big(\|e\|_{[0,t]}\Big)$$

$$\leq \beta(|x_0|, t) + \gamma_1\Big(\tau\gamma_2\Big(\|x\|_{[t-2\tau,t]}\Big)\Big) + \gamma_3\Big(\|e\|_{[t-2\tau,t]}\Big)$$

Small gain: if $\boxed{\gamma_1(\tau\gamma_2(r)) < r}$ $\quad \forall\, r > 0$

then we recover ISS w.r.t. e [Teel '98]

FINAL RESULT

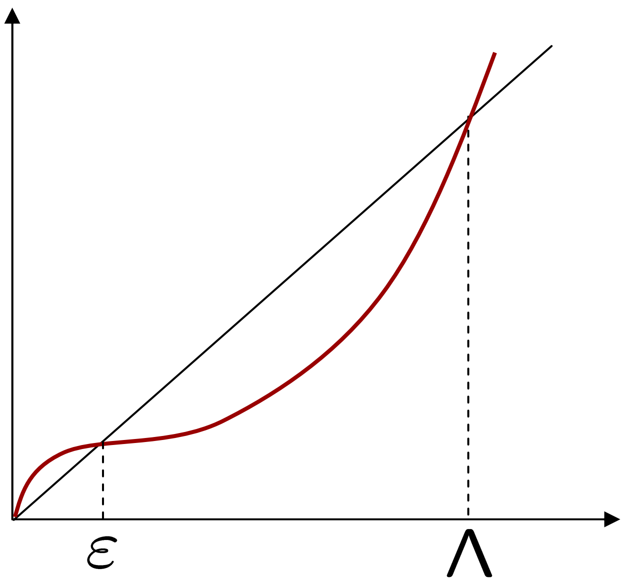


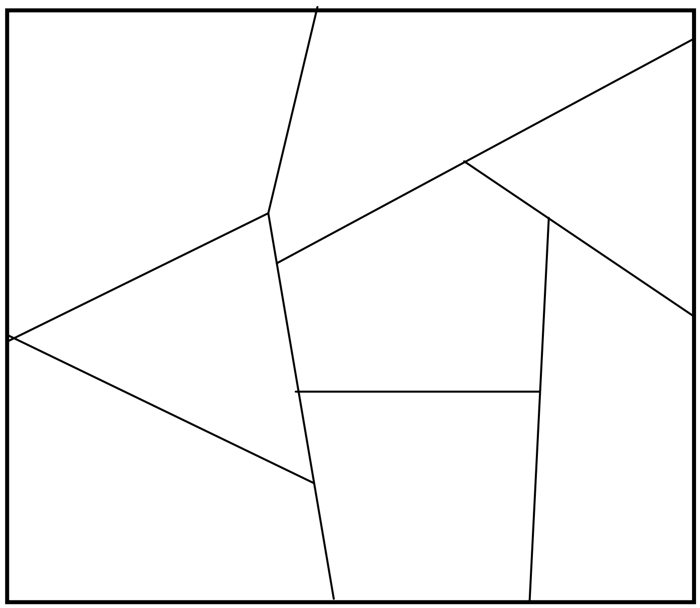
Need: $\gamma_1(\tau\gamma_2(r)) < r$

$\forall\, \Lambda > \varepsilon > 0 \quad \exists\, \tau^* > 0:$

small gain true $\forall\, \tau \leq \tau^*$

FINAL RESULT





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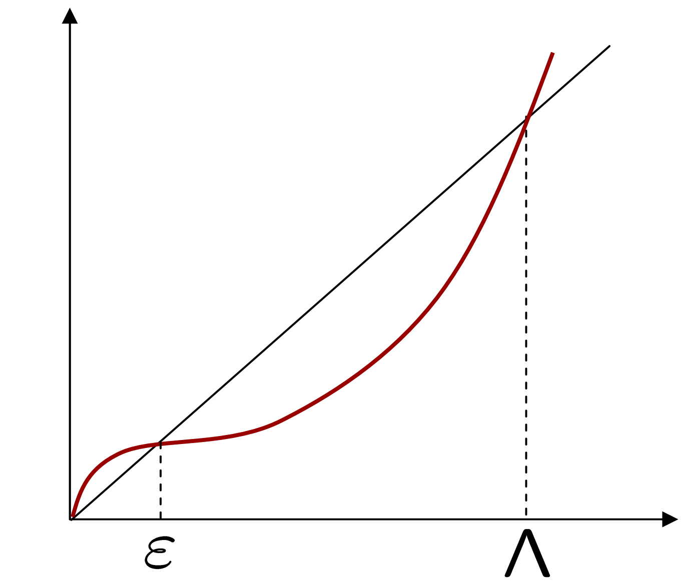

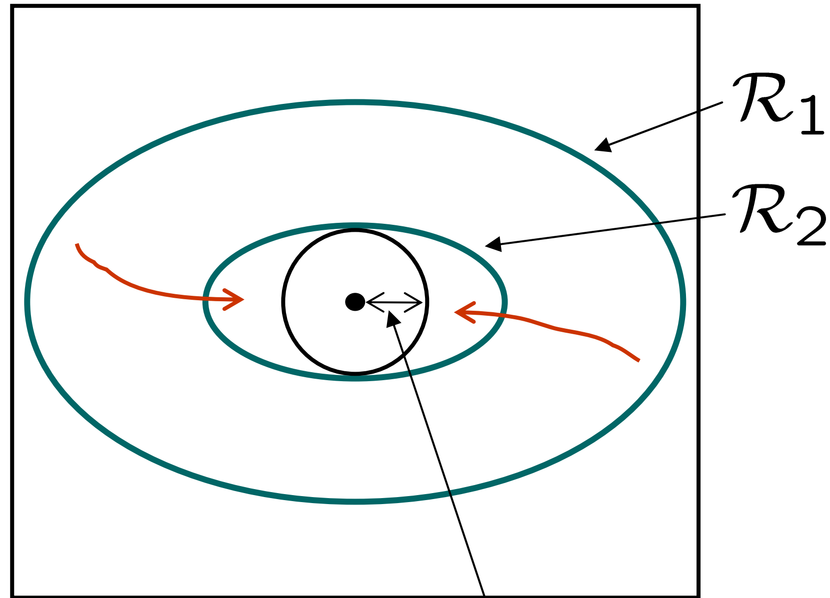

Need: $\gamma_1(\tau\gamma_2(r)) < r$

$\forall\, \Lambda > \varepsilon > 0 \;\; \exists\, \tau^* > 0:$
small gain true $\;\forall\, \tau \leq \tau^*$

$\tau \leq \tau^* \Rightarrow$ solutions starting in $\mathcal{R}_1$ enter $\mathcal{R}_2$ and remain there

Can use “zooming” to improve convergence

EXTERNAL DISTURBANCES [Nešić–L]

State quantization and completely unknown disturbance

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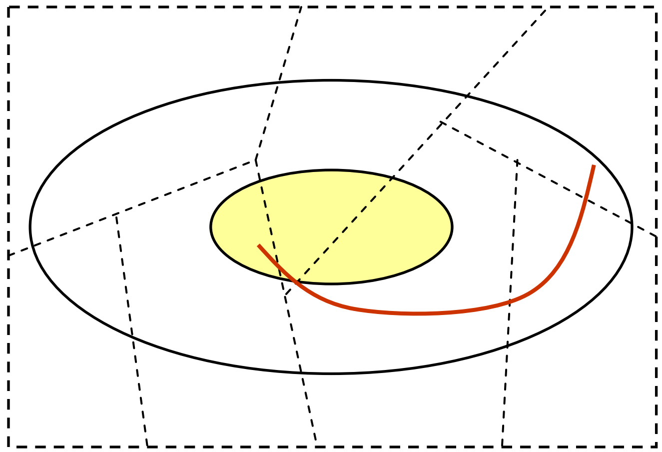
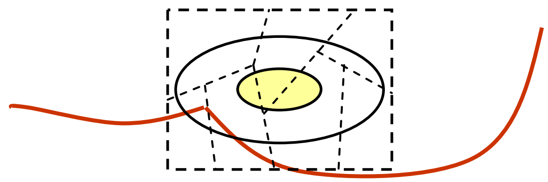
State quantization and completely unknown disturbance



EXTERNAL DISTURBANCES [Nešić–L]

State quantization and completely unknown disturbance

After zoom-in:



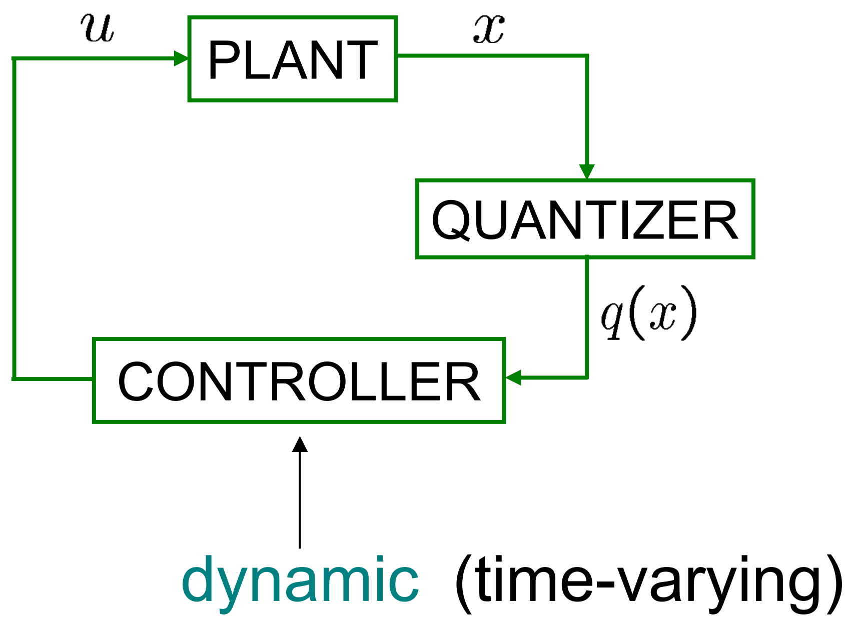
Issue: disturbance forces the state outside quantizer range

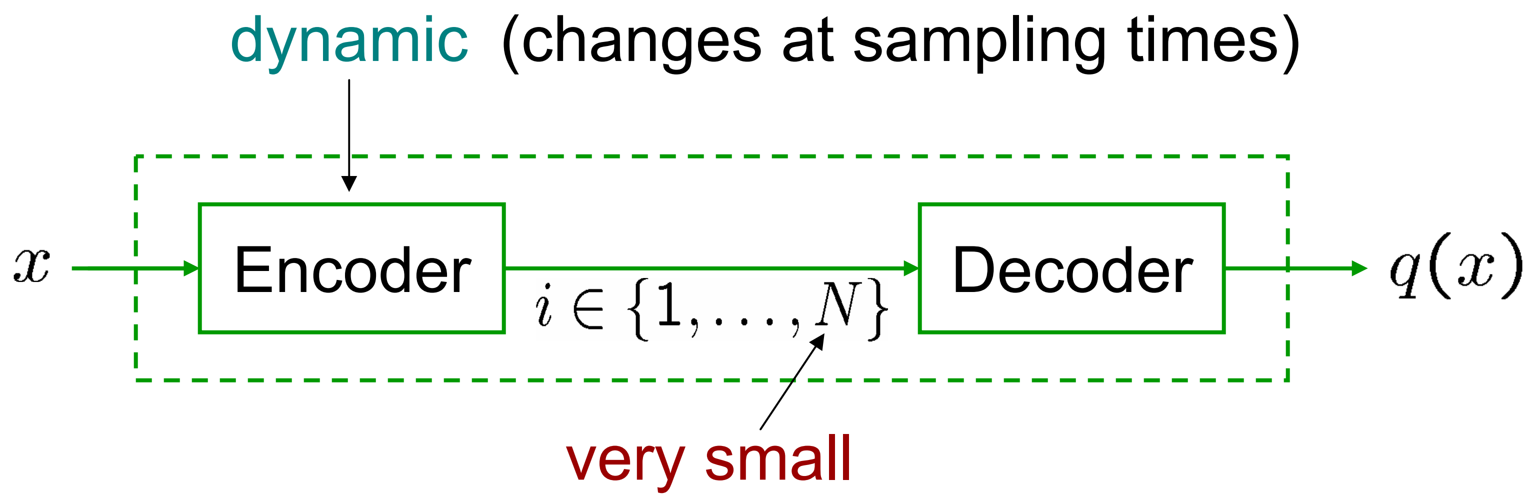
Must switch repeatedly between zooming-in and zooming-out

Result: for linear plant, can achieve ISS w.r.t. disturbance

(ISS gain is nonlinear although plant is linear; cf. [Martins])

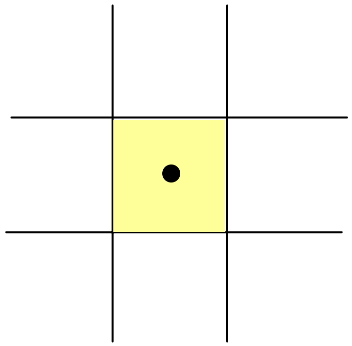
ACTIVE PROBING for INFORMATION





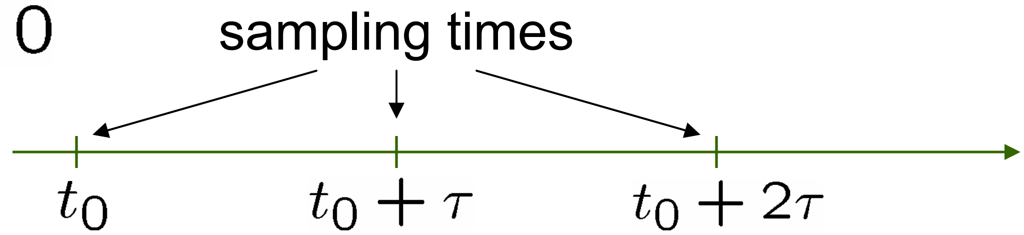
NONLINEAR SYSTEMS

Example: $\dot{x} = f(x, u),\ n = 2,\ N = 9 = 3^n$



Zoom out to get initial bound

$\hat{x}(t_0) := 0$



Between samplings $\dot{\hat{x}} = f(\hat{x}, u)$

NONLINEAR SYSTEMS

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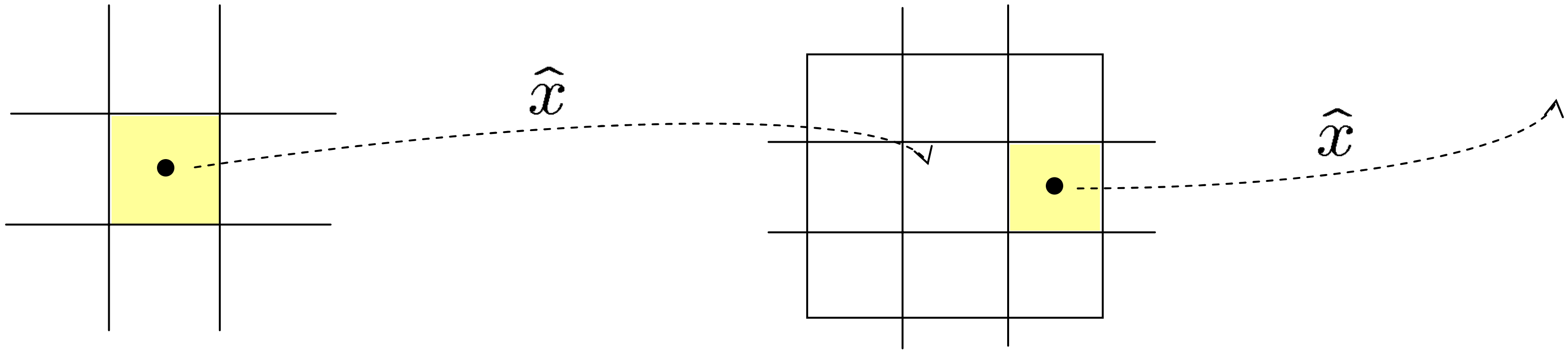

Between samplings $\quad \dot{\widehat{x}} = f(\widehat{x}, u)$

Let $e := \widehat{x} - x \qquad \dot{x} = f(x,u)$

$$\left.\begin{aligned}\dot{\widehat{x}} &= f(\widehat{x}, u)\\ \dot{x} &= f(x,u)\end{aligned}\right\} \Rightarrow \dot{e} = f(\widehat{x}, u) - f(x,u)$$

$\|f(\widehat{x}, u) - f(x,u)\|_\infty \leq L\|e\|_\infty$ on a suitable compact region (dependent on x_0)

The norm $\|e\|_\infty$:

- grows at most by the factor $\Lambda := e^{L\tau}$ in one period
- is divided by 3 at the sampling time

NONLINEAR SYSTEMS (continued)

$$e = \hat{x} - x$$

The norm $\|e\|_\infty$:

- grows at most by the factor $\Lambda := e^{L\tau}$ in one period
- is divided by 3 at each sampling time

Pick τ small enough s.t. $\Lambda < 3 \Rightarrow e \to 0$

$$u(t) = k(\hat{x}(t))$$

$$\dot{x} = f(x, k(\hat{x})) = f(x, k(x + e))$$

If this is ISS w.r.t. e as before, then $x \to 0$

ROBUSTNESS of the CONTROLLER

Option 1. $\dot{x} = f(x, k(x + e))$

ISS w.r.t. $e \Rightarrow x \to 0$

Same condition as before (restrictive, hard to check)

Option 2. Look at the evolution of $\widehat{x}$

$$\begin{cases} \dot{\widehat{x}} = f(\widehat{x}, k(\widehat{x})) & t \neq \text{sampl. time} \\ \widehat{x}(t) = \widehat{x}(t^-) + \Delta e(t), & t = \text{sampl. time} \end{cases}$$

ISS w.r.t. $\Delta e \Rightarrow \widehat{x} \to 0 \Rightarrow x \to 0$

$\exists$ checkable sufficient conditions ([Hespanha-L-Teel])

LINEAR SYSTEMS

$$\dot{x} = Ax + Bu$$

LINEAR SYSTEMS

Between sampling times, $\left.\begin{aligned}\dot{x} &= Ax + Bu \\ \dot{\hat{x}} &= A\hat{x} + Bu\end{aligned}\right\} \Rightarrow \dot{e} = Ae$

- $\|e\|_\infty$ grows at most by $\boxed{\Lambda := e^{\|A\|_\infty \tau}}$ in one period
- divided by 3 at each sampling time

global quantity: sampling frequency vs. open-loop instability

$$\Lambda < 3 \Rightarrow e \to 0$$

amount of static info provided by quantizer

$u(t) = K\hat{x}(t)$ where $A + BK$ is Hurwitz

$$\dot{x} = Ax + BK\hat{x} = (A + BK)x + BKe \Rightarrow x \to 0$$

0

[Baillieul, Brockett-L, Hespanha, Nair-Evans, Petersen-Savkin,Tatikonda]

RESEARCH DIRECTIONS

- Quantized output feedback
- Performance-based design
- Disturbances and coarse quantizers (with Y. Sharon)
- Modeling uncertainty (with L. Vu)
- Avoiding state estimation (with S. LaValle and J. Yu)
- Vision-based control (with Y. Ma and Y. Sharon)

http://decision.csl.uiuc.edu/~liberzon